Price Co-Movements and Investment Funds

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Abstract of the Dissertation

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This dissertation consists of two chapters, at the first chapter we analyze the rational expectation equilibria of a delegated portfolio model in which two risky assets have completely independent returns and liquidity shocks. The investment decision is delegated to risk neutral managers with reputational concerns. Some managers have perfect information on the assets' returns while others are uninformed and try to infer information from the prices. We show that in equilibrium there are always realizations of the shocks such that the returns are not revealed. In this region, the prices of the two assets exhibit a strong form of co-movement, as they must be identical. This occurs despite the fact that the two assets have different *ex ante* probabilities of repayment.

In the second chapter, we discuss price co-movements between fundamentally independent financial markets populated by risk neutral global funds and specialized funds. Similar to the first chapter, the investment decisions are delegated to risk neutral fund managers who are informed or uninformed of the state of the markets and have reputational concerns. Different from chapter one, these managers can be hired by three types of funds, specialized in one asset market or global. We show that in any equilibrium of the model, prices of the risky assets co-move with each other following any shock to ex-ante probabilities of default. The mechanism that generates this co-movement relies on two sources: the information asymmetry between fund managers and the reputational concerns of uninformed fund managers facing the threat of dismissal. The reputational channel reinforces the co-movement but it is not necessary to generate it. Information asymmetry induces co-movement even in the absence of reputational concerns.

To my parents.

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Chapter 1

Price Co-Movements and Reputational Concerns

1.1 Introduction

The growth of institutional trade in financial markets during the last decades has been widely documented. Since the management of mutual funds' portfolios is delegated to professionals, who typically have different incentives from investors, a large literature has studied the implications of the growth of institutional trade for the evolution of asset prices.

One phenomenon that has been observed is the tendency to comove for the returns of assets owned by the same financial institution or group of financial institutions (see e.g. Barberis and Shleifer (2003), Barberis et al. (2005), Coval and Stafford (2007), and Anton and Polk (2014)). This tendency to comove has been explained in two different ways. The simplest way is a mechanical effect that occurs when intermediaries which are 'large' with respect to the market try to rebalance their portfolios, when-for instance-they are hit by a liquidity shock. If a fund is forced to liquidate a significant part of its portfolio because of withdrawals and it does not want to change the composition of the portofolio then all the assets in the portfolio will experience simultanously a downward pressure on prices. Another potential explanation has to do with information transmission in a rational expectations equilibrium, as in Kodres and Pritsker (2002). They develop a model in which risk-averse investors trade assets whose returns are correlated and face liquidity shocks which may also be correlated. In their model a liquidity shock hitting one security will transmit to other securities because investor cannot separately observe liquidity and return shocks. In Kodres and Pritsker (2002) when the liquidity shocks and the returns shocks are independent, there is no contagion. However, in this model we have price co-movement even when liquidity and return shocks are entirely independent.

Vayanos and P. (2013) also consider a model in which the interaction of liquidity shocks and returns shocks generates price comovement across different assets. Their model considers a continuous time framework in which investors try to infer the ability of the managers running the funds and withdraw money from funds that perform poorly, although poor performance may be due to factors other than managerial competence. Withdrawals puts downward pressure on all the assets in the portfolio, thus producing comovement. Notice that the main goal of Vayanos and P. (2013) is to produce a model of momentum and reversal and these phenomena appear even if their model had only one asset. Comovement is a by-product when there are multiple assets.[1].

In this paper we analyze a rational expectations equilibrium in which comovement appears as an equilibrium phenomenon. Our model has competitive funds, so no single fund can exert a pressure on prices, and uncorrelated return and liquidity shocks, so that in a rational expectation equilibrium *a là* Kodres and Pritsker (2002) prices would be independent. Furthermore, we don't have inflows and outflows of funds: securities only last one period and then they are liquidated. A main point of departure from Kodres and Pritsker (2002) is that we assume that fund managers cannot take negative positions in the assets: fund managers are given a fixed amount of money to invest and they can only buy securities with that money. Furthermore, we assume that all market participants are risk neutral.

Our starting point is the delegated portfolio management model of Guerrieri and Kondor (2012). Their model has a single risky asset and a riskless asset. The supply of the risky asset is stochastic and it is only observed *ex*

[1]Our work is related to the literature on contagion and on the effects of managerial reputation, although most of this literature does not consider comovement. Contagion models are discussed for example in Basak and Pavlova (2011), Ilyin (2006) and Chakravorti and Lall (2005). The asset pricing implications of managerial reputational concerns are discussed in Cuoco and Kaniel (2006) , Dasgupta and Prat (2006), Dasgupta and Prat (2008), and Vayanos (2004). Scharfstein and Stein (1990) and Calvo and Mendoza (2000) are examples of papers linking contagion to herding Wagner (2012a) discusses contagion due to benchmark performance schemes.

post. Investors and managers are risk neutral and managerial compensation is exogenously given (it is a given fraction γ of the gross return). There are two types of fund managers, informed and uninformed; the types are private information and not observable by the investors. Informed fund managers know whether the risky assets are repaying or defaulting (they receive a perfect signal) while uninfomed fund managers don't have any information. Every investor has only one manager working for him at each period. At the end of the period, the investor observes the investment performance and updates beliefs about the manager's type. She then decides whether to fire or retain the manager.

Guerrieri and Kondor (2012) analyze a rational expectations equilibrium in which prices depend on both the signal of the informed managers and the realization of the random supply (which plays the role of a liquidity shock). The information about the value of the risky asset is revealed when the liquidity shock is sufficiently high or sufficiently low, while for intermediate values of the liquidity shock the price is non-revealing.

In this environment we introduce a second risky asset. Both the return and the liquidity shock on the second asset are independent of the first. In a rational expectations equilibrium it is the case that non-revelation occurs for some realizations of the returns and of the liquidity shock. Our main result is that in the non-revelation region we have an extreme form of comovement: risky assets end up having the same price and same expected return.

The intuition for the result goes as follows. A fully revealing equilibrium is impossible because of the presence of liquidity shocks which are not observed by any market participant and the impossibility of short-selling, which puts limits to arbitrage and prevents the informed managers from fully exploiting their information. Thus, in equilibrium there must be a region of realizations of returns and liquidity shocks such that the prices of the two assets do not reveal the true value. Suppose that in this region the prices of the two assets differ. When both assets repay, informed managers are going to demand exclusively the asset with the lowest price. This generates an adverse selection problem for the uninformed managers. They would get the same expected return on any of the two risky assets if such assets were actually randomly distributed, but this is not the case because whenever both assets repay the uninformed must be more likely to receive the highest priced asset. It is only when both prices are identical that this adverse selection phenomenon does not occur, as informed managers are indifferent between the two assets when they both repay.

The rest of the paper is organized as follows. In the next section we describe the model. In section 1.3 we define and characterize the equilibrium for the static case. We then embed the static equilibrium in a stationary equilibrium of a dynamic model in section 1.4. Section 1.5 contains concluding remarks and an appendix contains the proofs.

1.2 The Model

In each period each investor has a unit of funds to invest. Investors must hire fund managers in order to buy assets or have access to a risk-free technology. Both investors and managers are risk neutral and they discount the future at rate $\beta \in (0,1)$. There is a continuum of investors and managers and the measure of investors is N. Fund managers are infinitely lived and, whenever they are hired, at the beginning of each period they decide how to use the unit of capital provided by the investors. They can buy asset 0, with a safe return of R, or buy a risky asset i, with $i = 1, 2$. The return on risky asset i at time t is determined by the realization of a random variable $\widetilde{\chi}_{i,t}$ which takes values in the set $\{0,1\}$. The realization of $\widetilde{\chi}_t = (\widetilde{\chi}_{1,t}, \widetilde{\chi}_{2,t})$ is denoted $\chi_t = (\chi_{1,t}, \chi_{2,t})$. If $\chi_{i,t} = 0$ then the asset repays an amount of 1, while if $\chi_{i,t} = 1$ the asset defaults and pays zero. The random variables $\{\widetilde{\chi}_{i,t}\}_{t=0}^{\infty}$ are all independent and identically distributed, with $\Pr(\widetilde{\chi}_{i,t} = 1) = q_i$ and $q_2 > q_1$. Furthermore, each $\widetilde{\chi}_{i,t}$ is independent of all variables $\{\widetilde{\chi}_{j,\tau}\}_{\tau=0}^{\infty}$ with $j \neq i$.

Fund managers can be either informed or uninformed. Informed managers observe the realization of the random vector $\widetilde{\chi}_t$ at the beginning of period t, before trading takes place. Uninformed managers receive no information.

The supply of each risky asset is modeled as in Guerrieri and Kondor (2012). The supply of asset i at time t is determined by the random variable $\widetilde{b}_{i,t}$, with realization denoted $b_{i,t}$. When the realization is $b_{i,t}$ this means that there is a mass $b_{i,t}$ of agents who wants to finance one unit of consumption and have a technology that can produce unlimited units of risky asset i. The number of units produced by each agent is enough to finance one unit of consumption, so that the aggregate amount produced is $b_{i,t}/p_{i,t}$, as long as $p_{i,t} > 0$. If $p_{i,t} = 0$ then the supply is zero. The random variables $\widetilde{b}_{i,t}$ are independently and identically distributed, have a uniform distribution with support $[\underline{b}, \overline{b}]$ and are independent from all variables $\widetilde{\chi}_{j,\tau}$ for each j and τ. We denote as $\widetilde{b}_t = \left(\widetilde{b}_{1,t}, \widetilde{b}_{2,t}\right)$ the random vector determining the supply for the assets and with $b_t = (b_{1,t}, b_{2,t})$ its realization. The vector b_t is unobservable by fund managers and investors.

Again following Guerrieri and Kondor (2012), the sequence of events at each period t can be described separating what happens 'in the morning' and 'in the afternoon'.

In the 'morning' the labor market is cleared and investment decisions are made. More precisely:

- unemployed managers decide whether or not to search for a job. In order to search for a job an unemployed manager has to pay a cost κ;
- funds without a manager go to the labor market and randomly hire a manager among those who are searching.
- informed managers observe the realization of $\widetilde{\chi}_t$, while uninformed managers do not receive any information.
- both informed and uninformed managers who are employed submit vector demand schedules for the assets and the bond;
- given the realization of b_t the price vector $p_t = (p_{1,t}, p_{2,t})$ is determined to balance demand and supply in each market;
- given the prices, the assets are assigned to each fund manager according to their demand schedules.

In the 'afternoon' the realization χ_t is revealed and the investments of the managers are realized by their investors. At that point:

- managers receive a share γ of the returns;
- each investor receives an exogenous binary signal, σ_t^y about the type of the hired manager. If manager y is informed, σ_t^y is always zero, while if the manager is uninformed, $\sigma_t^y = 0$ with probability ω and $\sigma_t^y = 1$ with probability $1 - \omega$, with $\omega \in (0, 1)$;
- investors decide about firing or retaining their manager. There is also a probability $1 - \delta$, with $\delta \in (0, 1)$ that any given manager is exogenously separated from the job.

A general equilibrium of the model results from the interaction of the labor market and the asset markets. Decisions made in the labor market determine the measure of informed managers present in the asset markets. In turn, this determines how much information the equilibrium price function $p_t^e(b, \chi)$ reveals and therefore how profitable it is for the investor to have an informed, rather than an uninformed, manager. In turn this determines the optimal firing rule.

The choice variables in the labor market are the firing rule ϕ_t adopted by the investor and the search decision for unemployed managers. The choice variables in the asset markets are the demand functions submitted by the managers. The market mechanism generates matching between investors and managers and equilibrium price functions in the asset markets. The interaction of these forces determines at each time t the measure N_t^I of informed managers and the value W_t^U of being employed for an uninformed manager. In the following we will focus on stationary equilibria, i.e. situations in which $\phi_t = \phi$, $p_t^e = p^e$, $N_t^I = N^I$ and $W_t^U = W^U$ for each t. The existence of such an equilibrium requires some restrictions on the parameters, that we will discuss subsequently.

We now describe more in detail how the markets work.

1.2.1 Asset Markets

Employed fund managers are given one unit of funds to invest and submit a demand schedule, specifying for each price vector $p = (p_1, p_2)$ which assets they are willing to buy. To keep the notation similar to the one used by Guerrieri and Kondor (2012), we assume that the demand expressed by a fund manager at a given price vector is given by an element of the set

$$\Delta = \{(d_0, d_1, d_2) \,|d_i \in \{0, 1\}, \text{ for each } i = 0, 1, 2 \;\}$$

A vector $d = (d_0, d_1, d_2) \in \Delta$ is interpreted as stating the willingness (or lack of it) to buy a given asset at a certain price vector; $d_i = 1$ means that the manager is willing to buy asset i and $d_i = 0$ means that the manager is not willing to buy it (the subscript zero refers to the risk-free asset). For example, a vector $(0, 0, 1)$ indicates that the manager is willing to use the unit of funds available to buy risky asset 2 (up to an amount $1/p_2$) and nothing else. When a vector d has multiple elements equal to one then the manager is stating that she is equally happy with those assets. Thus, $d = (0, 1, 1)$

means that the manager is willing to buy either asset 1 up to an amount $1/p_1$ or asset 2 up to an amount $1/p_2$.

Let $\mathcal{D}$ be the set of functions from $\mathbb{R}^2_+$ into Δ. Then:

- an uninformed manager y chooses an element $d^y(p_1, p_2) \in \mathcal{D}$ and submits it to the auctioneer;

- an informed manager y observes the realization χ of the random vector $\widetilde{\chi}$ and submits a demand schedule $d^y(p_1, p_2 | \chi) \in \mathcal{D}$ to the auctioneer.

The aggregate demand vector will in general depend on the fraction of managers who are informed. In turn, the equilibrium price function will also depend on such fraction.

Let N^I and N^U be, respectively the mass of informed and uninformed managers employed in the period, with $N^I + N^U = N$. In equilibrium the price vector depends on the realizations of the random vectors $\widetilde{\chi}$ and $\widetilde{b}$. It can therefore be described as a price function $p^e : \left[\underline{b}, \overline{b}\right]^2 \times \{0,1\}^2 \to \left[0, \frac{1}{R}\right]^2$. Thus, in general, the realized price vector conveys information for the uninformed managers. Let $\phi(i, p^*, \chi, \sigma^y)$ be the probability that manager y is fired at the end of the period if asset i is bought when the price vector is p^*, the realization of the random vector is χ and the realization of the exogenous signal on competence is σ^y. Finally, let W^U be the value of being employed. Notice that $\delta(1 - \phi(i, p^*, \chi, \sigma^y))$ is the probability of being retained for a manager buying asset i. Define the expected utility of an uninformed manager who buys asset i when the price vector is p^* as

$$v^U(i, p^*) =$$

$$E\left[\gamma \frac{1 - \widetilde{\chi}_i}{p_i} + \beta\delta(1 - \phi(i, p^*, \widetilde{\chi}, \sigma^q)) W^U \middle| p^e\left(\widetilde{b}, \widetilde{\chi}\right) = p^*\right] \tag{1.1}$$

and notice that the function v^U depends on the equilibrium price function $p^e(b, \chi)$, the firing rule ϕ and the future utility W^U. Define

$$D_i^U(p) = \int_{y \in N^U} d_i^y(p)\, dy$$

and

$$D_i^I(p, \chi) = \int_{y \in N^I} d_i^y(p | \chi)\, dy.$$

The aggregate demand for asset i at price vector p and vector χ is given by

$$D_i(p,\chi) = D_i^U(p) + D_i^I(p,\chi).$$

The excess demand vector at price vector p and vector (b,χ) is given by

$$E(p,b,\chi) = \begin{bmatrix} D_1(p,\chi) \\ D_2(p,\chi) \end{bmatrix} - \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}.$$

The last definition refers to the set of players who demand a single asset. For a given (p,χ) let

$$\Xi_i(p,\chi) = \left\{ y |\, d_i^y = 1 \text{ and } d_j^y = 0 \text{ if } j \neq i \right\}$$

be the set of players who demand only asset i when the price vector is p and the realization of returns is χ, and

$$\xi_i(p,\chi) = \int_{\Xi_i(p,\chi)} d_i^y dy \qquad i = 1,2$$

as the mass of managers who demand exclusively asset i.

Let $D = (D_1, D_2) \in \mathbb{R}_+^2$ be a vector of demands and $\xi = (\xi_1, \xi_2)$ be a vector of 'exclusive' demands. A **feasible allocation rule** is a function $x(d^y, D, \xi, b) : \Delta \times \mathbb{R}_+^2 \times \mathbb{R}_+^2 \times \left[\underline{b}, \overline{b}\right]^2 \to [0,1]^3$ such that

$$\sum_{i=0}^{2} x_i(d^y, D, \xi, b)\, d_i^y = 1$$

for each $(d^y, D, \xi, b) \in \Delta \times \mathbb{R}_+^2 \times \mathbb{R}_+^2 \times \left[\underline{b}, \overline{b}\right]^2$ such that $d^y \neq (0,0,0)$. Thus, x_i can be interpreted as the probability of receiving asset i when the individual demand vector is d^y, the demand vector for the risky assets is D, the masses of managers demanding exclusively assets 1 and 2 are $\xi = (\xi_1, \xi_2)$ and the supply vector is b.

At this point we are ready to establish our equilibrium notion for the asset markets. In the following we use the convention that $\chi_0 = 0$ with probability 1 and $p_0 = \frac{1}{R}$.

Definition 1. *Take as given the collection* $\left(N^I, \phi, W^U\right)$. *A* ***rational expectations equilibrium*** *is a price function* $p^e : \left[\underline{b}, \overline{b}\right]^2 \times \{0,1\}^2 \to \left[0, \frac{1}{R}\right]^2$, *a feasible allocation rule* $x(d^y, D, \xi, b) : \Delta \times \mathbb{R}_+^2 \times \mathbb{R}_+^2 \times \left[\underline{b}, \overline{b}\right]^2 \to [0,1]^3$ *and demand functions* $d^y(p|\,\chi)$ *for each* $y \in N^I$ *and* $d^y(p)$ *for each for each* $y \in N^U$ *such that:*

1. *For each realization* (b,χ) *the price vector* $p(b,\chi) = p^*$ *is such that all markets clear, i.e. for each asset* i

$$\int_{y\in N^I} x_i\left(\widehat{d^y},\widehat{D},\widehat{\xi},b\right)\widehat{d^y_i}dy + \int_{y\in N^U} x_i\left(\widehat{d^y},\widehat{D},\widehat{\xi},b\right)\widehat{d^y_i}dy - b_i \le 0$$

$$p_i^*\left(\int_{y\in N_t^I} x_i\left(\widehat{d^y},\widehat{D},\widehat{\xi},b\right)\widehat{d^y_i}dy + \int_{y\in N_t^U} x_i\left(\widehat{d^y},\widehat{D},\widehat{\xi},b\right)\widehat{d^y_i}dy - b_i\right) = 0$$

where $\widehat{d^y} = d^y(p^*|\chi)$ *when* $y \in N^I$, $\widehat{d^y} = d^y(p^*)$ *when* $y \in N^U$, $\widehat{D} = D(p^*,\chi)$ *and* $\widehat{\xi} = \xi(p^*,\chi)$.

2. *The demand functions of informed traders maximize their expected utility at each price vector* p^* *and* χ, *i.e. if* $d_i^y(p^*|\chi) = 1$ *then* $\frac{1-\chi_i}{p_i^*} \ge \frac{1-\chi_j}{p_j^*}$ *each* $j \ne i$.

3. *The demand functions of uninformed traders maximize their expected utility at each price vector* p^*, *i.e. if* $d_i^y(p^*) = 1$ *then* $v^U(i,p^*) \ge v^U(j,p^*)$ *each* $j \ne i$, *where* $v^U(i,p^*)$ *is given by expression (1.1) using the equilibrium price function* $p^e(b,\chi)$, *the firing rule* ϕ *and the continuation value* W^U.

As it is clear from the definition, a rational expectations equilibrium in the assets market at time t is a static concept and it is defined for a given collection $\left(N^I,\phi,W^U\right)$. In the full dynamic analysis all these objects are endogenously determined, making sure that the labor market is in equilibrium and the firing rule is optimal.

In this paper we will focus on rational expectations equilibria that satisfy some restrictions. The most important is the one described in the following definition.

Definition 2. *A rational expectations price function* $p^e : \left[\underline{b},\overline{b}\right]^2 \times \{0,1\}^2 \to \left[0,\frac{1}{R}\right]^2$ *is* ***compatible with excess demand schedules*** *if the following condition is satisfied. Let* $p^* = p^e(b,\chi)$ *be the equilibrium price vector at* (b,χ). *Suppose that at a pair* (b',χ') *we have*

$$E(p^*,b,\chi) = E(p^*,b',\chi'), \tag{1.2}$$

i.e. the excess demand vector at p^* *is the same at the two pairs* (b,χ) *and* (b',χ'). *Then it has to be the case that* $p^e(b,\chi) = p^e(b',\chi')$.

The logic of the restriction is as follows. In principle, if we know the whole aggregate demand schedule (i.e. the aggregate demand at each possible price) it is possible to infer the realization of χ, since such realization determines the shape of the demand for informed managers. In fact, we do not really need to know the whole demand schedule, as χ can be inferred by looking at 2 points. For example, consider the price vector $\widehat{p} = \left(\frac{1}{R}, \frac{1}{2R}\right)$. Let $D_2^U(\widehat{p})$ be the aggregate demand for asset 2 by uninformed managers at the price vector $\widehat{p}$. Then, if the total demand for asset 2 at the price vector $\widehat{p}$ is $D_2^U(\widehat{p}) + N^I$ it can be inferred that $\chi_2 = 0$, while if the total demand is $D_2^U(\widehat{p})$ it can be inferred that $\chi_2 = 1$. A similar price vector can be used to find out the realization of χ_1.

This seems to disclose too much information. We would like the choice of the equilibrium price vector to be based only on the observed excess demand at the equilibrium price vector, rather than on some sophisticated procedure for extracting information even at price vectors that are never observed in equilibrium. Condition (1.2) is a way to ensure that. It requires that when p^* is an equilibrium price vector generating a certain excess demand under a given supply vector b and a given realization χ, then it should remain an equilibrium price vector whenever the price vector generates the same excess demand at a supply vector b' and realization χ'. If we were to allow something different, that would imply that the auctioneer can select the equilibrium price vector using information other than the excess demand at that price vector[2].

To better understand the restrictions imposed by condition (1.2), notice that the condition is equivalent to having p^* being an equilibrium price vector whenever (b, χ) and (b', χ') are such that

$$b - b' = D^I(p^*, \chi) - D^I(p^*, \chi'). \tag{1.3}$$

Consider two vectors b and b' such that $b'_1 = b_1 - N^I$ and $b'_2 = b_2$ and two vectors $\chi = (0, 1)$ and $\chi' = (1, 1)$. At any price vector the demand of the informed managers will be zero for each risky asset whenever χ' is observed, while it will be N^I for asset 1 and zero for asset 2 whenever χ is observed.

[2]It is worth observing that at an equilibrium price vector the excess demand is not necessarily zero. Remember that the convention is that demands are expressed by signalling all the assets that the agent is willing to buy. Equilibrium requires that there is a way of allocating the assets, through the feasible allocation function x, such that the conditions in Definition 1 are satisfied.

Now notice that in our example we have

$$b_1 - b_1' = D_1^I(p^*, \chi) - D_1^I(p^*, \chi') = N^I$$

and

$$b_2 - b_2' = D_2^I(p^*, \chi) - D_2^I(p^*, \chi') = 0.$$

Thus, condition (1.3) is satisfied. We should therefore have $p^e(b', \chi') = p^e(b, \chi) = p^*$, implying that observation of p^* does not fully reveal the vector χ.

In the rest of the paper we will focus on rational expectations equilibria which are compatible with excess demand schedules. Clearly, if such an equilibrium exists it cannot be fully revealing. Thus, in general, it will be valuable to employ informed rather than uninformed managers.

When the return of asset i is fully revealed by the price function then the price must be either 0 or $\frac{1}{R}$. However, non-revelation can come in many different ways. We will explore a class of equilibria similar to the one analyzed in Guerrieri and Kondor (2012) for the case of one risky asset, namely equilibria in which whenever there is no full revelation of the value of any risky asset then the price function always takes the same value. We call *simple* this class of equilibria.

Definition 3. *A rational expectations price function* $p^e : \left[\underline{b}, \overline{b}\right]^2 \times \{0,1\}^2 \to \left[0, \frac{1}{R}\right]^2$ *is* ***simple*** *if there is at most one pair* (p_1, p_2) *with* $p_i \in \left(0, \frac{1}{R}\right)$ $i = 1, 2$ *such that* $p^e(b, \chi) = (p_1, p_2)$.

In a simple equilibrium there is only one pair of prices which is realized in equilibrium when there is no full revelation for both assets. Notice that we still allow for the possibility that only the value of one asset is revealed while the other is not.

1.2.2 Labor Market

At the beginning of each period unemployed managers decide whether or not to search for a job. Search is costly: in order to be in the market a manager has to pay a cost κ. Let Z_t be the measure of managers who decide to be on the market at the beginning of time t, i.e. the supply of managers at time t. The previous history of a manager is not observable, so there is no information on whether a given manager may be informed or uninformed.

Let Z_t^I denote the mass of informed managers who are on the market at time t, and Z_t^U for the uninformed managers, with $Z_t = Z_t^I + Z_t^U$.

On the demand side we have the investors who don't have a manager, either because the previous one was fired or because there was an exogenous separation. They need to hire a new manager, since this is needed to invest their money. Let A_t be the measure of investors looking for a manager at time t. Since there is no price (managerial compensation is fixed at a fraction γ of gross return) and demand and supply are inelastic, the matching follows the Leontie f rule, that is a measure $\min\{A_t, Z_t\}$ ends up being employed. Define

$$\mu_t = \frac{\min\{A_t, Z_t\}}{Z_t}$$

as the probability that a manager searching for a job ends up being employed.

We assume that informed and uninformed managers are indistinguishable, so the probability of hiring an informed manager is $\epsilon_t = Z_t^I / \left(Z_t^I + Z_t^U\right)$. The value ϵ_t is important because it influences the firing decision of the investors. For a given probability assigned to the fact that the current manager is uninformed, whether or not it is optimal to fire the current manager depends on the probability of hiring an informed manager when going to the labor market.

After hiring has occurred, trade takes place. At the end of the period each investor observes the assets assigned to the fund manager and the realization χ_t of the vector of returns. At that point investors have to decide whether to retain or fire the manager. Typically, they will want to fire managers who are believed to be uninformed and retain managers who are believed to be informed. The new information observed by the investors at time t is the price vector p_t, the investment actually made by the manager, the return on the investment χ_t and the signal σ_t^y for manager y. The information is used to update beliefs about the managers and decide about firing or retaining them. Furthermore, there is a probability $1-\delta$ that the manager is separated from the fund for exogenous reasons (for example, the manager may relocate for family reasons). The firing rule used at time t is summarized by the function $\phi_t(i, p_t, \chi_t, \sigma_t^y)$, giving the probability of firing a manager who invested in asset i when the price vector was p_t, the realized return vector was χ_t and the exogenous signal was σ_t^y.

1.2.3 Assumptions

In the rest of the paper we will maintain the following assumptions that will ensure the existence of a stationary equilibrium in the asset and labor markets.

Assumption 1. *Let M^I be the measure of informed managers, M^U the measure of uninformed managers and N the measure of investors. Then $M^I < \min\left\{\underline{b}, \left(\bar{b} - \underline{b}\right)\right\}$ and $M^U > N > 2\bar{b} + M^I$.*

The assumption says that there are relatively few informed manager and in particular it is never the case that a market can clear with a demand coming only from informed managers (since $M^I < \underline{b}$). Furthermore, the mass of money to be invested is large compared to the supply of risky assets, $N > 2\bar{b} + M^I$, so that there will always be investment in the riskless bond. This will simplify the equilibrium condition, as it implies that uninformed managers have to be indifferent between the riskless bond and any risky asset with a strictly positive price. Finally, $M^U > N$ makes sure (together with Assumption 3 below) that investors are always on the short side of the managerial labor market.

Assumption 2. $q_1\left(1 - \frac{M^I}{\Delta b}\right)^2 > \frac{\delta\omega\beta}{1+\delta\omega\beta}$

This assumption ensure that unrevealing prices are always less that $\frac{1}{R}$. It is satisfied when the probabilities of default are large enough. As in Guerrieri and Kondor (2012), risky assets may have an expected rate of return lower than the risk-free asset because uninformed managers see the risk-less asset as actually risky (it leads to firing if one of the two risky assets repays). Condition **??** makes sure that this effect is sufficiently counterbalanced by a high risk of default for the risky asset, so that the price of the risky asset remains inferior to $1/R$ in equilibrium.

Assumption 3. *$\kappa < \gamma R$.*

To see the role of this assumption, consider a situation in which the mass of investors trying to hire managers is greater than the mass of managers looking for a job, so that the probability of finding a job is $\mu = 1$. This situation cannot be an equilibrium because an uninformed manager could pay the search cost κ and, once hired, he would get at least γR (this can be obtained simply by investing in the riskless asset and then quitting). Since

$\gamma R > \kappa$ all uninformed managers would want to enter. But then, since $M^U > N$, the matching probability would have to be less than one. Thus, in equilibrium we must have $\mu < 1$, i.e. the mass of managers searching for a job has to be greater than the mass of funds trying to hire a manager. The presence of the exogenous probability of separation $(1 - \delta)$ makes sure that in equilibrium $\mu > 0$, since firms which have exogenously lost their manager will be on the market.

Assumption 4. *Let $1 - \omega$ be the probability that an uninformed manager is exogenously revealed as such. Then* $1 - \omega > \frac{\delta}{1+\delta}$.

The assumption ensures that the probability assigned to the fact that a manager is informed increases in a sufficiently rapid way when the manager makes choices that are optimal *ex post* and is not revealed uninformed. In equilibrium we want this type of manager to be retained, and this happens if the probability assigned to the fact that the manager is informed is higher than the probability assigned to the fact that a manager randomly picked from the unemployment pool is informed. When $1 - \omega$ is sufficiently large the probability that a manager is informed increases at a sufficiently rapid pace. For example, if $\omega = 0$, so that the signal is perfect, then the probability assigned to the fact that the manager is informed after an *ex post* optimal choice and a favorable signal would go to 1.

1.3 Rational Expectations Equilibria in Asset Markets

In this section we analyze the structure of rational expectations equilibria compatible with excess demand. We will take as given the quantities N^I and W^U and we will assume that the optimal firing rule ϕ is to fire any manager who does not make the *ex post* optimal choice or is revealed uninformed by the signal σ^y. We will later prove that this firing rule is optimal and we will pin down the values of N^I and W^U compatible with a stationary equilibrium.

1.3.1 Information Revelation in Equilibrium

An equilibrium price function $p^e(b, \chi)$ can in principle carry information about the pair (b, χ). So, the question is: how much information is revealed in a rational expectation equilibrium compatible with excess demand? Our focus will be on revelation of χ, the payoff-relevant variable.

The two extreme possibilities are that at each equilibrium price vector the actual value of χ is revealed or that nothing is revealed. We will show that neither of these cases can occur when we look at equilibria which are compatible with excess demand.

Consider the case of full revelation first. We say that a rational expectations equilibrium is **fully revealing** if the price function $p^e(b,\chi)$ is such that $p_i^e(b,\chi) = \frac{1-\chi_i}{R}$ for each i. It is worth noting that when the equilibrium is fully revealing investors are indifferent between hiring an informed or an uninformed manager, as they end up with the same allocation. In such situations managers and investors separate only for exogenous reasons and the ratio of informed and uninformed managers in the unemployment pool is irrelevant. It turns out that a fully revealing equilibrium always exists, but it is not compatible with excess demand.

Proposition 1. *For each collection $\left(N^I, \phi, W^U\right)$ a fully revealing rational expectations equilibrium exists. The equilibrium is not compatible with excess demand.*

The existence of a fully revealing equilibrium does not depend on the number of assets. In particular, such an equilibrium exists when $n = 1$, the case considered by Guerrieri and Kondor Guerrieri and Kondor (2012), although they focus their analysis on a partially revealing equilibrium which is simple (see Definition 3). The fully revealing equilibrium however appears quite implausible, as it requires that different equilibrium price vectors be selected at different values of χ even if the excess demand vectors are the same; thus, the fully revealing rational expectations equilibrium is *not* compatible with the excess demand schedules.

The other extreme case is the one in which no information is ever revealed. We say that a rational expectations equilibrium is **completely unrevealing** if there is a price vector p^* such that $p^e(b,\chi) = p^*$ for each pair (b,χ), i.e. the equilibrium price vector is constant. It turns out that there is no price vector p^* for which this is possible.

Proposition 2. *A completely unrevealing equilibrium does not exist.*

The intuition for the result is relatively simple and it can be better understood in the case in which there is a single risky asset. Since in equilibrium there must be demand both for the risky and the non-risky asset, some uninformed managers must demand both the risky asset and the non-risky asset. Furthermore, the price of the risky asset has to be strictly less than $\frac{1}{R}$ and the informed managers will demand the risky asset only when it repays and the safe asset only when the risky asset defaults. The expected utility of investing in the two activities must be the same, but when an uninformed manager demands both assets an adverse selection problem arises: the probability of receiving the risky asset is lower when the asset actually repays. This adverse selection phenomenon implies that it cannot be optimal for an uninformed manager to demand both assets, thus destroying the equilibrium.

The consequence of Propositions 1 and 2 is that all equilibria compatible with excess demand must be partially revealing: in equilibrium some information is always leaked. Since the equilibrium is not fully revealing, informed managers will perform better on average than uninformed ones. On the other hand, since there is some revelation of information, uninformed managers can do better than just choosing which assets to buy at random.

1.3.2 General Properties of $p^e(b, \chi)$

We now investigate some general properties of any partially revealing equilibrium[3]. The next proposition states some properties that any equilibrium price function must display.

Proposition 3. *Take as given the collection $\left(N^I, \phi, W^U\right)$. In every rational expectations equilibrium the following must be true:*

1. *if $p_i^e(b, \chi) = \frac{1}{R}$ then $\chi_i = 0$;*
2. *if $p_i^e(b, \chi) = 0$ then $\chi_i = 1$;*
3. *if $p_i^e(b, \chi) \in \left(0, \frac{1}{R}\right)$ then $v^U(i, p(b, \chi)) \geq \max_{j \neq i} v^U(j, p(b, \chi))$;*
4. *at each vector p which can be obtained as a realization of $p^e(b, \chi)$ it must be the case that $v^U(0, p) \geq \max_{j \neq 0} v^U(j, p)$.*
5. *If $p^e(b, \chi)$ is an equilibrium price function there is no pair (b, χ) such that $p_i^e(b, \chi) = \frac{1}{R}$ for some $i \geq 1$ and $p_j^e(b, \chi) \in \left(0, \frac{1}{R}\right)$ for $j \neq i$.*

[3]Notice that the equilibrium discussed in Guerrieri and Kondor Guerrieri and Kondor (2012) is partially revealing.

Notice that points (3) and (4) imply that whenever $p_1 \in \left(0, \frac{1}{R}\right)$ and $p_2 \in \left(0, \frac{1}{R}\right)$ then $v^U(0,p) = v^U(1,p) = v^U(2,p)$. In other words, the uninformed managers must be indifferent between all assets whose value is not revealed and the risk-free asset. Furthermore, point (1) implies that whenever the equilibrium price of asset i is $p_i = \frac{1}{R}$ we have $v^U(i,p) = v^U(0,p)$. Thus at any equilibrium price vector the uninformed managers must be indifferent between all the assets which have a price different from zero. The intuition for point (5) is that in order to have $p_i = \frac{1}{R}$ there must be a strictly positive demand on the part of the informed managers for asset i. But this must mean that the informed managers are unwilling to buy the other risky asset despite the fact that it has a strictly lower price. This reveals that the other asset has a return of zero and therefore the price cannot be strictly positive. Notice that it is possible to have equilibrium price vectors in which $p_i = 0$ for some asset i (so that it is revealed that $\chi_i = 1$) while the other risky asset has a price in the interval $\left(0, \frac{1}{R}\right)$, and is thus non-revealing. However, if $p_i = \frac{1}{R}$ for some i then full revelation must occur: all prices are either 0 or $\frac{1}{R}$. We can now establish the first important result.

Proposition 4. *There is no simple equilibrium in which, for some* (b,χ)*, it holds that* $\frac{1}{R} > p_1^e(b,\chi) > p_2^e(b,\chi) > 0$ *or* $\frac{1}{R} > p_2^e(b,\chi) > p_1^e(b,\chi) > 0$.

The intuition is very similar to the one behind Proposition 2. When $p_1 > p_2$ the uninformed managers tend to receive asset 1 with higher probability when $\chi = (0,0)$ rather than when $\chi = (0,1)$. This adverse selection problem implies that the expected value of asset 1 conditional on receiving asset 1 is lower than the expected value of receiving asset 0. Thus, uninformed managers are better off not demanding asset 1 at the pair of prices (p_1, p_2). But this makes it impossible for the market for asset 1 to clear.

1.3.3 Building a Partially Revealing Equilibrium

Proposition (4) implies that a simple equilibrium can exist only if the two risky assets have the same price whenever there is no full revelation. In other words, there will be a set of values of (b,χ) for which the price vector is (p,p), with $p \in \left(0, \frac{1}{R}\right)$, while outside the set there will be revelation of at least one asset. In order to further explore the nature of equilibrium we start from a somewhat obvious observation that we state without proof.

Lemma 1. *Suppose that there is an equilibrium in which the price function* $p^e(b,\chi)$ *is such that* $p_1^e(b,\chi) = p_2^e(b,\chi) = p$ *and* $p \in \left(0, \frac{1}{R}\right)$ *for some subset of* $\left[\underline{b}, \overline{b}\right]^2 \times \{0,1\}^2$*. Then it must be the case that*

$$\Pr\left(\chi_1 = 0 | (p,p)\right) = \Pr\left(\chi_2 = 0 | (p,p)\right).$$

In equilibrium the two assets must generate the same expected utility for the uninformed managers when the price vector is (p,p). Since the two assets have the same price, so they are both the lowest priced risky asset, the expected utility of each asset is $(1-f_i)\left(\frac{\gamma}{p} + \beta\omega\delta W^U\right)$, where

$$f_i = \Pr\left(\chi_i = 0 | (p,p)\right).$$

To generate the same utility we must have $f_1 = f_2$. If we call f the common probability we must also have

$$(1-f)\left(\frac{\gamma}{p} + \beta\omega\delta W^U\right) = \gamma R + \beta\omega\delta f^2 W^U.$$

Define now

$$r = \frac{N^I}{\overline{b} - \underline{b}}$$

and observe that, because of Assumption 1, we have $r \in (0,1)$. The ratio r measures the impact of informed traders and we will see that it plays an important role in the construction of the equilibrium.

The key observation is that when the price pair (p,p) occurs in equilibrium then the informed traders are indifferent between the two assets when $\chi = (0,0)$. If they were to choose both assets as part of their demand then the posterior probabilities could not be equal. But by Lemma 1 they must be equal. The equilibrium therefore requires that when $\chi = (0,0)$ there is some asymmetry in the demand for the two assets, so that posterior beliefs end up being the same. Let α be the fraction of informed traders demanding asset 1 when the prices are (p,p) and $\chi = (0,0)$, and β the similar fraction for asset 2. The values α and β are chosen so that $\Pr\left(\chi_1 = 0 | (p,p)\right) = \Pr\left(\chi_2 = 0 | (p,p)\right)$. The existence of the values α and β that makes this possible is not obvious and requires conditions on the parameters. This leads to the following proposition.

Proposition 5. *A simple equilibrium in which* $p_1^e(b,\chi) = p_2^e(b,\chi) = p$ *and* $p \in \left(0, \frac{1}{R}\right)$ *for some subset of* $\left[\underline{b}, \overline{b}\right]^2 \times \{0,1\}^2$ *is possible only if*

$$rq_2(1-q_1) \geq q_2 - q_1. \tag{1.4}$$

Condition (1.4) is satisfied when r is sufficiently high or when the difference $q_2 - q_1$ is sufficiently small. When $r = 1$ the condition is always satisfied, and so it is when $q_2 = q_1$. The economic intuition is as follows. We want to move from a situation in which the probabilities of repayment are different ($q_2 > q_1$) to a situation in which the probabilities are equal ($f_1 = f_2$). In order to do that we need a sufficient mass of informed managers. When the priors are very close then a slight asymmetry in the demands for the assets by informed managers is sufficient to achieve equality. When the initial difference is high we need a more robust presence of informed managers, which is equivalent to a higher r.

We are now in a position to characterize a simple equilibrium. In equilibrium there are values of (b,χ) for which no revelation occurs and in that case the two assets will have the same price. There will also be areas in which full revelation occurs, with both assets having prices reflecting their fundamentals, and areas in which one asset is revealed not paying while the other is not fully revealed.

Proposition 6. *Suppose that inequality (1.4) is satisfied. Then there is a continuum of simple equilibria compatible with excess demand in which the equilibrium price function takes the following values:*

- *no revelation and perfect comovement: the prices are given by* (p,p), *where* $p = \frac{1-f}{R+(f^2-(1-f))\beta\omega\delta\frac{W^U}{\gamma}}$;
- *partial revelation: the prices are given either by* $(0,p_2)$ *or* $(p_1,0)$, *where* $p_i = \frac{1-q_i}{R+(2q_i-1)\beta\omega\delta\frac{W^U}{\gamma}}$;
- *full revelation.*

In equilibrium, when $\chi = (0,0)$ *a fraction* α *of informed traders demand asset 1 and a fraction* β *demand asset 2, with* $\alpha \in [0,1]$, $\beta \in [0,1]$ *and* $\alpha + \beta \in [1,2]$. *The value* f *is the conditional probability of failure for an asset when the realized price pair is* (p,p) *and it depends on* α *and* β.

In the appendix we explain in detail the shapes of the different regions. The basic idea is to generalize the equilibrium structure in Guerrieri and Kondor (2012). In their setting with a single risky asset, non-revelation occurs when either the asset repays and the supply of bond is sufficiently high ($\chi = 0$ and $b \in \left[\underline{b} + N^I, \overline{b}\right]$) or the asset does not repay and the supply of the bond is sufficiently low ($\chi = 1$ and $b \in \left[\underline{b}, \overline{b} - N^I\right]$).

With two assets we have a similar structure but we have to make sure that the regions of non-revelation are determined in such a way that the posterior probabilities are equal. So, for example, when both assets repay ($\chi = (0,0)$) the non-revelation region is given by $\left[\underline{b} + \alpha N^I, \overline{b}\right] \times \left[\underline{b} + \beta N^I, \overline{b}\right]$, while when both assets fail the non-revelation region is $\left[\underline{b}, \overline{b} - \alpha N^I\right] \times \left[\underline{b}, \overline{b} - \beta N^I\right]$. The choice of α and β, i.e. the weight put in the demand of asset 1 and asset 2 by informed managers when $\chi = (0,0)$, determines the boundaries of the non-revelation region and therefore the conditional probability of default for each asset when (p, p) is observed. Condition (1.4) makes sure that it is possible to choose α and β so that the probability of default is the same for the two assets.

It is worth noticing that when there is partial revelation, i.e. one asset is revealed as defaulting while uncertainty remains on the other asset, the price of the asset is exactly the same as in Guerrieri and Kondor (2012).

1.4 Stationary Equilibrium

Up to now the analysis has taken the mass of informed managers N^I, the firing rule and the present value W^U of the utility of being employed for uninformed managers as given. The values N^I and W^U can be endogenized and the firing rule made optimal in a stationary equilibrium, by slightly adjusting the analysis in Guerrieri and Kondor (2012). This section shows how this is done, so that the paper is self contained.

1.4.1 Labor Market and Determination of N^I and W^U

Let W^U be the present value of being employed for an uninformed manager. If the uninformed manager stays out of the labor market, current period utility is zero. If instead the manager decides to search for a job then, as explained in subsection 1.2.2, the probability of finding one is $\mu_t = \min\left\{A_t, Z_t\right\}/Z_t$. Call $\widehat{W}_t^U$ the present value of an *unemployed* uninformed manager making the optimal search decision at time t. The the following relation must hold:

$$\widehat{W}_t^U = \max\left\{\beta\widehat{W}_{t+1}^U, \mu_t W_t^U + (1-\mu_t)\,\beta\widehat{W}_{t+1}^U - \kappa\right\} \tag{1.5}$$

where $\beta\widehat{W}_{t+1}^U$ is the utility obtained if no search is made in the current period (thus not paying the cost κ and obtaining zero in the current period).

In a stationary equilibrium $W_t^U = W^U$ and $\widehat{W}_t^U = \widehat{W}^U$for each t. Furthermore, in a stationary equilibrium in which the probability of finding employment is μ whenever the search cost κ is paid we must have

$$\mu W^U + (1-\mu)\,\beta\widehat{W}^U - \kappa = 0. \tag{1.6}$$

The reason is that with $\mu W^U + (1-\mu)\,\beta\widehat{W}^U - \kappa > 0$ we would have all uninformed managers entering, which is impossible as there is a large mass of uninformed managers and the probability of obtaining employment would be too low. On the other hand if $\mu W^U + (1-\mu)\,\beta\widehat{W}^U - \kappa < 0$ then only informed managers can possibly be in the market and the firing rule cannot be optimal (any manager who has a probability of being informed less than 1 should be fired).

Equation (1.5) and (1.6) imply that in a stationary equilibrium $\widehat{W}^U = 0$ and therefore

$$\mu W^U = \kappa. \tag{1.7}$$

Notice further that the utility of being employed for an informed manager must be higher than for an uninformed manager, since the probability of losing the job is lower. Thus $W^I > W^U$. Since the probability of getting hired is the same for informed and uninformed managers, equation (1.7) implies $\mu W^I - \kappa > 0$, so that it is always optimal for an unemployed informed manager to search for a job.

Let λ be the probability that an uninformed manager makes a choice that does not lead to being fired. This will happen either because the equilibrium is revealing or because the equilibrium is unrevealing but the uninformed manager makes by chance the correct choice. Furthermore, in both cases, the uninformed manager needs the message σ^y not to reveal that she is uninformed. The probability λ is an increasing function of N^I, the number of informed managers, since the larger is N^I the higher is the probability that the equilibrium price vector will be fully revealing and therefore the uninformed managers will be able to make the correct choice. Furthermore, in a stationary equilibrium the number N^I must satisfy

$$(1-\delta) N^I = \mu \left(M^I - \delta N^I\right).$$

The reason is that, as previously observed, in an equilibrium characterized by the free entry condition (1.7) for the uninformed manager, all informed managers must have a strictly positive utility from searching. In any given moment, the number of unemployed informed managers is M^I minus the ones who were employed in the previous period and retained their job, i.e. δN^I. Thus $M^I - \delta N^I$ is the mass of informed unemployed managers who search and $\mu \left(M^I - \delta N^I\right)$ is the number of informed managers hired in any given period. In order to keep the mass of informed managers constant, this number must equal the number of informed managers departing in each period. Since an informed manager departs only for exogenous reasons, the number is $(1-\delta) N^I$. Thus, when the matching probability is μ we have

$$N^I = \frac{\mu M^I}{1-\delta+\delta\mu} \tag{1.8}$$

Thus, N^I is an increasing function of μ. This in turn implies that λ is also an increasing function of μ.

Let $\lambda(\mu)$ denote the function that describes the probability of being retained for an uninformed manager when buying the riskless asset. Notice that $\lambda(\mu)$ is increasing in μ and $\lambda(\mu) \in [0,1]$ whenever $\mu \in [0,1]$

Since choosing the riskless asset must always be an optimal choice for the uninformed manager we must have

$$W^U = \gamma R + \beta\delta\omega\lambda(\mu) W^U \qquad \to \qquad W^U = \frac{\gamma R}{1-\beta\delta\omega\lambda(\mu)}. \tag{1.9}$$

Using the free-entry condition (1.7) and the expression for W^U in (1.9) we obtain the following equation to be solved for μ:

$$\mu\gamma R = \kappa\left(1-\beta\delta\omega\lambda(\mu)\right). \tag{1.10}$$

The LHS is continuous and strictly increasing in μ while the RHS is continuous and decreasing in μ. Furthermore, at $\mu = 0$ the LHS is strictly lower than the RHS and at $\mu = 1$ the LHS is strictly higher than the RHS (since we assumed $\gamma R \geq \kappa$). Thus, a unique value $\mu^* \in (0,1)$ exists. In turn, this determines a unique value N^I from equation (1.8) and a unique value for W^U from equation (1.9).

1.4.2 Firing Rule Optimality

The last step is to show that the firing rule (i.e. fire a manager only when she is revealed uninformed, retain the manager otherwise) is optimal. It is obvious that, as long as there is a strictly positive percentage of informed managers looking for a job, it is optimal to fire a manager who is considered uninformed with probability 1. What we need to prove is that it is never the case that it is optimal to fire a manager who has not been revealed uninformed. This is the case if the probability that a manager is informed is higher than the fraction of informed in the unemployment pool, as stated by the following prposition.

Proposition 7. *The firing rule is optimal.*

In a stationary equilibrium there is always a strictly positive fraction of informed managers in the unemployment pool, due to the exogenous rate of separation δ and to the fact that the free-entry condition for uninformed implies that it is stricty optimal for informed managers to search for a job. This immediately implies that it is optimal to fire a manager who has been proved uninformed. To complete the prove of optimality we also need to show that it is never optimal to fire a manager who has *not* been proved uninformed. This is true if the probability assigned to the fact that a manager (who has made no mistakes) is informed is greater than the fraction of informed managers present in the unemployment pool. Essentially, what is required is that the probability that a manager is informed goeas up sufficiently fast when she does not make a mistake. The proof shows that when Assumption 4 is satisfied this is the case.

1.5 Conclusion

This paper explores a rational expectation model in which two assets with independent returns and liquidity shocks are present. The equilibrium exhibits a strong form of price comovement: unless there is full revelation of the information on the assets' returns, the prices of the two assets are the same. This happens despite the fact that *ex ante* the two assets have different distributions.

The intuition for the result is that in an equilibrium in which the assets have non-revealing but different prices, informed managers will buy exclusively the asset with the lower price when both assets are repaying. Thus, the uninformed managers face an adverse selection problem that prevents the existence of such equilibrium. It is only when the informed managers are indifferent between the two assets when they both repay, something that happens only if they have the same price, that an equilibrium becomes possible.

This somewhat extreme form of comevement occurs because all the fund managers are evaluated looking at the performance of all assets. An extension of the model may consider the case in which there are both specialized and general funds. Specialized funds are restricted to buy certain classes of assets and the managers are evaluated only looking at the *ex post* performance of those assets. In such a model the managers of the specialized funds do not face the same type of adverse selection problem that managers of the general funds face, so equilibria with different unrevealing prices become possible.

1.6 Appendix

Proof of Proposition 1. In a fully revealing rational expectations equilibrium the price function $p^e(b,\chi)$ is given by $p_i^e(b,\chi) = \frac{1-\chi_i}{R}$, so that in equilibrium any asset has a price which is either 0 or $\frac{1}{R}$. Prices different from 0 or $\frac{1}{R}$ are not observed in equilibrium, so that the probability distribution held by uninformed agents at a price $p_i \notin \left\{0, \frac{1}{R}\right\}$ is undetermined. We specify that whenever $p_i \notin \left\{0, \frac{1}{R}\right\}$ uninformed managers believe $\chi_i = 1$ with probability 1, i.e. they are certain that the asset will default.

Given this price function and beliefs, a demand function that maximizes the expected utility of an uninformed manager is given by

$$d_i^U(p) = \begin{cases} 1 & \text{if } p_i = \frac{1}{R} \\ 0 & \text{otherwise} \end{cases}$$

for each asset $i = 0, 1, 2$. The demand function of the informed can be described as follows. For any given realization χ, let

$$P(\chi) = \{i \,|\chi_i = 0\}$$

be the set of assets which are going to repay. Then

$$d_i^I(p) = \begin{cases} 1 & \text{if } \chi_i = 0 \text{ and } p_i \le p_j \text{ for each } j \in P(\chi) \\ 0 & \text{otherwise} \end{cases} \tag{1.11}$$

i.e. the informed manager demands asset i if and only if the asset is not in default and it has the lowest price among the assets which are not in default (we maintain the convention that $\chi_0 = 0$ with probability 1).

It is clear that, given the price function and the specified beliefs for out-of-equilibrium prices, the demand functions maximize the expected utility of both informed and uninformed managers. With these demand functions:

- since $N^I \le M^I < \underline{b}$, no market can be in equilibrium unless there is demand on the part of the uninformed managers;
- the price cannot be 0 for a repaying asset, since in that case the demand on the part of the informed would be strictly positive and the supply would be zero;

- thus, the only possible equilibrium price vector is such that $p_i = \frac{1-\chi_i}{R}$ for each i; no other equilibria are possible.

To show that the price function $p_i = \frac{1-\chi_i}{R}$ is in fact an equilibrium we have to specify the allocation rule x. For a given vector $d^y \in \Delta$ define $\delta^1(d^y) = \{i \,|d^y_i = 1\}$ and let $\{i_1, i_2, \ldots, i_m\}$ be an enumeration of $\delta^1(d^y)$. We set $x_i = 0$ if $i \notin \delta^1(d)$. For assets in the set $\delta^1(d^y)$ we define x_i as follows:

- If $\delta^1(d^y)$ has a single element i_1 then $x_{i_1} = 1$.
- If $\delta^1(d^y)$ has multiple elements we define recursively x_{i_k} as follows:
  - $x_{i_m} = \frac{b_{i_m}}{N}$;
  - $x_{i_k} = \min\left\{\frac{b_{i_k}}{N}, 1 - \sum_{j=k+1}^{m} x_{i_j}\right\}$;
  - $x_{i_1} = 1 - \sum_{j=2}^{m} x_{i_j}$.

It can be readily checked that the allocation rule satisfies $\sum_{i=0}^{2} x_i d^y_i = 1$ for each d^y and b.

We can now check that this allocation rule clears the markets at each possible equilibrium price vector. Remember that in equilibrium we have $d^U_i = 1$ only when $p_i = \frac{1}{R}$, and $p_i = \frac{1}{R}$ if and only if $\chi_i = 0$. All managers, informed and uninformed have the same demand at an equilibrium point. Thus the aggregate demand is N (the entire mass of traders) if $p_i = \frac{1}{R}$ and 0 if $p_i = 0$. Furthermore, the set $\delta^1(d^y)$ has always as first element the safe asset, i.e. $i_1 = 0$ and includes the assets with price $\frac{1}{R}$. Since, by assumption $N > 2\bar{b}$ we have that at an equilibrium point $x_i = \frac{b_i}{N} \in (0, 1)$ when $p_i = \frac{1}{R}$ and $x_i = 0$ when $p_i = 0$, thus clearing all markets.

That this equilibrium is not compatible with excess demand can be seen immediately considering two vectors (b, χ) and (b', χ') such that $b'_1 = b_1 - N^I$, $\chi_1 = 0$ and $\chi'_1 = 1$ (see the discussion after Definition 2).

Proof of Proposition 2. Suppose that the equilibrium price function is such that $p^e(b, \chi) = p^*$ for each vector (b, χ) for some vector p^*. In an equilibrium in which prices are constant and do not depend on (b, χ) the probability conditional on prices must be equal to the prior probabilities, i.e. $\Pr(\chi_i = 1 | \, p^*) = q_i$ for each i. In order to have positive demand for each asset, the expected utility of investing in each asset must be the same. We have

$$\begin{aligned} v^U(i,p^*) &= E\left[\gamma\frac{1-\widetilde{\chi}_i}{p_i^*} + \beta\delta\left(1-\phi\left(i,\widetilde{\chi},p^*,\sigma^y\right)\right)W^U \middle| p^e\left(\widetilde{b},\widetilde{\chi}\right) = p^*\right] \\ &= \gamma\frac{1-q_i}{p_i} + \beta\delta\omega\Pr\left(i \text{ ex post optimal}\right)W^U \end{aligned}$$

where we have assumed that the firing rule is that a manager is retained only if the exogenous signal σ^y does not reveal that the manager is uninformed (probability ω) and the choice of i turns out to be optimal *ex post*. For $i = 0$ the probability that the choice is *ex post* optimal is q_1q_2, i.e. the probability that all risky assets will fail. Thus, we have

$$v^U(0,p^*) = \gamma R + \beta\delta\omega q_1q_2W^U$$

Consider now the possible pricing of the risky assets. We start observing that we can rule out the case $p_1^* = p_2^*$. In this case $\Pr\left(i \text{ ex post optimal}\right) = 1-q_i$, so the prices must be

$$p_1^* = \frac{1-q_1}{R + \left(q_1q_2 - \left(1-q_1\right)\right)\frac{\beta\delta\omega}{\gamma}W^U}$$

$$p_2^* = \frac{1-q_2}{R + \left(q_1q_2 - \left(1-q_2\right)\right)\frac{\beta\delta\omega}{\gamma}W^U},$$

but this implies $p_1^* \neq p_2^*$, a contradiction.

Thus suppose $p_1^* > p_2^*$ (the case $p_2^* > p_1^*$ is symmetric). In this case

$$v^U(1,p^*) = \gamma\frac{1-q_1}{p_1} + \beta\delta\omega\left(1-q_1\right)q_2W^U$$

$$v^U(2,p^*) = \gamma\frac{1-q_2}{p_2} + \beta\delta\omega\left(1-q_2\right)W^U$$

and the prices must be

$$p_1^* = \frac{1-q_1}{R + \left(q_1q_2 - \left(1-q_1\right)q_2\right)\frac{\beta\delta\omega}{\gamma}W^U} \tag{1.12}$$

$$p_2^* = \frac{1-q_2}{R + \left(q_1q_2 - \left(1-q_2\right)\right)\frac{\beta\delta\omega}{\gamma}W^U} \tag{1.13}$$

If the prices given by (1.12) and (1.13) are such that $p_1^* \leq p_2^*$ then there is no equilibrium of this sort. Thus, suppose that in fact the parameters are such that $p_1^* > p_2^*$. In equilibrium the demand functions of the informed managers are given by (1.11). Since there are no ties among prices, informed managers always demand at most one risky asset. For a given vector (b, χ) the *ex post* utility of manager y with demand d^y is

$$u\left(d^y, (b, \chi)\right) = x_0\left(d^y, b, \chi\right)\left(\gamma R + \beta\delta\omega\chi_1\chi_2 W^U\right)$$

$$+x_1\left(d^y, b, \chi\right)\left(1-\chi_1\right)\left(\frac{\gamma}{p_1^*} + \beta\delta\omega\chi_2 W^U\right) + x_2\left(d^y, b, \chi\right)\left(1-\chi_2\right)\left(\frac{\gamma}{p_2^*} + \beta\delta\omega W^U\right),$$

where $x_i\left(d^y, b, \chi\right)$ is the probability of receiving asset i when (b, χ) occurs and the demand is d^y.

Now notice that whenever asset i is the lowest priced repaying asset, a quantity N^I must be allocated to the informed managers, as this is the only asset that they demand. Integrating over N^U the quantity $x_1\left(d^y, b, \chi\right)$ we therefore have

$$\int_{y \in N^U} x_1\left(d^y, b, \chi\right) dy = b_1 - N^I\left(1-\chi_1\right)\chi_2 \tag{1.14}$$

and integrating over N^U the quantity $x_2\left(d^y, b, \chi\right)$ we have

$$\int_{y \in N^U} x_2\left(d^y, b, \chi\right) dy = b_2 - N^I\left(1-\chi_2\right). \tag{1.15}$$

Finally, the quantity of riskless bond allocated to uninformed managers is determined residually as

$$\int_{y \in N^U} x_0\left(d^y, b, \chi\right) dy = N^U - \left(b_1 - N^I\left(1-\chi_1\right)\chi_2\right) - \left(b_2 - N^I\left(1-\chi_2\right)\right) \tag{1.16}$$

Using (1.14), (1.15) and (1.16), by integrating $u\left(d^y, b, \chi\right)$ over N^U we obtain

$$\int_{y \in N^U} u\left(d^y, b, \chi\right) dy =$$

$$\left(N^U - \left(b_1 - N^I\left(1-\chi_1\right)\chi_2\right) - \left(b_2 - N^I\left(1-\chi_2\right)\right)\right)\left(\gamma R + \beta\delta\omega\chi_1\chi_2 W^U\right)$$

$$+\left(b_1 - N^I\left(1-\chi_1\right)\chi_2\right)\left(1-\chi_1\right)\left(\frac{\gamma}{p_1^*} + \beta\delta\omega\chi_2 W^U\right)$$

$$+\left(b_2 - N^I\left(1-\chi_2\right)\right)\left(1-\chi_2\right)\left(\frac{\gamma}{p_2^*} + \beta\delta\omega W^U\right)$$

Taking expectation with respect to χ, and using the fact that b and χ are independent and $v^* = v^U(i, p^*)$ for each i, we have

$$\int_{y\in N^U} E_\chi\left[u\left(d^y, b, \chi\right)\right]dy = N^U v^*$$

$$-N^I E\chi\left[\left(1-\chi_1\right)\chi_2\left(\left(1-\chi_1\right)\left(\frac{\gamma}{p_1^*} + \beta\delta\omega\chi_2 W^U\right) - \left(\gamma R + \beta\delta\omega\chi_1\chi_2 W^U\right)\right)\right]$$

$$-N^I E\chi\left[\left(1-\chi_2\right)\left(\left(1-\chi_2\right)\left(\frac{\gamma}{p_2^*} + \beta\delta\omega W^U\right) - \left(\gamma R + \beta\delta\omega\chi_1\chi_2 W^U\right)\right)\right]$$

It can be readily checked that the second and third term on the right hand side are negative. For example, the second term is non-zero only when $\chi_1 = 0$ and $\chi_2 = 1$. In that case the term in the square parenthesis is $\frac{\gamma}{p_1^*} + \beta\delta\omega W^U - \gamma R$ which is strictly positive since $1/R > p_1^*$, and multiplication by $-N^I$ yields a negative value.

This means that there must be a positive mass of uninformed managers who obtain an expected utility lower than v^*. This cannot be the case in equilibrium.

Proof of Proposition 3. Remember that the supply of risk-free bonds is infinitely elastic at the price $\frac{1}{R}$ and that in equilibrium $N^I \le M^I < \underline{b}$, so that equilibrium at any non-zero price for a risky asset is possible only if there is demand from the uninformed managers.

1. Suppose that at a vector (b, χ) the equilibrium price vector is $p^e(b, \chi)$ with $p_i^e(b, \chi) = \frac{1}{R}$. If $\Pr\left(\chi_i = 0 \middle| p^e(b, \chi)\right) < 1$ then the investment is strictly dominated by the investment in the riskless asset for the uninformed managers, so their demand for asset i at that price vector is zero. But then $p_i^e = \frac{1}{R}$ cannot be part of an equilibrium price vector since the demand for asset i is at most N^I and it is therefore strictly less than supply.

2. Suppose that at a vector (b, χ) the equilibrium price vector is $p^e(b, \chi)$ with $p_i^e(b, \chi) = 0$. If $\Pr\left(\chi_i = 1 \middle| p^e(b, \chi)\right) < 1$ then the demand on asset i by the uninformed manager would be infinity, thus violating the equilibrium condition.

3. Suppose that at a vector (b, χ) the equilibrium price vector is $p^e(b, \chi)$ with $p_i^e(b, \chi) \in \left(0, \frac{1}{R}\right)$. Then demand must be equal to supply for asset i and this is possible only if there is a strictly positive demand by uninformed managers. In turn, this is possible only if $v^U(i, p) \geq v^U(j, p)$ for each $j \neq i$.

4. Suppose that at a vector (b, χ) the equilibrium price vector is $p^e(b, \chi)$ and $\max_{j \neq 0} v^U(j, p) > v^U(0, p)$. Then demand for the risk-free asset can only come from the informed managers and the uninformed managers will only demand risky assets. But since $N - M^I > 2\bar{b}$, it is impossible to reach equilibrium in all markets for risky assets.

5. Suppose that at a vector (b, χ) the equilibrium price vector is $p^* = p^e(b, \chi)$ with $p_i^* = \frac{1}{R}$ for some $i \geq 1$ and $p_j^* \in \left(0, \frac{1}{R}\right)$ for $j \notin \{0, i\}$. It must be the case that the informed managers are demanding asset i, thus revealing that $\chi_i = 0$. If $p_j^* \in \left(0, \frac{1}{R}\right)$ this means that informed managers must demand asset j with strictly positive probability, i.e. that $\chi_j = 0$ with strictly positive probability when the price vector is p^*. But this is impossible, since in this case the informed managers would not demand asset i (which has a higher price), thus making it impossible to have $p_i^* = \frac{1}{R}$. So p_j^* must be either 0 or $\frac{1}{R}$.

Proof of Proposition 4. We break down the proof in two steps.

Step 1. If $b_i \geq \underline{b} + N^I$ then $\chi_i = 0$ cannot be revealed. This is proved by contradiction. Consider wlog asset 1 and suppose there is a pair $((b_1, b_2), (\chi_1, \chi_2))$ such that $b_1 \geq \underline{b} + N^I$, $\chi_1 = 0$, and $p_1 = \frac{1}{R}$. First notice that by point (5) of Proposition (3), full revelation of χ_2 must also occur. Suppose first $(\chi_1 = 0, \chi_2 = 1)$ and consider the pair $(b', \chi') = \left(\left(b_1 - N^I, b_2\right), (1, 1)\right)$. By compatibility with excess demand the price must be the same at (b', χ') and at (b, χ). But this is impossible, since we would have $p_2 = \frac{1}{R}$ and $\chi_2 = 1$. Next suppose $(\chi_1, \chi_2) = (0, 0)$. Let (α, β) be the demand for the risky assets on the part of the informed managers when prices are $\left(\frac{1}{R}, \frac{1}{R}\right)$ and $(\chi_1, \chi_2) = (0, 0)$, with $\alpha \in \left[0, N^I\right]$ and $\beta \in \left[0, N^I\right]$. We distinguish two cases.

a) β is such that $b_2 \geq \underline{b} + \beta$. In that case the excess demand would be exactly the same at $(b', \chi') = ((b_1 - \alpha, b_2 - \beta), (1, 1))$.

b) β is such that $b_2 < \underline{b} + \beta$. In that case the excess demand would be exactly the same at $(b', \chi') = \left(\left(b_1 - \alpha, b_2 + N^I - \beta\right), (1, 0)\right)$. Notice that $b'_2 = b_2 + N^I - \beta$ is in the interval $\left[\underline{b}, \underline{b} + N^I\right]$ and it is therefore feasible.

We conclude that whenever $b_i \geq \underline{b} + N^I$ and $\chi_i = 0$ the price function must be unrevealing, i.e. there must be vector (b', χ') with $\chi'_i = 1$ such that $p^e(b, \chi) = p^e(b', \chi')$.

Step 2. Let

$$u_1\left(\chi | (p_1, p_2)\right) = (1 - \chi_1)\left(\frac{\gamma}{p_1} + \chi_2 \beta \delta \omega W^U\right)$$

be the *ex post* utility of an uninformed manager who receives asset 1 when the prices are (p_1, p_2) with $p_1 > p_2$ (the case $p_1 < p_2$ is similar) and (χ_1, χ_2) realizes; observe that $u_1\left((1, \chi_2) | (p_1, p_2)\right) = 0$. Let

$$f_{ij} = \Pr\left(\widetilde{\chi}_1 = i, \widetilde{\chi}_2 = j | (p_1, p_2)\right)$$

be the conditional probability of $\widetilde{\chi} = (i, j)$ when the prices are (p_1, p_2). In equilibrium it has to be

$$f_{00} u_1\left((0, 0) | (p_1, p_2)\right) + f_{01} u_1\left((0, 1) | (p_1, p_2)\right) = v^*$$

where v^* is the expected utility obtained demanding the safe asset only.

Let $x_1\left(b, \chi | (p_1, p_2)\right)$ be the quantity of asset 1 given to uninformed managers when the supply of the risky assets is $b = (b_1, b_2)$, the realization is χ and prices are (p_1, p_2). Since when $\chi = (0, 0)$ the informed managers demand asset 2 only, in equilibrium it has to be the case that $x_1\left(b, (0, 0) | (p_1, p_2)\right) = b_1$. On the other hand, when $\chi = (0, 1)$ the informed managers demand asset 1 only, so $x_1\left(b, (0, 1) | (p_1, p_2)\right) = b_1 - N^I$. Define

$$\overline{x}_1^{00} = E\left[x_1\left(b, (0, 0) | (p_1, p_2)\right)\right]$$

and

$$\overline{x}_1^{01} = E\left[x_1\left(b, (0, 1) | (p_1, p_2)\right)\right]$$

We want to show that

$$\overline{x}_1^{00} > \overline{x}_1^{01}.$$

We first observe that

$$\overline{x}_1^{00} = E\left[x_1\left(b,(0,0)\right|(p_1,p_2)\right)] = E\left[b_1\right|\chi=(0,0),(p_1,p_2)] \geq \frac{\underline{b}+\overline{b}}{2}$$

since, by step 1, the no-revelation region must include the upper interval $\left[\underline{b}+N^I,\overline{b}\right]$. Furthermore

$$\overline{x}_1^{01} = E\left[x_1\left(b,(0,1)\right|(p_1,p_2)\right)] = E\left[b_1-N^I\right|\chi=(0,1),(p_1,p_2)\right] \leq \frac{\underline{b}+\overline{b}-N^I}{2}$$

since $E\left[b_1|\chi=(0,1),(p_1,p_2)\right] \leq \frac{\underline{b}+\overline{b}+N^I}{2}$, where the highest value is attained when the no-revelation region is exactly $\left[\underline{b}+N^I,\overline{b}\right]$. We conclude

$$\overline{x}_1^{00} > \overline{x}_1^{01}$$

Let

$$\overline{x}_1 = f_{00}\overline{x}_1^{00} + f_{01}\overline{x}_1^{01}$$

be the expected quantity of asset 1 received by uninformed managers who are willing to buy asset 1 when prices are (p_1,p_2) and $\chi_1=0$. The expected utility *conditional on receiving asset* 1 by an uninformed manager is

$$\frac{f_{00}\overline{x}_1^{00}}{\overline{x}_1}u_1\left((0,0)|(p_1,p_2)\right) + \frac{f_{01}\overline{x}_1^{01}}{\overline{x}_1}u_1\left((0,1)|(p_1,p_2)\right) <$$

$$f_{00}u_1\left((0,0)|(p_1,p_2)\right) + f_{01}u_1\left((0,1)|(p_1,p_2)\right) = v^*.$$

But this means that for at least some uninformed agents demanding asset 1 at prices (p_1,p_2) receive an expected utility strictly inferior to v^*. This cannot happen in equilibrium.

Proof of Proposition 5. In equilibrium it must be the case that

$$v^U\left(0,(p,p)\right) = v^U\left(1,(p,p)\right) = v^U\left(2,(p,p)\right)$$

and the last equality implies

$$\Pr\left(\chi_1=0|(p,p)\right) = \Pr\left(\chi_2=0|(p,p)\right). \tag{1.17}$$

Since

$$\Pr\left(\chi_i=0|(p,p)\right) = \Pr\left(\chi_i=0,\chi_{-i}=0|(p,p)\right) + \Pr\left(\chi_i=0,\chi_{-i}=1|(p,p)\right)$$

the condition boils down to

$$\Pr\left(\chi_1 = 0, \chi_2 = 1 | (p,p)\right) = \Pr\left(\chi_1 = 1, \chi_2 = 0 | (p,p)\right)$$

which in turn is equivalent to

$$\Pr\left(\chi_1 = 0, \chi_2 = 1 \text{ and } (p,p)\right) = \Pr\left(\chi_1 = 1, \chi_2 = 0 \text{ and } (p,p)\right). \tag{1.18}$$

How can this be achieved? We start observing that in each equilibrium, whenever the prices for the risky assets are (p,p) with $0 < p < \frac{1}{R}$ and the realization of $\widetilde{\chi}$ is different from $(0,0)$ then the optimal demand of an informed manager y is unique and given by:

$$d^y\left((p,p)|\,\chi\right) = \begin{bmatrix} \chi_1\chi_2 \\ (1-\chi_1)\,\chi_2 \\ \chi_1\,(1-\chi_2) \end{bmatrix} \tag{1.19}$$

When $(\chi_1, \chi_2) = (0,0)$ and the prices are (p,p) then informed managers are indifferent between the three vectors $(0,1,0)$, $(0,0,1)$ and $(0,1,1)$. Suppose that whenever $\chi_1 = \chi_2 = 0$ and prices are (p,p) then a fraction α of informed managers demands asset 1 and a fraction β demands asset 2, with $\alpha \in [0,1]$, $\beta \in [0,1]$, $\alpha + \beta \in [1,2]$. For example, if $\alpha = \beta = 1$ this means that all informed managers submit the demand vector $(0,1,1)$ when $\chi = (0,0)$ and the price vector is (p,p). The aggregate demand of informed managers is therefore

$$\int_{y \in N^I} d^y\left((p,p)|\,(0,0)\right) dy = \begin{bmatrix} 0 \\ \alpha N^I \\ \beta N^I \end{bmatrix} \tag{1.20}$$

We have now to find the subset in the space $\left[\underline{b}, \overline{b}\right]^2 \times \{0,1\}^2$ for which $p^e(b,\chi) = (p,p)$. We will go through the four possible realizations of χ.

<u>Case $\chi = (0,0)$.</u>

We first show that if $b_1 < \underline{b} + \alpha N^I$ or $b_2 < \underline{b} + \beta N^I$ then prices are fully revealing, i.e. $p^e(b,\chi) = \left(\frac{1}{R}, \frac{1}{R}\right)$. Suppose not. Take $b_1 < \underline{b} + \alpha N^I$ and suppose that the price vector is not fully revealing, so that $p^e(b,\chi) = (p,p)$. Then there must be vectors $\chi' = (1, \chi_2')$ and b' such that the excess demand is the same as at (b,χ) when the price pair is (p,p). This implies

$$\left(b_1 - \alpha N^I, b_2 - \beta N_I\right) = \left(b_1', b_2' - N^I\left(1 - \chi_2'\right)\right).$$

In particular this requires $b_1' = b_1 - \alpha N^I < \underline{b}$, which is impossible. A similar reasoning applies when $b_2 < \underline{b} + \beta N^I$.

Next we show that if the two inequalities $b_1 \geq \underline{b} + \alpha N^I$ and $b_2 \geq \underline{b} + \beta N^I$ are satisfied then the price is not fully revealing, i.e. $p^e(b,\chi) = (p,p)$. Suppose not. Then the price vector is fully revealing, so that $p^e(b,\chi) = \left(\frac{1}{R}, \frac{1}{R}\right)$. Now consider the vectors $b' = \left(b_1 - \alpha N^I, b_2 - \beta N^I\right)$ and $\chi' = (1,1)$. It is easy to check that at (b',χ') the excess demand when the prices are $\left(\frac{1}{R}, \frac{1}{R}\right)$ is the same as at (b,χ). This is a contradiction, since $\left(\frac{1}{R}, \frac{1}{R}\right)$ can be an equilibrium price vector only when $\chi = (0,0)$.

We conclude that when $\chi = (0,0)$ we should have $p^e(\chi, b) = (p,p)$ whenever $(b_1, b_2) \in \left[\underline{b} + \alpha N^I, \overline{b}\right] \times \left[\underline{b} + \beta N^I, \overline{b}\right]$. In particular, the probability of the event

$$E_{pp}^{00} = (\chi = (0,0) \text{ and } (p,p) \text{ observed})$$

is given by

$$\Pr\left(E_{pp}^{00}\right) = (1-q_1)(1-q_2)\frac{\Delta b - \alpha N^I}{\Delta b}\frac{\Delta b - \beta N^I}{\Delta b} =$$

$$(1-q_1)(1-q_2)(1-\alpha r)(1-\beta r).$$

Case $\chi = (0,1)$.

We first show that if $b_1 < \underline{b} + N^I$ or $b_2 > \overline{b} - \beta N^I$ then the price is fully revealing, i.e. $p^e(b,\chi) = \left(\frac{1}{R}, 0\right)$. Suppose first that $b_1 < \underline{b} + N^I$ and the price vector is not fully revealing, i.e. $p^e(b,\chi) = (p,p)$. To make sure that $\chi_1 = 0$ is not revealed, there must be a pair (b',χ') with $\chi' = (1, \chi_2')$ such that the excess demand is the same as at (b,χ). This requires

$$\left(b_1 - N^I, b_2\right) = \left(b_1', b_2' - (1-\chi_2') N^I\right)$$

and in particular $b_1' = b_1 - N^I < \underline{b}$, which is impossible. Suppose now $b_2 > \overline{b} - \beta N^I$. There must be a pair (b',χ') with $\chi' = (\chi_1', 0)$ such that the excess demand is the same as at (b,χ). If $\chi_1' = 0$ then the condition becomes

$$\left(b_1 - N^I, b_2\right) = \left(b_1' - \alpha N^I, b_2' - \beta N^I\right)$$

or $b_2' = b_2 + \beta N^I > \overline{b}$, which is impossible. If $\chi_1' = 1$ then the condition becomes

$$\left(b_1 - N^I, b_2\right) = \left(b_1', b_2' - N^I\right)$$

or $b_2' = b_2 + N^I > \bar{b}$, which is also impossible.

We next show that if $b_1 \geq \underline{b} + N^I$ and $b_2 \leq \bar{b} - \beta N^I$ then the price vector is not fully revealing, that is $p^e(b,\chi) = (p,p)$. Suppose not, so that the price vector is revealing and $p^e(b,\chi) = \left(\frac{1}{R}, 0\right)$. Suppose first $b_1 \geq \underline{b} + N^I$ and consider the vectors $\chi' = (1,1)$ and $b' = \left(b_1 - N^I, b_2\right)$. Then, at $\left(\frac{1}{R}, 0\right)$ the excess demand is the same as at (b,χ), a contradiction. Similarly, if $b_2 \leq \bar{b} - \beta N^I$ consider the vectors $\chi' = (0,0)$ and $b' = \left(b_1 - (1-\alpha) N^I, b_2 + \beta N^I\right)$. Again the excess demand is the same at $\left(\frac{1}{R}, 0\right)$, a contradiction.

We conclude that when $\chi = (0,1)$ we should have $p^e(\chi, b) = (p,p)$ whenever $(b_1, b_2) \in \left[\underline{b} + N^I, \bar{b}\right] \times \left[\underline{b}, \bar{b} - \beta N^I\right]$. The probability of the event

$$E_{pp}^{01} = (\chi = (0,1) \text{ and } (p,p) \text{ observed})$$

is given by

$$\Pr\left(E_{pp}^{01}\right) = (1-q_1)\, q_2 \left(\frac{\Delta b - N^I}{\Delta b}\right) \frac{\Delta b - \beta N^I}{\Delta b}$$
$$= (1-q_1)\, q_2\, (1-r)\,(1-\beta r)$$

<u>Case $\chi = (1,0)$.</u>
This case is symmetric to the previous one, so applying the same reasoning we can conclude that when $\chi = (0,1)$ we should have $p^e(b,\chi) = (p,p)$ whenever $(b_1, b_2) \in \left[\underline{b}, \bar{b} - \alpha N^I\right] \times \left[\underline{b} + N^I, \bar{b}\right]$. The probability of the event

$$E_{pp}^{10} = (\chi = (1,0) \text{ and } (p,p) \text{ observed})$$

is

$$\Pr\left(E_{pp}^{10}\right) = q_1\,(1-q_2)\,(1-\alpha r)\,(1-r)\,.$$

<u>Case $\chi = (1,1)$.</u>
Again, applying the same reasoning as above we can conclude that when $\chi = (1,1)$ we should have $p^e(b,\chi) = (p,p)$ whenever $(b_1, b_2) \in \left[\underline{b}, \bar{b} - \alpha N^I\right] \times \left[\underline{b}, \bar{b} - \beta N^I\right]$. The probability of this event

$$E_{pp}^{11} = (\chi = (1,1) \text{ and } (p,p) \text{ observed})$$

is

$$\Pr\left(E_{pp}^{11}\right) = q_1 q_2\,(1-\alpha r)\,(1-\beta r)$$

After examining the 4 cases we can conclude that the probability of observing (p, p) in equilibrium is strictly positive and condition (1.18) becomes equivalent to

$$(1 - q_1)\, q_2\, (1 - \beta r) = q_1\, (1 - q_2)\, (1 - \alpha r)$$

or

$$\frac{(1 - q_1)\, q_2}{(1 - q_2)\, q_1} = \frac{1 - \alpha r}{1 - \beta r}. \tag{1.21}$$

An equilibrium exists if we can find feasible values α and β such that (1.21) is satisfied Since feasibility requires $1 \geq \alpha \geq 0$, $1 \geq \beta \geq 0$ and $2 \geq \alpha + \beta \geq 1$, the lowest possible value attainable by the ratio $\frac{1-\alpha r}{1-\beta r}$ is $1 - r$, while the highest possible value is $\frac{1}{1-r}$. Since $q_2 > q_1$ the LHS in (1.21) is strictly greater than 1. We conclude that a solution exists if

$$\frac{(1 - q_1)\, q_2}{(1 - q_2)\, q_1} \leq \frac{1}{1 - r}$$

or

$$r q_2\, (1 - q_1) \geq q_2 - q_1.$$

The probability of failure for each risky asset condition on observing (p, p) is

$$f = \frac{\Pr\left(E^{11}_{pp}\right) + \Pr\left(E^{10}_{pp}\right)}{\Pr\left(E^{11}_{pp}\right) + \Pr\left(E^{10}_{pp}\right) + \Pr\left(E^{01}_{pp}\right) + \Pr\left(E^{00}_{pp}\right)} \tag{1.22}$$

and it can be computed from the formulas above. The price p has to satisfy

$$p = \frac{1 - f}{R + \left(f^2 - (1 - f)\right) \beta \omega \delta \frac{W^U}{\gamma}} \tag{1.23}$$

so that $v^U\left(i, (p, p)\right) = v^U\left(0, (p, p)\right)$ for $i = 1, 2$.

Proof of Proposition 6. In any simple equilibrium the structure of the non-revealing region conforms to the one described in the proof of Proposition 5. We now show how this structure can be embedded in a rational expectations equilibrium compatible with excess demand. We will first describe the price function $p^e(b, \chi)$ for each pair (b, χ) and then describe the demand and allocation functions supporting the equilibrium.

The price function.

For each of the possible values of the pair (χ_1, χ_2) we will show how the space $\left[\underline{b}, \overline{b}\right]^2$ is partitioned and the values taken by the price function.

Case $\chi = (0, 0)$. The prices are non-revealing when both supply shocks are sufficiently high, and they are fully revealing otherwise. Figure 1.1a shows the price function for this case.

It is never the case that the value of one asset is revealed while the other is not, as this may happen only when $\chi_i = 1$ for some i. The value of p is given by (1.22) and (1.23).

Case $\chi = (0, 1)$. In this case there is an area in which the value of risky asset 2 is revealed but the value of risky asset 1 is not. Figure 1.1b shows these regions. For this to be possible, the excess demands in the region must be indistinguishable from excess demands when $\chi = (1, 1)$ and the true value of risky asset 2 is revealed. The value of p_1 is given by

$$p_1 = \frac{1 - q_1}{R + (2q_1 - 1)\,\frac{\beta\omega\delta}{\gamma} W^U}.$$

Case $\chi = (1, 0)$. Figure 1.1c shows the values of price function for this case. This case is symmetric to the previous one.

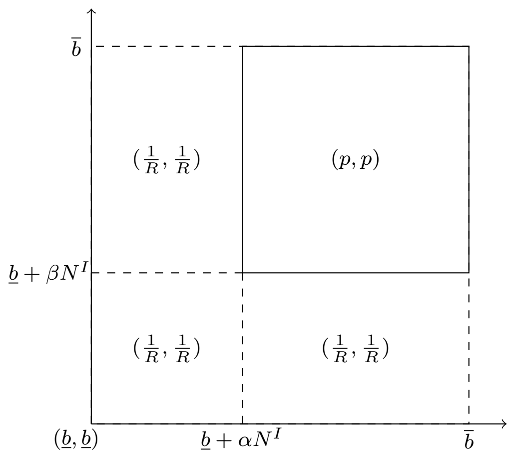
Now there is an area in which it is revealed that asset 1 is defaulting while there is uncertainty on asset 2. The value of p_2 is given by

$$p_2 = \frac{1 - q_2}{R + (2q_2 - 1)\,\frac{\beta\omega\delta}{\gamma} W^U}$$

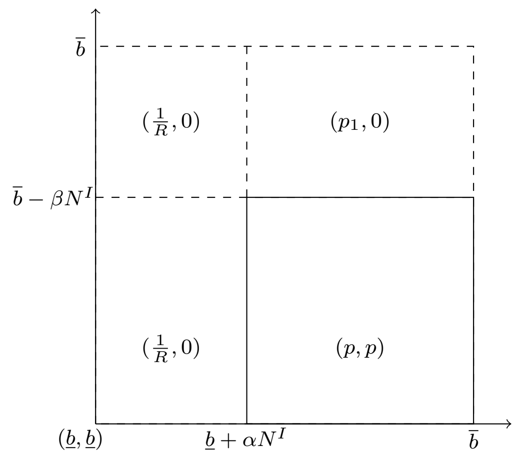
Case $\chi = (1, 1)$. This is the most complex case. Figure 1.1d shows the revelation and non-revelation regions for this case. Besides the area of non-revelation there are two areas of partial revelation, in which either asset 1 or asset 2 is revealed in default, as well as an area in which there is full revelation and both assets are revealed in default.

The values of p_1 and p_2 are determined as follows. In equilibrium it has to be the case that

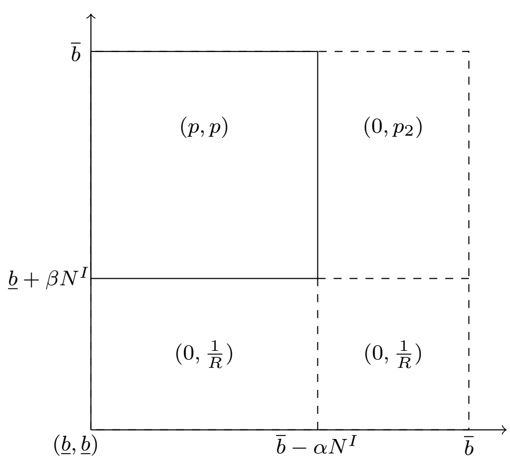
$$(1 - f_i)\left(\frac{\gamma}{p_i} + \beta\omega\delta W^U\right) = \gamma R + f_i \beta\omega\delta W^U.$$



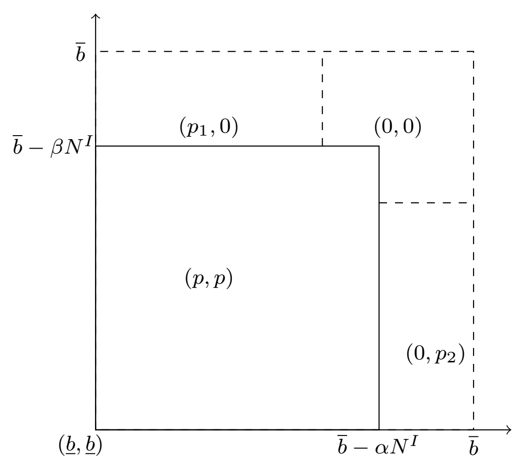
(a) $\chi = (0,0)$



(b) $\chi = (0,1)$



(c) $\chi = (1,0)$



(d) $\chi = (1,1)$

Figure 1.1: Price function at (χ_1,χ_2)

Observe now that

$$f_1 = \Pr\left(\chi_1 = 1|\left(p_1, 0\right)\right) = \frac{\Pr\left(\chi_1 = 1|\left(p_1, 0\right)\right)}{\Pr\left(\chi_1 = 1|\left(p_1, 0\right)\right) + \Pr\left(\chi_1 = 0|\left(p_1, 0\right)\right)}$$
$$= \frac{q_1\left(1-\beta r\right)\left(1-r\right)}{q_1\left(1-\beta r\right)\left(1-r\right) + \left(1-q_1\right)\left(1-\beta r\right)\left(1-r\right)} = q_1.$$

A similar reasoning establishes that $f_2 = q_2$.

The demand functions.
The demand functions for informed managers supporting this equilibrium are the ones described in Proposition 5. For uninformed managers the demand is described as follows:

- when the prices are (p, p) or $\left(\frac{1}{R}, \frac{1}{R}\right)$the demand is $d^y = (1, 1, 1)$;
- when the prices are $(p_1, 0)$ or $\left(\frac{1}{R}, 0\right)$ the demand is $d^y = (1, 1, 0)$;
- when the prices are $(0, p_2)$ or $\left(0, \frac{1}{R}\right)$ the demand is $d^y = (1, 0, 1)$;
- for all other pairs $(\widehat{p}_1, \widehat{p}_2)$ the demand is $d^y = (1, 0, 0)$.

The optimality of the demand functions for the informed is immediate. For the uninformed, given the equilibrium price function above optimality is clear when prices are observed in equilibrium. If a pair $(\widehat{p}_1, \widehat{p}_2)$ is not observed in equilibrium we specify that uninformed managers assume that the risky assets are defaulting and demand the riskless asset only.

The feasible allocation rule.
If d^y has a single non-zero element i then feasibility requires $x_i = 1$ and $x_j = 0$ for $j \neq i$. Suppose d^y has multiple non-zero elements and enumerate the elements as $\{i_1, \ldots, i_m\}$. Then x_{i_k} is defined recursively as

- $x_{i_m} = \frac{b_{i_m} - \xi_{i_m}}{D_{i_m} - \xi_{i_m}}$;
- $x_{i_k} = \min\left\{\frac{b_{i_k} - \xi_{i_k}}{D_{i_k} - \xi_{i_k}}, 1 - \sum_{j=k+1}^{m} x_{i_j}\right\}$;
- $x_{i_1} = 1 - \sum_{j=2}^{m} x_{i_j}$.

To see how the allocation rule works, consider an equilibrium in which $\beta = 1-\alpha$. If $\chi = (0,0)$ and the realized price pair is (p,p) then all uninformed traders have a demand vector $(1,1,1)$, while a fraction αN^I of informed managers has demand vector $(0,1,0)$ and a fraction $(1-\alpha) N^I$ has demand vector $(0,0,1)$. We therefore have $\xi_1 = \alpha N^I$, $\xi_2 = (1-\alpha) N^I$, $D_1 = N^U + \alpha N^I$ and $D_2 = N^U + (1-\alpha) N^I$. The amount of asset 2 allocated is

$$N^U \frac{b_2 - (1-\alpha) N^I}{N^U} + (1-\alpha) N^I = b_2,$$

so that all the supply is allocated. For asset 1 we can check that

$$\frac{b_1 - \xi_1}{D_1 - \xi_1} < 1 - \frac{b_2 - \xi_2}{D_2 - \xi_2}$$

so that the quantity allocated is

$$N^U \frac{b_1 - \alpha N^I}{N^U} + \alpha N^I = b_1,$$

and again al the supply is allocated. The rest of the uninformed managers receive the riskless asset. Other cases are treated similarly.

Proof of Proposition 7. Let η_t be the probability that a manager is informed at time t and let $\zeta_t(b,\chi)$ be the equilibrium probability that an uninformed manager is fired at time t when the realization of the random variable is (b,χ). The probability is zero when (b,χ) is such that the price vector is fully revealing and it is strictly positive otherwise.

In a stationary equilibrium an investor is separated from an informed manager only for exogenous reasons. Thus, the mass of informed managers losing the job in every period is

$$A^I = (1-\delta) N^I \tag{1.24}$$

and it does not depend on t or the realization (b,χ). The measure of uninformed managers who lose their job at period t depends on (b_t,χ_t) and it is given by

$$A_t^U(b_t,\chi_t) = ((1-\delta) + \delta\zeta_t(b_t,\chi_t)) N^U \tag{1.25}$$

Thus at any given time t the mass of available positions is

$$A_t(b_t,\chi_t) = (1-\delta) N^I + ((1-\delta) + \delta\zeta_t(b_t,\chi_t)) N^U \tag{1.26}$$

To maintain the values N^I and N^U constant, the mass of informed managers who lose the job must be replaced by an equal mass of informed managers who are hired, and the same is true for uninformed managers. Thus we must have

$$\mu^* Z^I = A^I$$
$$\mu^* Z_t^U (b_t, \chi_t) = A_t^U (b_t, \chi_t)$$

which yields

$$\frac{Z_t^U (b_t, \chi_t)}{Z^I} = \frac{A_t^U (b_t, \chi_t)}{A^I} = \frac{((1-\delta) + \delta \zeta_t (b_t, \chi_t)) N^U}{(1-\delta) N^I}.$$

The probability of getting an informed manager at time t is

$$\epsilon_t = \frac{Z^I}{Z^I + Z_t^U}$$

The equilibrium condition is therefore that it is always the case, for each possible history, that the belief η_t following a history in which the manager picked the 'right' investment and was not revealed uninformed by the exogenous signal is such that $\eta_t \geq \epsilon_t$, i.e. the probability of being informed assigned to a manager making no mistakes is higher than the probability of hiring an informed manager on the labor market.

In equilibrium, the Bayes rule implies

$$\eta_{t+1} = \frac{\eta_t}{\eta_t + (1-\zeta_t)(1-\eta_t)} \tag{1.27}$$

for a manager who has not been hired at the end of period t, where the dependence of ζ_t on (b_t, χ_t) has been omitted for simplicity. For a newly hired manager we have $\eta_t = \epsilon_{t-1}$. By (1.27) we have

$$\eta_{t+1} = \frac{\epsilon_{t-1}}{\epsilon_{t-1} + (1-\zeta_t)(1-\epsilon_{t-1})}.$$

We first prove that $\eta_{t+1} > \epsilon_t$. Note that this is equivalent to proving

$$\frac{1-\epsilon_t}{\epsilon_t} > \frac{1-\epsilon_{t-1}}{\epsilon_{t-1}} (1-\zeta_t) \tag{1.28}$$

Since $\epsilon_t = Z^I / \left(Z^I + Z_t^U\right)$ we have,

$$\frac{1-\epsilon_t}{\epsilon_t} = \frac{Z_t^U}{Z^I} = \frac{\left((1-\delta)+\delta\zeta_t\right) N^U}{(1-\delta)\, N^I}$$

Using an analogous expression for $\frac{1-\epsilon_{t-1}}{\epsilon_{t-1}}$ we can write inequality (1.28) as

$$\zeta_t > \delta\zeta_{t-1}\left(1-\zeta_t\right).$$

Since $\zeta_{t-1} \leq 1$, a sufficient condition for the inequality to be satisfied is $\zeta_t > \delta\left(1-\zeta_t\right)$, or $\zeta_t > \frac{\delta}{1+\delta}$. Since $\zeta_t^U \geq 1-\omega$, the inequality is satisfied by Assumption 4, and therefore $\eta_{t+1} > \epsilon_t$.

When the manager has been employed for more than one period the reasoning is similar. Suppose the manager was hired at the end of $t'-1$ and has not made any mistake and σ_t^y for $t \geq t'$ has been always 0. This implies that the updated belief of the investor at the end of any time period t is not less than his belief at the beginning of t for $t \geq t'$ and therefore it is also greater than his initial belief at t'. That is

$$\eta_{t+1} = \frac{\eta_t}{\eta_t + (1-\zeta_t^U)(1-\eta_t)} \geq \eta_{t'} = \frac{\epsilon_{t'-1}}{\epsilon_{t'-1} + (1-\zeta_{t'}^U)(1-\epsilon_{t'-1})}$$

for any $t \geq t'$. Hence, a sufficient condition for $\eta_{t+1} > \epsilon_t$ is having

$$\frac{\epsilon_{t'-1}}{\epsilon_{t'-1} + (1-\zeta_{t'}^U)(1-\epsilon_{t'-1})} \geq \epsilon_t$$

But by the same argument, this inequality holds by assumption 4.

Chapter 2

Price Co-Movements, Heterogeneous Funds and Reputational Concerns

2.1 Introduction

According to New York Stock Exchange Factbook, in 2003 institutional investors held almost 50% of corporate equity in NYSE. In 1950, this number was only 7%. Investors reward fund managers according to some measure of their success in generating returns and withdraw their funds if they deem the manager incompetent and unsuccessful. So the managers incentives are two fold; they want to maximize the return on their portfolio and build up a reputation for themselves as competent managers. There is a growing literature on the general equilibrium implications of institutional trading that discusses the price distortions generated by the incentives of fund managers.[1]

Alongside the shift from individual investors to institutional investors, there have been episodes of the spread of financial crisis between emerging markets that had no common fundamentals. A good example is the 1998 Russian Flu that spread to Brazil. The common notion in the literature about these episodes has been multiplicity of equilibrium due to financial vulnerability and market incompleteness. Interestingly, the affected markets were all populated by institutional investors such as global hedge funds. At

[1]For example, Cuoco and Kaniel (2006), Dasgupta and Prat (2008), Basak and Pavlova (2011)

the same time, there has also been a rise in interdependence and co-movement between stock prices all over the world that is not explained by common fundamentals, global shocks and changes in volatility.[2]

Our aim is to address the equilibrium consequences of having specialized and global investment funds, delegating the investment decision to reputationally motivated managers for price co-movement between fundamentally independent markets. Our key assumption is the asymmetric information among managers. Managers can be *informed* of the true state of the assets or *uninformed*. We show that in any equilibrium of the model, prices co-move with each other following any shock to the prior beliefs about the markets. Our model builds on Sami and Brusco (2014) and Guerrieri and Kondor (2012). There are two fundamentally independent risky assets and a risk less bond. We have three types of funds; specialized in market one, specialized in market two, and global. Risk neutral fund managers are either informed or uninformed and are hired to invest the money of risk neutral investors. Types of funds are observable but types of managers are private information. Also, there are independent masses of liquidity traders at each risky asset market. Managers are paid a fixed share of return they have made and are retained by the funds if they have made the highest possible return feasible for them. This means that specialized funds retain the manager if he has bought the repaying asset or the risk-less bond when the asset defaults. However, global funds retain their managers if they have bought the repaying risky asset with the lowest price or risk free bond when both assets default.

As in Sami and Brusco (2014), we consider partially revealing rational expectation equilibria. We focus on equilibria in which if p^* is an equilibrium price at a certain value of the liquidity and return shock realization, and at p^* the excess demand is identical for another shock realization, then the equilibrium price must be the same.

If asset i repays, p_i reveals it's repaying when it is equal to $\frac{1}{R}$, where R is the return on riskless bond with a price normalized to 1. If asset i defaults, p_i reveals the default if it is less than or equal to $\overline{p}_i$ which clears the market with the demands of liquidity traders[3]. In Sami and Brusco (2014), we showed

[2]Forbes (2012) surveys empirical and theoretical literature on contagion and documents significant rise in co-movement between stocks within advanced countries, Euro region and all over the world controlling for global shocks and changes in volatility. Anton and Polk (2014) identifies a significant increase in the return co-movement of the stocks held by the same mutual funds.

[3]We will later on solve for $\overline{p}_i$ in equilibrium.

that there is no equilibrium at which prices don't reveal any information about the true state of the assets. Besides, when all the funds are global, there is no partially revealing equilibrium with different unrevealing prices, i.e., in any equilibrium, unrevealing prices must be equal. In this paper, I first prove that as long as there are global funds in the market, prices are interdependent in any equilibrium. Consequently, interdependent prices co-move with each other following any shock to the priors on the assets. The result is obtained despite the fact that all agents are risk-neutral. This co-movement is magnified by reputational concerns of managers but does not go away if there is no reputational concern. Moreover, we show that when there are heterogeneous funds, we have both types of equilibria, equilibria with equal and unequal unrevealing prices. The analysis in Sami and Brusco (2014) was only limited to one type of equilibria-*simple equilibria*- while in this paper we characterize both simple and non-simple equilibria.[4]

The mechanism that generates the interdependence and co-movement relies on two sources, the information asymmetry between fund managers and the reputational concerns of uninformed fund managers facing the threat of dismissal by funds. Informed managers are perfectly informed and have strict demands for the repaying asset. I also assume that there are very few informed managers and a lot of uninformed managers in the market so that the demands of informed manages can't clear the market so uninformed managers must have positive demand for the assets for the market to get cleared. Now, imagine that the risky assets are Russian bond and Brazilian bond. Suppose that Russian bond defaults and Brazilian bond repays. Global informed managers know this and they all demand Brazilian bond. But then the probability that uninformed managers receive Brazilian bond is less than receiving Russian bond simply because all the informed managers demand Brazilian bond. This shows a couple of things. First uniformed managers face an adverse selection problem, with higher probability they receive the defaulting bond(Russian bond). Second, the probability of receiving Brazilian bond depends on the state of the Russian bond and decreases with the increase in the default probability of Russian bond. So uninformed managers demand the bonds if prices compensate them for this adverse selection problem. Also, prices must co-move with each other following any change in ex-ante default probability of one of the bonds. To see this, suppose that the ex-ante default probability of the Russian bond increases. Clearly, price

[4] An equilibrium is simple if only one unrevealing price vector occurs in equilibrium.

of Russian bond suffers. But at the same time, uninformed managers would rationally believe that if Brazilian bond has repaid, the probability of receiving it is now even less. So to compensate the uninformed for the rise in the risk of not receiving the repaying Brazilian bond, the price of Brazilian bond must also go down.

Since I assume that the total mass of informed managers and liquidity traders is never enough to clear the markets, unrevealing prices are clearing the markets only if there is a positive demand from uninformed managers. Uninformed managers have positive demand for risky assets if prices compensate them for the risk of being dismissed. This means that they ask for premia over the return of the risk free bond which are not independent of each other. This premia increases the price co-movements, however, the co-movement doesn't disappear if there is no reputational concern and the premium is zero. In other words, if instead of delegating the investment, investors directly invest in the markets, uninformed traders face the same signal extraction problem of uninformed managers. They have to learn the signals of informed traders from prices. Since prices reflect the signals of informed global traders, they are interdependent and co-move with each other following any change in the ex-ante probabilities of the default of any asset.

Literature Review. This paper is an extension of Guerrieri and Kondor (2012). In a model with only one risky asset, one risk free bond and one type of investors, they show that the reputationally concerned managers distort the price of the risky asset by asking a premium over the risk free bond that compensates them for the risk of getting fired and makes the price more volatile. Our paper contributes to the literature on general equilibrium models of contagion with information asymmetry and delegation. The closest models to ours are the models that discuss information channels of contagion and the contagion due to delegation. Calvo (1999) has a rational expectation model at which uninformed traders see the actions of informed traders but face a signal extraction problem; when informed traders don't buy an asset, uninformed traders don't know if this is because of a negative idiosyncratic shock to their demand or it is because of a negative shock to the valuation of the assets. Thus, when the volatility of the returns in emerging markets are relatively higher than the volatility of the idiosyncratic shocks, following a negative shock to one market uninformed traders attach higher probability to the low return for other market as well. The main difference between Calvo (1999) and us is the pricing mechanism; at his model uniformed traders first observe the actions of informed ones and then choose to buy or sell emerging markets. In our model, all the traders move simultaneously and it's only the price that reveals information to the market.

Chakravorti and Lall (2005) have a general equilibrium model of delegated portfolio management. They have dedicated and opportunist managers. Dedicated managers only invest in emerging markets and are compensated based on the excess return that they make over a benchmark index of emerging markets. Opportunist managers are allowed to short sell and are payed a fixed share of the total return made on the portfolio. They show that price co-movement between emerging markets is the result of the portfolio re-balancing by managers following a shock to one market. Our model differs from them in having asymmetric information as the main source of price co-movement. Dasgupta and Prat (2008) is a sequential trading model with one risky asset that extends Glosten and Milgrom (1985) model by introducing career concerned traders. They show that managers with reputational concerns distort the price so that it never reveals the true state of the asset. Kodres and Pritsker (2002) has a rational expectations model of contagion with asymmetric information where fundamentally unrelated markets can experience contagion due to the cross-market re-balancing. There is no contagion in their model when fundamentals and liquidity shocks are uncorrelated. Finally, our model is related to the big literature on contagion due to herding. In Scharfstein and Stein (1990) managers follow each other to avoid being regarded dumb and share the blame if the things go wrong. In a more recent paper, Wagner (2012b) shows that the threat of dismissal by investors induces the managers to fire sales and run when they suspect others would do the same to avoid selling the assets at lower prices later evenif they are not going to be evaluated in the future.

The rest of this paper is organized as follows. Next section presents the model. In section 3, we characterize the equilibrium. Section 4 contains concluding remarks.

2.2 Model

There are two risky assets and one risk free bond paying $R > 1$. The return on risky asset i at time t is determined by the realization of a random variable $\widetilde{\chi}_{i,t}$ which takes values in the set $\{0, 1\}$. The realization of $\widetilde{\chi}_t = (\widetilde{\chi}_{1,t}, \widetilde{\chi}_{2,t})$ is denoted $\chi_t = (\chi_{1,t}, \chi_{2,t})$. If $\chi_{i,t} = 0$ then the asset repays an amount of 1, while if $\chi_{i,t} = 1$ the asset defaults and pays zero. The random variables

$\{\widetilde{\chi}_{i,t}\}_{t=0}^{\infty}$ are all independent and identically distributed, with $\Pr(\widetilde{\chi}_{i,t} = 1) = q_i$ and $q_2 > q_1$. Furthermore, each $\widetilde{\chi}_{i,t}$ is independent of all variables $\{\widetilde{\chi}_{j,\tau}\}_{\tau=0}^{\infty}$ with $j \neq i$.

Risky assets are sold at prices $p_i \leq \frac{1}{R}$. They are supplied in fixed inelastic amounts of b_1 and b_2. Let $\mathbf{b} = (b_1, b_2)$ be the vector of supply. There is also a perfectly elastic supply of risk free bonds at price $\frac{1}{R}$.

We have three kinds of agents; investors, fund managers and liquidity traders. Investors are endowed with one unit of capital but they can't invest it themselves and have to hire fund managers. Investors are of three types, only investing in asset 1 and bond, I_1, investing in asset 2 and bond, I_2, or investing in both assets and bond, I_3. We assume that the mass of I_j investors is also I_j. We can think of each type of investor as a type of fund. Fund managers are also of two types; informed (I) and uninformed (U). Informed managers observe the realizations of $\widetilde{\chi}_{i,t}$ for $i = 1, 2$. Uninformed managers only observe prices of the assets. The types of investors are observable while the types of managers are private information.

The mass of informed managers(M^I) is less than the mass of any fund I_j. Liquidity traders are only demanding risky assets for random reasons. Let y_1 and y_2 be the masses of liquidity traders at each asset market. We assume that y_is are independently and identically distributed according to the uniform distribution over $[\underline{y}, \overline{y}]$.

At the beginning of each day, funds with no manager are randomly matched with a manager in the unemployment pool. We assume that funds looking for a manager are not observing the previous history of any employment of the managers in the unemployment pool. Funds offer the matched manager a contract that pays a fixed share of return γ, and retains him only if the manager has made the highest possible return. We will discuss the asset and labor markets in detail later but before that we present the time line of the model.

2.2.1 Timing

The timeline of the model is as follows;

- In the morning
    - Unemployed managers decide to pay the search cost κ and enter the unemployment pool or stay out of market.

- Funds with no manager randomly pick a fund manager from unemployment pool.
- Informed managers observe the realization of return shocks $\chi_{\mathbf{t}}$.
- Managers choose their demand of the assets and the bond.
- Equilibrium prices $p_t = (p_{1t}, p_{2t})$ are determined and the assets are allocated.

- In the evening,
  - $\chi_{\mathbf{t}}$ is publicly observed and the investments of the managers are realized by their investors.
  - Managers receive a share γ of the returns.
  - Any fund receives an exogenous binary signal, σ_t^l, about the type of manager l. If the manager is informed, then σ_t^l is always zero. Otherwise, $\sigma_t^l = 0$ with probability ω and $\sigma_t^l = 1$ with probability $1 - \omega$.
  - Funds decide to fire or retain their managers.
  - With probability $1 - \delta$ any manager is exogenously separated from the job.

2.2.2 Labor Market

To hire a manager each fund randomly picks a manager from the pool of unemployed managers Let $Z_t = Z_t^I + Z_t^U$ be the total mass of unemployed managers of both types and A_t the mass of funds looking for a manager at any time t. Also define μ_t as the probability of matching. Since funds and managers are matched randomly the probability that a manager is matched is:

$$\mu_t = \frac{\min\{A_t, Z_t\}}{Z_t} \tag{2.1}$$

Clearly, funds decision to fire or retain any manager after observing managers returns depends on the matching probability μ_t, the fraction of informed unemployed managers out of all unemployed managers $\frac{Z_t^I}{Z_t}$, and their updated probability about the managers competence. Let N_j^i be the set of the managers of type $i = I, U$ hired by the funds of type I_j. Let also

$\phi_j(\theta_j^q, \sigma^q, p_j, \chi_j) \in \{0,1\}$ denote the retention decision of fund I_j after observing the investment decision θ_j^q of the manager q, the exogenous separation signal σ^q, equilibrium price(s) and the true value of the asset(s) χ_j. By the same argument, let $\phi_3(\theta_3^q, \sigma^q, \mathbf{p}, \chi) \in \{0,1\}$ be the firing decision for I_3 funds. Then, $\phi_j = 0$ if the manager is retained and $\phi_j = 1$, otherwise.

The investments of managers in $I_1(I_2)$ funds are successful if they buy risky asset 1(2) when it repays and buy risk free bond when it defaults. For managers in I_3 funds, the investment is successful whenever they buy risk free bond when both assets default or they buy the cheapest asset that is repaying.

2.2.3 Asset Markets

Each manager submits a demand schedule. Managers in I_1 and I_2 funds can demand risk free bond, the risky asset the fund specializes in, or state indifference between them. Managers hired by I_3 funds can demand each of the risky assets, risk free bond or be indifferent for a subset of assets. The auctioneer collects the demand schedules, sets the market clearing prices and allocates the assets to managers and liquidity traders. Given the submitted demands of managers, the auctioneer first assigns the managers with the strict demand of asset 1, asset 2 or risk free bond and then assigns to the managers stating indifference between the investment opportunities at prices that clear markets.

N_1^I and N_2^I managers submit the demand schedules $d_j^I(p_j|\chi_j) : [0, \frac{1}{R}] \times \{0,1\} \to \{0,1\}^2$, $j = 1,2$ to the auctioneer. If $d_1^I = (0,1)$ for some χ_1 and p_1, then the manager demands no bond and $1/p_1$ units of risky asset 1 while $d_1^I = (1,1)$ means that the manager is indifferent between 1 unit of bond or $1/p_1$ units of risky asset. Given $\chi = (\chi_1, \chi_2)$ and $\mathbf{p} = (p_1, p_2)$, N_3^I managers submit $d_3^I(\mathbf{p}|\chi) : [0, \frac{1}{R}]^2 \times \{0,1\}^2 \to \{0,1\}^3$ to the auctioneer. Finally, uninformed managers hired at I_j funds, N_j^U, have no private signal so when hired by I_1 or I_2 funds, their demand schedules are given by $d_j^U(p_j) : [0, \frac{1}{R}] \to \{0,1\}^3$ where $d_{jk}^U = 0$ for $k \notin \{0, j\}$. If hired by I_3 funds, uninformed managers demand is given by $d_3^U(p_1, p_2) : [0, \frac{1}{R}]^2 \to \{0,1\}^3$. Like managers, liquidity traders are also endowed one unit of capital that they invest it entirely on a risky asset. At any price p_{it} liquidity traders in market i buy $1/p_i$ units of asset i. Throughout the paper, we assume that $b_i > \overline{y}$ so there is always sufficient supply to cover the demands of liquidity traders. Now, assume that asset i is expected to default and the only agents that still demand the auctioneer to assign them asset i are liquidity traders. The auctioneer clears the market by assigning the entire b_i units of asset i to liquidity traders at $p_{it}(y_{it}) = \frac{y_{it}}{b_i}$. Note that $p_{it}(y_{it}) \in [\frac{\underline{y}}{b_i}, \frac{\overline{y}}{b_i}]$. This means that in equilibrium, any price below $\frac{\underline{y}}{b_i}$ automatically reveals that the asset is defaulting. From this point on, let $\overline{p}_i = \frac{\overline{y}}{b_i}$ and $\overline{p} = \max\{\overline{p}_1, \overline{p}_2\}$.

Define W_j^U, $j = 1, 2, 3$, as the continuation payoff for an uniformed manager of being employed at fund I_j. Also define $v_j^U(k, p_j)$ as the expected payoff of N_j^U manager, $j = 1, 2$, buying asset $k = 0, j$. We have,

$$v_j^U(k,p) = E[\gamma e_j + (1 - \phi_j(\theta_j^q, \sigma^q, p_j, \chi_j))\beta W_j^U | \mathbf{p^e} = (p_1, p_2)] \tag{2.2}$$

where

$$e_j = \begin{cases} R & \text{if } k = 0 \\ \frac{1-\chi_j}{p_j} & \text{if } k = j \end{cases}$$

Now let $v_3^U(k, p_1, p_2)$ be the payoff of N_3^i manager buying asset $k = 0, 1, 2$. Then,

$$v_3^U(k, p_1, p_2) = E[\gamma e_3 + (1 - \phi_3(\theta_j^q, \sigma^q, \mathbf{p}, \chi))W_3^U | \mathbf{p^e} = (p_1, p_2)] \tag{2.3}$$

where,

$$e_3 = \begin{cases} R & \text{if } k = 0 \\ \frac{1-\chi_1}{p_1} & \text{if } k = 1 \\ \frac{1-\chi_2}{p_2} & \text{if } k = 2 \end{cases}$$

The payoffs for informed managers are defined the same as (2.2) and (2.3) but note that N_1^I and N_2^I managers receive single perfect signals χ_j and their payoff is $v_j^I(p_j|\chi_j)$. N_3^I managers observe $\chi = (\chi_1, \chi_2)$ and hence their payoff is denoted by $v_3^I(\mathbf{p}|\chi)$ and is given by

$$v_3^I(k, \mathbf{p}, \chi)) = E(\gamma r_3 + (1 - \phi_j(\theta, \sigma, p, \chi))\beta W_3^I \mid \chi) \tag{2.4}$$

Define the set of all possible demand vectors as

$$\Delta = \{(d_0, d_1, d_2) | \sum_{i=0}^{2} d_i \geq 1\} \tag{2.5}$$

Let $A(p) = \big(A_1(p), A_2(p)\big)$ be the aggregated demand vector at price p, that is

$$A(p) = \int_{q \in \mathcal{N}} d^q(p) dq \tag{2.6}$$

where $\mathcal{N}$ is the set of all traders. Let $x_k(d^q; A_k) : \Delta \to [0,1]$ where $\sum_{k=0}^{2} x_k d^q = 1$ denotes the feasible allocation to a manager with demand d^q.

We have now defined all the elements of the equilibrium and are ready to define the equilibrium.

Definition 4. *Given any collection of* (N_j^I, N_j^U, W_j^U), $j = 1, 2, 3$, *the rational expectations equilibrium consists an equilibrium price mapping* $\mathbf{p} : \{0,1\}^2 \times [\underline{y}, \overline{y}]^2 \to [\frac{y}{b_1}, \frac{1}{R}] \times [\frac{y}{b_2}, \frac{1}{R}]$; *equilibrium demand schedules* d_j^i, *for* $i = I, U$ *and* $j = 1, 2, 3$; *and feasible allocation mapping* $x_k(d_j^i) \in [0,1]$, *for each asset* k *such that,*

1. *the price vector* $p(\chi, y) = (p_1(\chi, y), p_2(\chi, y))$ *clears the markets. That is, for asset* $k = 1, 2$,

$$\int_{q \in N_k^I} x_k(\hat{d}^q)\hat{d}_k^q dq + \int_{q \in N_3^I} x_k(\hat{d}^q)\hat{d}_k^q dq + \int_{q \in N_k^U} x_k(\hat{d}^q)\hat{d}_k^q dq + \int_{q \in N_3^U} x_k(\hat{d}^q)\hat{d}_k^q dq = p_k b_k - y_k \qquad (2.7)$$

   where $\hat{d}^q = d_j^i(p, .)$.

2. *the demand schedules of* N_j^U *managers are optimal given* $p(\chi, y)$. *That is, if* $d_{kj}^i = 1$ *then* $v_j^U(k, p) \geq v_j^U(k', p)$ *for all* $k' \neq k$.

3. *the demand schedules of* N_j^I, $j = 1, 2$, *and* N_3^I *managers are optimal given* $p(\chi, y)$, *i.e.,* $d_j^I(p_j|\chi_j) = 1$ *and* $d_{j3}^I(p|\chi) = 1$ *for* $\chi_j = 0$, *and* $d_j^I(p_j|\chi_j) = 0$ *and* $d_{j3}^I(p_j|\chi_j) = 0$ *for* $\chi_j = 1$; $j = 1, 2$.

Let $D_k(p)$ be the set of all the traders with strict demands for asset k at price p. That is, for $q \in D_k(p)$, $d_k^q = 1$ and $d_j^q = 0$ for $j \neq k$. Let $Z_k(\chi, \mathbf{y})$ be the mass of all d^q, $q \in D_k(p)$ at $(\chi, \mathbf{y})$ That is,

$$Z_k(\chi, \mathbf{y}) = \int_{q \in D_k(p)} d_k^q dq \qquad (2.8)$$

The equilibrium that we construct satisfies the following *belief consistency* condition.

Definition 5. *Let* $\mathbf{p^e}$ *be a rational expectations equilibrium price mapping and* $\mathbf{p^e}(\chi, \mathbf{y})$ *be the equilibrium price vector at* $(\chi, \mathbf{y})$. *Also assume that there exists* $(\chi', \mathbf{y}')$ *such that* $\mathbf{Z}(\chi, \mathbf{y}) = \mathbf{Z}(\chi', \mathbf{y}')$. *Then,* $\mathbf{p^e}$ *is* ***belief consistent*** *if* $\mathbf{p^e}(\chi, \mathbf{y}) = \mathbf{p^e}(\chi', \mathbf{y}')$.

Definition 5 restricts the set of equilibria to the partially revealing equilibria. If the equilibrium price vector is belief consistent there are some values of $\mathbf{y}$ that price vector is not revealing χ. Hence, the equilibrium price vector that always reveals the repay or default of the assets is not belief consistent.

Before moving on to the next section, we introduce another feature of our equilibrium.

Definition 6. *An equilibrium price mapping $p^e(\chi, y)$ is **simple** if there is at most one pair (p_1, p_2) with $p_i \in (\overline{p}, \frac{1}{R})$, $i = 1, 2$, such that $p^e(\chi, y) = (p_1, p_2)$.*

An equilibrium is *non-simple* if the equilibrium price mapping $p^e(\chi, y)$ takes more than one value in $(\overline{p}, \frac{1}{R})^2$.

2.3 Equilibrium

We construct a class of stationary equilibria at which $N^i_{jt} = N^i_j$, $\mu_{jt} = \mu_j$, and $W^U_{jt} = W^U_j$. In none of these equilibria prices are fully revealing, so funds have higher expected payoff if they have an informed manager. This suggests that reputation is valuable for uninformed managers as well, because any mistake leads to dismissal and loss of W^U_j. Hence, from this point on we take $(\{N^I_j, N^U_j, W^U_j, \mu_j\}_{j=1,2,3})$ as given and discuss the existence and properties of the rational expectations equilibrium at asset markets. We also assume that it is optimal for the funds to fire any manager who hasn't made the highest possible return. Later on we solve for equilibrium N^i_j and W^U_j and prove the optimality of the firing rule.

The existence of this class of equilibria is guaranteed under the following assumptions;

$$M^I < \min\{\overline{y}, \overline{y} - \underline{y}\} \quad , \quad M^I + \overline{y} < \underline{C} \quad , \quad \frac{\max\{b_1, b_2\}}{R} < \underline{y} + \min\{I_1, I_2, I_3\} \tag{2.9}$$

where $\underline{C}$ is given in the Appendix. The first part ensures that the mass of informed managers is small relative to noise traders, making the equilibrium not always fully revealing. The second part ensures that the total investments of informed managers and noise traders are never enough to clear the markets so there is always some amount of each asset that is allocated to uninformed managers. However, by the third part the supply is never enough to allocate risky assets to all uninformed managers. Thus, in equilibrium uninformed managers are always indifferent between risky asset(s) and riskfree bond.

- $\omega > \frac{1}{1+\delta}$

This assumption is identical to the assumption made by Guerrieri and Kondor (2012). After observing a right decision by a fund manager at the end of each day, funds attach a higher probability to the event that the manager is informed than uninformed. This assumption ensures that in equilibrium, the beliefs of funds about successful managers grows at the high enough speed so that they are retained after a right decision.

- $\kappa < \gamma R$

This assumption ensures that the search cost is not more than the expected payoff of getting hired for uninformed managers. It excludes equilibria which all unemployed uninformed managers are matched with probability 1.

Given these assumptions, we discuss how information spreads into the market from the demands of informed managers. Before characterizing the equilibria we need to introduce the concept of marginal traders that is going to play a central role in the characterization of each equilibrium.

2.3.1 Marginal Traders

Definition 7. *Suppose $p^e(\chi, y)$ is an equilibrium price mapping with $p^e(\chi, y) = (p_1, p_2) \in (\overline{p}, \frac{1}{R})^2$ for some (χ, y). Also assume $f_j = Pr(\chi_j = 1 \mid p^e = (p_1, p_2))$. Then, N_j^U managers $j = 1, 2, 3$ are **marginal traders** at $p = (p_1, p_2)$ if*

$$v_j^U(j, p) = v_j^U(0, p)$$

By Definition 7, if N_j^U $j = 1, 2, 3$, are marginal traders at $p = (p_1, p_2)$, their expected payoff of buying asset j is equal to the expected payoff of buying risk free bond. This means that N_j^U are indifferent between asset j and risk free bond at p_j . This also implies that p_j is the maximum price that marginal traders are willing to pay for asset j. At any price above p_j they never demand asset j. Rewriting the condition in Definition 7 for $j = 1, 2$, we have

$$(1 - f_j)(\frac{\gamma}{p_j} + \delta\omega\beta W_j^U) = \gamma R + \delta\omega f_j \beta W_j^U \qquad (2.10)$$

The right hand side of (2.10) is the expected payoff of buying asset j. Recall that uniformed managers in specialized funds are paid γ share of the return and are only retained if they buy risky asset when it repays and riskless bond when risky asset defaults. Thus, their expected payoff of buying asset j is the expected retrun on asset j, $\frac{\gamma(1-f_j)}{p_j}$, plus the expected payoff of being retained. But the probability of being retained for N_j^U when buying asset j is the probability of the repay of asset j, $1-f_j$, times the probability that he is not exogenously separated, δ, times the probability that his type is not revealed ω. The left hand side of (2.10) is the expected payoff of buying the safe asset. Now, the manager buying riskless assset is retained only if risky asset j has defaulted, hence the expected payoff of being retained is $\delta\omega f_j \beta W_j^U$.

Now suppose N_3^U managers are marginal traders at (p_1, p_2) and $p_1 > p_2$. This means that

$$(1-f_1)(\frac{\gamma}{p_1} + \delta\omega f_2 \beta W_3^U) = \gamma R + \delta\omega f_1 f_2 \beta W_3^U \tag{2.11}$$

$$(1-f_2)(\frac{\gamma}{p_2} + \delta\omega \beta W_3^U) = \gamma R + \delta\omega f_1 f_2 \beta W_3^U \tag{2.12}$$

Note that when $p_1 > p_2$, N_3^U managers buying asset 1 are only retained when asset 1 repays and asset 2 defaults, because if asset 2 repays, the return on asset 2 is higher than the return on asset 1 and N_3^U are only retained when they buy the asset that pays the highest return. Therefore, the probability of the retainment for a manager buying asset 1 is $\delta\omega(1-f_1)f_2$. However, the manager that buys asset 2 is retained whenever this asset repays irrespective of the default or repay of asset 1 and his probability of retainment is equal to $\delta\omega(1-f_2)$. Any N_3^U manager who buys risk free bond is only retained if both assets default. This means that the probability of the retainment is $\delta\omega f_1 f_2$. Note that if $p_1 = p_2$, both assets are paying the same return if they repay. Thus, the indifference conditions for N_3^U managers are

$$(1-f_j)(\frac{\gamma}{p_j} + \delta\omega \beta W_3^U) = \gamma R + \delta\omega f_1 f_2 \beta W_3^U \tag{2.13}$$

where $j = 1, 2$.

Note that, N_3^U managers continuation payoff of being employed, W_3^U, is less than the continuation payoff of uninformed managers of specialized funds. This is because if both risky assets repay, N_3^U managers who buy the more expensive asset are fired. Nevertheless, N_j^U ,$j = 1, 2$, managers buying the repaying risky asset are always retained. Notice that if $W_3^U \leq W_j^U$,

$$\gamma R + \delta\omega f_j \beta W_j^U > \gamma R + \delta + \omega f_1 f_2 \beta W_3^U \tag{2.14}$$

If $p = (p_1, p_2)$, $p_1 > p_2$, occurs in equilibrium and marginal traders are N_j^U managers, (2.14) and indifference conditions (2.11)-(2.12) imply that at $p = (p_1, p_2)$ the payoff to N_3^U managers of buying riskless bond is less than the payoff of buying asset j. When N_j^U are marginal traders at $p = (p_1, p_2)$ the maximum price that N_3^U are willing to pay for asset j is always higher than p_j. Therefore N_3^U managers are not marginal traders at $p = (p_1, p_2)$ and strictly demand the cheapest risky asset.

Let P_{jj}^U and P_{j3}^U denote the maximum prices that N_j^U and N_3^U pay for asset j. This means that P_{jj}^U and P_{j3}^U are solved from (2.10) and (2.11)-(2.12) and are given as

$$P_{jj}^U = \frac{\gamma(1 - f_j)}{\gamma R + (2f_j - 1)\delta\omega\beta W_j^U} \tag{2.15}$$

$$P_{j3}^U = \frac{\gamma(1 - f_j)}{\gamma R + (f_1 f_2 - f_j - 1)\delta\omega\beta W_3^U} \tag{2.16}$$

2.3.2 Information Revelation in Asset Market Equilibrium

In equilibrium, price is a mapping from the space of stochastic shocks $(\chi_1, \chi_2) \times (y_1, y_2)$ to the interval $[\frac{\underline{y}}{b_1}, \frac{1}{R}] \times [\frac{\underline{y}}{b_2}, \frac{1}{R}]$. Clearly, the inverse mapping $(p^e)^{-1}(p)$ at any $p \in [\frac{\underline{y}}{b_1}, \frac{1}{R}] \times [\frac{\underline{y}}{b_2}, \frac{1}{R}]$ is a subset of $\{0, 1\}^2 \times [\underline{y}, \overline{y}]^2$. So any $p = p^e(\chi, y)$ is in principle revealing information about (χ, y). Now the question is, how much information is revealed at a belief consistent equilibrium? Is there any belief consistent equilibrium at which price does not reveal any information, that is $p^e(\chi, y) = p$ for all (χ, y)? Is there any equilibrium that is fully revealing, i.e, $p_j^e(\chi_j, y) \in \{\frac{1}{R}, \underline{p}_j\}$ for any (χ_j, y_j)? Is there any equilibrium that is revealing for some $(\chi, \mathbf{y})$ and unrevealing for other values of $(\chi, \mathbf{y})$? Before answering these questions, let us first state the following proposition about the properties of the equilibrium price mappings.

Proposition 8. *Any rational expectations equilibrium price mapping* $\mathbf{p}^{\mathbf{e}}(\chi, \mathbf{y})$ *satisfies the following conditions;*

(i). If $p_i^e = \frac{1}{R}$*, then* $\chi_i = 0$.

(ii). If $p_i^e \in [\frac{\underline{y}}{b_i}, \frac{\overline{y}}{b_i}]$*, then* $\chi_i = 1$.

The proof of the above result is very simple and is omitted. If there is any equilibrium at which $p_i^e = \frac{1}{R}$ when $\chi_i = 1$, there would be no demand from informed or uninformed managers to buy asset i and only noise traders demand asset i at $\frac{1}{R}$. But then to clear the market p_i^e must be $\frac{y_i}{b_i}$ and not $\frac{1}{R}$.

Proposition 9. *Under assumption* (2.9),

1. *There is no belief consistent unrevealing equilibrium.*
2. *Suppose* $\frac{\max\{b_1,b_2\}}{R} < \underline{y} + \min\{I_1, I_2\}$. *There exists a revealing equilibrium. This equilibrium is not belief consistent.*

These results are similar to the results in Sami and Brusco (2014) and their proof is presented in the on-line appendix of the paper.

We know by the above proposition that none of the two extremes, no revelation and full revelation are possible or plausible. So the equilibrium must always be partially revealing; something is always leaked to the market. Indeed, this is the case in the base line model of Guerrieri and Kondor (2012). We show that there exist simple and non-simple partially revealing equilibria with a common property; when prices are not fully revealing they are *interdependent.*

Definition 8. *Suppose* $\mathbf{p^e}(\chi, \mathbf{y}) = (p_1^e(\chi, \mathbf{y}), p_2^e(\chi, \mathbf{y}))$ *is an equilibrium price mapping. Then* $p_1^e(\chi, \mathbf{y})$ *and* $p_2^e(\chi, \mathbf{y})$ *are* ***interdependent*** *if there is at least one pair* $(p_1, p_2) \in (\overline{p}, \frac{1}{R})^2$; $p_i^e(\chi, \mathbf{y}) = p_i$ *for some* $(\chi, \mathbf{y})$ *such that* $Pr(p_1^e = p_1, p_2^e = p_2 \mid \chi_1, \chi_2) \neq Pr(p_1^e = p_1 \mid \chi_1) Pr(p_2^e = p_2 \mid \chi_2)$.

We call $p_1^e(\chi, \mathbf{y})$ and $p_2^e(\chi, \mathbf{y})$ independent if they aren't interdependent. To understand Definition 8, suppose $\mathbf{p^e}(\chi, \mathbf{y})$ is a simple equilibrium price function where $p_1^e(\chi, \mathbf{y})$ and $p_2^e(\chi, \mathbf{y})$ are independent. Also assume that all the funds are global and there is no specialized fund. Let N^I be the mass of informed managers. Suppose $\mathbf{p^e}(\chi, \mathbf{y}) = (p_1, p_2) \in (\overline{p}, \frac{1}{R})^2$ for the following values of $(\chi, \mathbf{y})$;

- $(\chi_1, \chi_2) = (0, 0)$ and $(y_1, y_2) \in [\underline{y}, \overline{y} - N^I]^2$
- $(\chi_1, \chi_2) = (0, 1)$ and $(y_1, y_2) \in [\underline{y}, \overline{y} - N^I] \times [\underline{y} + N^I, \overline{y}]$.
- $(\chi_1, \chi_2) = (1, 0)$ and $(y_1, y_2) \in [\underline{y} + N^I, \overline{y}] \times [\underline{y}, \overline{y} - N^I]$.
- $(\chi_1, \chi_2) = (1, 1)$ and $(y_1, y_2) \in [\underline{y} + N^I, \overline{y}]^2$

Suppose, $(\chi_1, \chi_2) = (0, 1)$. The probability of $\mathbf{p^e}(\chi, \mathbf{y}) = (p_1, p_2)$ is equivalent to the probability that $(y_1, y_2) \in [\underline{y}, \overline{y} - N^I] \times [\underline{y} + N^I, \overline{y}]$ and is equal to $(1 - \frac{N^I}{\overline{y} - \underline{y}})^2$. But note that $Pr\big(p_1^e(\chi, \mathbf{y}) = p_1, p_2^e(\chi, \mathbf{y}) = p_2 \mid \chi_1, \chi_2\big)$ is equal to $(1 - \frac{N^I}{\overline{y} - \underline{y}})^2$ for any (χ_1, χ_2). Furthermore, $Pr\big(p_i^e(\chi, \mathbf{y}) = p_i \mid \chi_i, \chi_j, p_j^e = p_j\big) = 1 - \frac{N^I}{\overline{y} - \underline{y}}$ for any value of χ_i, χ_j and $p_j \in (\overline{p}, \frac{1}{R})$. Hence, by Definition 8, p_1^e and p_2^e are independent of each other.

When prices are independent, conditional on χ_1 the probability of p_1^e being unrevealing is independent of p_2^e and χ_2. Therefore, if p_1^e is unrevealing, uninformed managers at all funds know that price of asset 2 reveals nothing about the state of asset 1.

Observing $p_1^e(\chi, \mathbf{y}) = p_1$ and $p_2^e(\chi, \mathbf{y}) = p_2$, uninformed managers at **all** funds try to figure out the probability of the repayment of the assets by learning the actions of informed managers of **both** specialized and global funds. As long as the mass of I_3 funds is non-zero, uninformed managers form their posteriors about asset i taking into account both p_i and p_j. Recall that $1 - f_j(p_1, p_2) \equiv Pr(\chi_j = 0 | \underline{p} < p_1 < \frac{1}{R}, \underline{p} < p_2 < \frac{1}{R})$, i.e., $1 - f_j(p_1, p_2)$ is the posterior of the uninformed managers funds after observing a price pair (p_1, p_2). Next result shows that prices are interdependent in any equilibrium.

Proposition 10. *As long as there are some global funds in the market, prices are interdependent in any equilibrium.*

To understand the intuition behind this result, suppose asset 2 defaults and p_2 is unrevealing. When asset 1 repays, informed managers of global funds and informed managers of funds specializing in market 1 are all demanding asset 1. However, informed managers of global funds can demand either asset 1 or asset 2 when both assets repay. So when asset 1 repays and asset 2 defaults the demand for asset 1 is higher than when both assets repay. This implies that the probability that $p_1 = \frac{1}{R}$ and the repay of asset 1 is revealed is higher when asset 2 defaults and p_2 is unrevealing. Therefore, default or repay of asset 2 changes the probability that p_1 is revealing the repay of asset 1. Therefore, p_1 can not be independent of the repay or default of asset 2 and p_2.

But since prices are interdependent by the demands of N_3^I managers, uninformed managers face an adverse selection problem. When prices are unrevealing, uninformed managers receive asset 1 with a lower probability when asset 2 defaults and with a higher probability when asset 2 repays or asset 1 is defaulting. In equilibrium, price of asset 1 must compensate uninformed managers for this adverse selection problem and must decrease following any shock to the ex-ante default probabilities of both asset 1 and asset 2. When q_2 increases, the default of asset 2 is more likely and price of asset 2 suffers. Also, the adverse selection problem in market 1 is more severe because the probability of receiving the repaying asset 1 decreases. Hence, price of asset 1 must also decrease to compensate uninformed managers for the risk of not receiving the repaying asset 1. When there is no global fund, there is no adverse selection problem and there is no co-movement.

As long as there are some global funds and the mass of N_3^I managers is not zero, their trades contain information regarding both assets and in equilibrium market clearing prices reveal this information to all uninformed managers. When there is no I_3 fund- and no N_3^I manager- the price of asset 1 only contains the information revealed by the demands of N_1^I managers. Since N_1^I managers never demand asset 2, p_1 has no information regarding the repay or default of asset 2. When the investment strategy is specialization in one market managers are evaluated only based on the returns of that particular market. But when the investment strategy is to seek investment opportunity in as many markets as possible managers returns are compared with the highest return among all the markets. Therefore, even a small mass of global funds is enough to induce the rest of managers in I_1 (I_2) funds to extract information about market 1 from the actions of N_3^I managers at market 2. Notice that the interdependence is amplified by the continuation payoff of being employed. To see this better, let $\mathbf{p^e}(\chi, \mathbf{y}) = (P_{11}^U, P_{22}^U)$, that is N_1^U and N_2^U managers are marginal traders in equilibrium. This means that

$$P_{jj}^U = \frac{\gamma(1 - f_j)}{\gamma R + (2f_j - 1)\delta\omega\beta W_j^U} \tag{2.17}$$

and

$$\frac{1 - f_j}{P_{jj}^U} - R = (2f_j - 1)\frac{W_j^U}{\gamma} \tag{2.18}$$

This premium is similar to the reputational premium in Guerrieri and Kondor (2012) and disappears as soon as $W_j^U = 0$. However, evenif $W_j^U = 0$, $p_1^e(\chi, y)$ and $p_2^e(\chi, y)$ are still interdependent. This is because at (P_{11}^U, P_{22}^U), uniformed traders face the same signal extraction problem of uniformed managers. Hence, the posteriors of uninformed traders, f_j, are not independent of $p_i^e = P_{ii}^U$ and the repay or default of asset i. Thus, P_{jj}^U are functions of q_1 and q_2 and any change in q_1 and q_2 shifts both P_{11}^U and P_{22}^U. The following Corollary summarizes the discussion.

Corollary 1. *As long as there are some global funds in the market, prices are co-moving in any equilibrium following any shock to priors.*

2.3.3 Simple Equilibria

In this section we characterize some simple partially revealing equilibria. The following Lemma gives the posteriors of uninformed managers at any simple equilibrium with unequal unrevealing prices. In all the following results assume $r_j = \frac{N_j^I}{\overline{y}-\underline{y}}$.

Lemma 2. *Assume* $\frac{r_3}{1-r_1} < \frac{q_2-q_1}{(1-q_1)q_2}$*. In any simple equilibrium at which with positive probability* $(p_1^e, p_2^e) = (p_1, p_2)$ *where* $\overline{p} < p_2 < p_1 < \frac{1}{R}$*, posteriors of uninformed managers about risky assets at* (p_1, p_2) *are given as*

$$1 - f_1 = \frac{(1-q_1)(1-r_1-q_2r_3)}{1-r_1-(1-q_1)q_2r_3} \tag{2.19}$$

$$1 - f_2 = \frac{(1-q_2)(1-r_1)}{1-r_1-(1-q_1)q_2r_3} \tag{2.20}$$

Next, we derive the posteriors of uninformed managers when unrevealing prices are equal. When unrevealing prices are equal, informed managers of I_3 funds are indifferent between risky assets when both repay. So we can assume that α fraction of them only asks asset 1 and $1-\alpha$ fraction of them asks asset 2 where α is determined in equilibrium so that the posteriors of uninformed managers at $p^e = (p, p)$ are equal. The following Lemma is giving these equal posteriors in an equilibrium with equal prices.

Lemma 3. *Assume* $\frac{r_3}{1-r_1} > \frac{q_2-q_1}{(1-q_1)q_2}$*. In any simple equilibrium at which with positive probability* $(p_1^e, p_2^e) = (p, p)$*;* $\overline{p} < p < \frac{1}{R}$ *is realized the posterior beliefs of uninformed managers about risky assets are equal and given as follows*

$$1 - f_1 = \frac{(1-q_1)(1-r_2-(1-\alpha^*)r_3)(1-r_1-r_3(q_2+(1-q_2)\alpha^*))}{(1-q_1)G_0(r_1,r_2,r_3,\alpha^*)+q_1G_1(r_1,r_2,r_3,\alpha^*)} \tag{2.21}$$

$$1 - f_2 = \frac{(1-q_2)(1-r_1-\alpha^*r_3)(1-r_2-r_3+\alpha^*(1-q_1)r_3)}{(1-q_1)G_0(r_1,r_2,r_3,\alpha^*)+q_1G_1(r_1,r_2,r_3,\alpha^*)} \tag{2.22}$$

where

$$G_0(r_1, r_2, r_3, \alpha^*) = (1 - r_2 - (1-\alpha^*)r_3)(1 - r_1 - r_3(q_2 + (1-q_2)\alpha^*)) \tag{2.23}$$

$$G_1(r_1, r_2, r_3, \alpha^*) = (1 - r_1 - \alpha^* r_3)(1 - r_2 - r_3(1 - q_2 + (1-\alpha^*)q_2)) \tag{2.24}$$

and α^* *is the solution to*

$$\begin{aligned}(1-q_1)(1-r_2-(1-\alpha)r_3)(1-r_1-r_3(1-q_2)\alpha - r_3q_2)\\ -(1-q_2)(1-r_1-\alpha r_3)(1-r_2-r_3+\alpha(1-q_1)r_3) = 0\end{aligned} \tag{2.25}$$

Lemmas 2 and 3 show clearly that at any non-revealing price, the posteriors of uninformed managers are not the same as their priors. The difference between f_j and q_j is the information leaked to the market at any equilibrium. Note that Lemmas 2 and 3 put mutually exclusive conditions on $\frac{r_3}{1-r_1}$ so the posteriors of uninformed managers are always well defined for any value of N_1^U and N_3^U.

In Sami and Brusco (2014), we proved that as long as there are no specialized funds in the market, there is no equilibrium with unequal unrevealing prices. Next proposition shows the existence of such equilibrium at this model. In this equilibrium, the information that is revealed to the market is not enough to convince uniformed managers that the probability of the repay of asset 2 is as high as asset 1.

Proposition 11. *Assume* $\frac{r_3}{1-r_1} < \frac{q_2-q_1}{(1-q_1)q_2}$ *and*

(i). $I_3 < \underline{C} - M^I - \overline{y}$

(ii). $\min\{I_1, I_2\} > \overline{C} + M^I$

where $\underline{C}$ *and* $\overline{C}$ *are given in the Appendix. There exists an equilibrium at which* $p^e(\chi, y)$ *takes the following values;*

- *some revelation;*

$$p_j^e = P_{jj}^U = \frac{\gamma(1-f_j)}{\gamma R + (2f_j - 1)\delta\omega\beta W_j^U} \tag{2.26}$$

where $1 - f_1$ *and* $1 - f_2$ *are given in Lemma 2.*

- *partial revelation at which either $p_j^e \leq \overline{p}_j$ or $p_j^e = \frac{1}{R}$.*

$$p_i^e = \frac{\gamma(1-q_i)}{\gamma R + (2q_i - 1)\delta\omega\beta W_i^U} \tag{2.27}$$

- *full revelation at which for each $i = 1, 2$ we have either $p_i \leq \overline{p}_i$ or $p_i = \frac{1}{R}$.*

Moreover, $\underline{p} < P_{22}^U < P_{11}^U < \frac{1}{R}$ and marginal traders at (P_{11}^U, P_{22}^U) are uninformed managers of specialized funds.

When there are no specialized funds, the demands for any asset is only coming from the managers of global funds. At $p_1 > p_2$, uninformed managers are marginal traders; and again face the same adverse selection problem; the probability of receiving asset 1 when it defaults or asset 2 repays is higher than the probability of receiving it when it repays and asset 2 defaults. Therefore, an equilibrium with unequal non-revealing prices exists only if we have enough specialized funds in the market so that the demands of N_j^U managers can clear the market and make them marginal traders.

When the informed managers hired at I_3 funds are relatively less than the informed managers hired at I_1 funds (so that $\frac{r_3}{1-r_1} < \frac{q_2-q_1}{(1-q_1)q_2}$), the investments of N_3^I managers are not enough to change the prior belief of uninformed managers about asset 1 being the highest repaying asset. Moreover, assumptions (i) and (ii) imply that $I_3 < \min\{I_1, I_2\}$. This means that the total investments made by global funds is less than the investments of specialized funds. When the total investments of global funds are low, markets are cleared only if there is positive demands from specialized funds and equilibrium prices are set by uninformed managers of specialized funds. Thus, prices do not contain as much information as they would if there were more informed managers hired at I_3 funds and the size of I_3 funds were.

When marginal traders are uninformed managers of specialized funds and $p_j^e = P_{jj}^U$, maximum prices that N_3^U managers pay are solved from equations (2.11)-(2.12). Note that N_3^U managers continuation payoff of being employed , W_3^U, is less than the continuation payoff of uninformed managers of specialized funds. This is because if both risky assets repay, N_3^U managers who buy the more expensive asset are fired. Nevertheless, N_j^U ,$j = 1, 2$, managers buying the repaying risky asset are always retained. Therefore, when N_j^U managers are marginal traders at p_j^e their payoff of buying risk less bond is $\gamma R + \delta\omega f_j \beta W_j^U$ while the payoff to N_3^U managers of buying risk free bond is

$\gamma R + \delta\omega f_1 f_2 W_3^U$. Clearly, at $p_j^e = P_{jj}^U$ the payoff to N_3^U managers of buying riskless bond is less than the payoff of buying asset j, therefore $P_{j3}^U > P_{jj}^U$ and it is not optimal for N_3^U managers to be indifferent between risky assets and risk less bond at $p_j^e = P_{jj}^U$.

The next result characterizes the equilibrium when N_3^I is large. When the size of global funds is large relative to specialized funds, the demands of N_3^U managers clear the market and the only possible equilibrium is the one with equal unrevealing prices. In this equilibrium marginal traders are uninformed managers of global funds. When $p_1^e = p_2^e$ both assets are paying the same expected return, so managers hired at I_3 funds are fired if they buy the defaulting risky asset or risk free bond when at least one of the assets is repaying. Thus, P_{j3}^U now solves

$$(1-f_j)\left(\frac{\gamma}{P_{j3}^U} + \delta\omega\beta W_3^U\right) = \gamma R + \delta\omega f_1 f_2 W_3^U \tag{2.28}$$

where $1-f_1$ and $1-f_2$ are given in Lemma 3. The following proposition summarizes this discussion.

Proposition 12. *Assume that* $\frac{r_3}{1-r_1} > \frac{q_2-q_1}{(1-q_1)q_2}$ *and*

(i). $I_3 > \frac{b_1+b_2}{R} + M^I$.

(ii). $\min\{I_1, I_2\} - M^I > \frac{\max\{b_1,b_2\}}{R}$.

Then, there exists an equilibrium at which $\mathbf{p^e}(\chi, \mathbf{y})$ *takes the following values;*

- *some revelation;*

$$p_j^e = P_{j3}^U = \frac{\gamma(1-f_j)}{\gamma R + (f_1 f_2 - f_j - 1)\delta\omega\beta W_3^U} \tag{2.29}$$

  where $P_{13}^U = P_{23}^U \in (\overline{p}, \frac{1}{R})$*, and* $1-f_1$ *and* $1-f_2$ *are given in Lemma 3.*

- *partial revelation at which either* $p_j^e \leq \overline{p}_j$ *or* $p_j^e = \frac{1}{R}$ *and,*

$$p_i^e = \frac{\gamma(1-q_i)}{\gamma R + (2q_i - 1)\delta\omega\beta W_i^U} \tag{2.30}$$

- *full revelation at which for each* $i = 1, 2$ *we have either* $p_i \leq \overline{p}_i$ *or* $p_i = \frac{1}{R}$.

Moreover, marginal traders at P_{j3}^U *are uninformed managers of global funds.*

2.3.4 Non-Simple Equilibrium

Up to now we just focused on simple equilibria where p^e only gets a unique value in $(\overline{p}, \frac{1}{R})^2$. Suppose $(\chi_1, \chi_2) = (0, 0)$. Suppose also that I_1 and I_3 funds have hired very few informed managers. Then, the mass of informed managers in I_3 funds is not enough to reveal enough information to convince the uninformed managers to bid the same price for both assets in equilibrium. Moreover, for some values of liquidity trading equilibrium prices do not reveal any information to uninformed managers and the posteriors of uninformed managers are the same as their priors, i.e. $1 - f_i = 1 - q_i$. As we discussed in Sami and Brusco (2014) because of adverse selection problem that arises for marginal N_3^U managers when prices are different, the equilibrium only exists if the marginal traders are uninformed managers of specialized funds.

Proposition 13. *Suppose*

$$\frac{(r_1 + r_2 + 3r_3) + (r_2 + r_3)[r_1 - 4(r_1 + r_3)]}{(r_1 + r_2 + 2r_3) - 3(r_1 + r_3)(r_2 + r_3)} < \frac{(1 - q_1)q_2}{q_1(1 - q_2)}$$

Assume also,

(i). $I_3 < \underline{C} - M^I - \overline{y}$.

(ii). $\min\{I_1, I_2\} > \overline{C} + M^I$.

There exists a partially revealing, non-simple equilibrium at which $\mathbf{p^e}(\chi, \mathbf{y})$ *takes the following values;*

- *no revelation;*

$$p_j^e = P_{jj}^U(q_j) = \frac{\gamma(1 - q_j)}{\gamma R + (2q_j - 1)\delta\omega\beta W_j^U} \tag{2.31}$$

  where $j = 1, 2$.

- *some revelation;*

$$p_j^e = P_{jj}^U(f_j) = \frac{\gamma(1 - f_j)}{\gamma R + (2f_j - 1)\delta\omega\beta W_j^U} \tag{2.32}$$

  where $j = 1, 2$*, and* f_j *is given in the appendix.*

- *partial revelation at which either* $p_j^e \leq \overline{p}_j$ *or* $p_j^e = \frac{1}{R}$ *and,*

$$p_i^e = \frac{\gamma(1-q_i)}{\gamma R + (2q_i - 1)\delta\omega\beta W_i^U} \tag{2.33}$$

- *full revelation at which for each* $i = 1, 2$ *we have either* $p_i \leq \overline{p}_i$ *or* $p_i = \frac{1}{R}$.

Moreover, marginal traders at P_{jj}^U *are uninformed managers of specialized funds.*

$p_1^e((\chi, \mathbf{y}))$ and $p_2^e((\chi, \mathbf{y}))$ co-move because at $(P_{11}^U(f_1), P_{22}^U(f_2))$, the probability of $p_j^e = P_{jj}^U$ is not independent of χ_i and $p_i^e = P_{ii}^U$. However, when no information is revealed through prices at $(P_{11}^U(q_1), P_{22}^U(q_2))$, the probability that $p_1^e = P_{11}^U(q_1)$ is independent of the repay or the default of asset 2 and $p^e = P_{22}^U$.

Price co-movements only disappear when there is no I_3 fund in the market. But in that case, less information leaks to the market as well.

2.3.5 Optimal Retention Rule

The optimal behavior of funds in our model is identical to the one in Guerrieri and Kondor (2012). We have to prove that the firing rule of funds are optimal. Specialized funds fire their managers when they buy the defaulting asset or riskless bond when asset repays. Global funds fire the managers if they don't achieve the highest ex-post return. Given that the return signals to informed managers are perfect, any manager with wrong investment decision is immediately revealed uninformed with probability 1. If the percentage of the informed managers in unemployment pool is always non-zero, it is optimal to fire an uninformed manager. But recall that a fraction δ of informed managers is always separated from the funds. Separated or unemployed informed managers always search for a job because by free entry condition for uninformed managers, informed managers get a positive expected pay-off if they look for a job. Thus, unemployment pool is never empty of informed managers and it is optimal to fire a manager that is revealed uninformed. It remains to show that funds retain a manager who has made the right investment decision and is not revealed uninformed by exogenous signal. This is the case when the updated belief of funds about manager being informed is higher than the probability that a just hired manager is informed, i.e.,

$$\eta_{t+1} > \epsilon_t = \frac{\tilde{L}_t^I}{\tilde{L}_t^I + \tilde{L}_t^U} \tag{2.34}$$

But (2.34) holds given the assumption $\omega > \frac{1}{1+\delta}$, by exactly the same arguments in the proof of Proposition 1 of Guerrieri and Kondor (2012) and Proposition 7 of Sami and Brusco (2014) and assuming . This assumption ensures that when a manager is not revealed uninformed and has not made any mistake the beliefs of funds improves with a high enough speed that surpasses the probability of hiring an informed manager from the unemployment pool.

2.4 Conclusion

This paper discussed price co-movement between two financial markets in a risk neutral world with independent liquidity and return shocks. The investment decisions of funds are delegated to fund managers who are informed or uninformed on the return of the assets and face dismissal if they don't make the highest possible return. We showed that in any equilibrium of the model prices co-move with each other following a shock to the priors on any asset. In equilibrium, market clearing prices reflect all the information available in the market. As long as there are some global funds in the market, the demands of informed managers hired at these funds reveal information about both assets. When the total mass of informed managers is so low that market is not cleared, the equilibrium price must make uninformed managers marginal traders. But as long as there are some global funds in the market, with higher probability uninformed managers receive the defaulting asset or riskfree bond when both prices are unrevealing. In equilibrium, prices must compensate uninformed managers for this adverse selection problem and are functions of the ex-ante default probabilities of both assets. Hence, any shock to the ex-ante default probability of one asset changes the price of both assets.

The reputationally concerned managers always ask for a premium over the risk free rate that compensates them for the risk of being dismissed. This premium magnifies the co-movement between prices. However, the co-movement doesn't disappear if there is no repuatational concern. This means that even if investors were directly investing their capital and weren't delegating the investment decision to fund managers, prices would still co-move with each other. Co-movement only disappears when there is no global fund in the market. But if there is no global fund, less information is revealed to the market. This suggests that there is a trade-off between market stability and information revelation. Global funds increase the price co-movement but reveal more information to the markets. Without global funds there is no price co-movement and more market stability, but less information revelation as well.

2.5 Appendix

In all the following results assume $q_2 > q_1$. Note that this leads to $P_{22}^U < P_{11}^U$. Besides, note that by (2.19), in the equilibrium with unequal prices $1 - f_2 > 1 - q_2$ and $1 - f_1 < 1 - q_1$, hence,

$$P_{11}^U < \overline{P} = \frac{\gamma(1-q_1)}{\gamma R + \delta\omega\beta(2q_1 - 1)W_1^U} \tag{2.35}$$

$$P_{22}^U > \frac{\gamma(1-q_2)}{\gamma R + \delta\omega\beta(2q_2 - 1)W_2^U} > \underline{P} = \frac{(1-q_2)(1-\frac{M^I}{\Delta y})^2}{\gamma R + \delta\omega\beta W_3^U} \tag{2.36}$$

Let $\overline{C} = \frac{\max\{b_1, b_2\}}{R}$ and $\underline{C} = \min\{b_1, b_2\}\underline{P}$. We rewrite the supply assumption as follows;

$$M^I + \overline{y} < \min\{b_1, b_2\}.\underline{P} \tag{2.37}$$

Claim 1. *Suppose $p^e(\chi, y)$ is an equilibrium price mapping. Define*

$$\mathcal{I} = \left\{(p_1, p_2) \in (\overline{p}, \frac{1}{R})^2 \mid p_1 > p_2,\ p^e(\chi, y) = (p_1, p_2) \quad \textit{for some} \quad (\chi, y)\right\}$$

Also, define

$$\Phi_{(\chi_1,\chi_2)} = \{y = (y_1, y_2) | p^e(\chi, y) = (p_1, p_2) \quad \textit{for some} \quad (p_1, p_2) \in \mathcal{I}\} \tag{2.38}$$

Then,

1. $\Phi_{00} = \{(y_1, y_2) | \underline{y} \le y_1 \le \overline{y} - N_1^I, \underline{y} \le y_2 \le \overline{y} - N_2^I - N_3^I\}$.
2. $\Phi_{01} = \{(y_1, y_2) | \underline{y} \le y_1 \le \overline{y} - N_1^I - N_3^I, \underline{y} + N_2^I + N_3^I \le y_2 \le \overline{y}\}$.

3. $\Phi_{10} = \{(y_1, y_2) | \underline{y} + N_1^I \leq y_1 \leq \overline{y}, \underline{y} \leq y_2 \leq \overline{y} - N_2^I - N_3^I\}$.

4. $\Phi_{11} = \{(y_1, y_2) | \underline{y} + N_1^I \leq y_1 \leq \overline{y}, \underline{y} + N_2^I + N_3^I \leq y_2 \leq \overline{y}\}$

Proof. (i). Let $y \in [\underline{y}, \overline{y} - N_1^I] \times [\underline{y}, \overline{y} - N_2^I - N_3^I]$, then $z_i((0,0),(y_1,y_2)) = z_i((1,1),(y_1+N^I, y_2+N_2^I+N_3^I))$. Thus, by belief consistency condition we must have $p_i^e((0,0),(y_1,y_2)) = p_i^e((1,1),(y_1+N^I, y_2+N_2^I+N_3^I))$. But this is possible only if equilibrium prices are unrevealing and $p_2^e \leq p_1^e$. Therefore, $[\underline{y}, \overline{y} - N_1^I] \times [\underline{y}, \overline{y} - N_2^I - N_3^I] \subset \Phi_{00}$.

Let $(y_1, y_2) \in \Phi_{00}$ but $(y_1, y_2) \notin [\underline{y}, \overline{y} - N_1^I] \times [\underline{y}, \overline{y} - N_2^I - N_3^I]$. This means that $z_2((0,0),(y_1,y_2)) \geq \overline{y}$ which reveals repay for asset 2 and hence p_2^e must be $1/R$. Contradiction.

(ii). Let $y \in [\underline{y}, \overline{y} - N_1^I - N_3^I] \times [\underline{y} + N_1^I + N_3^I, \overline{y}]$, then $z_i((0,1),(y_1,y_2)) = z_i((1,1),(y_1+N^I+N_3^I, y_2))$. Thus, by belief consistency condition we must have $p_i^e((0,1),(y_1,y_2)) = p_i^e((1,1),(y_1+N^I+N_3^I, y_2))$ which is possible only if prices are unrevealing. Therefore, $[\underline{y}, \overline{y} - N_1^I - N_3^I] \times [\underline{y} + N_1^I + N_3^I, \overline{y}] \subset \Phi_{01}$.

If $(y_1, y_2) \in \Phi_{01} / [\underline{y}, \overline{y} - N_1^I - N_3^I] \times [\underline{y} + N_1^I + N_3^I, \overline{y}]$, then $y_1 > \overline{y} - N_1^I - N_3^I$ and $z_1((0,1),(y_1,y_2)) > \overline{y}$ which implies $p_1 = 1/R$. Contradiction.

(iii). Similar to (2).

(iv). Let $y \in [\underline{y} + N_1^I, \overline{y}] \times [\underline{y} + N_2^I + N_3^I, \overline{y}]$, then $z_i((1,1),(y_1,y_2)) = z_i((0,0),(y_1+N^I, y_2+N^I+N_3^I))$ and by belief consistency $p_i(1,1),(y_1,y_2)) = p_i((0,0),(y_1+N^I, y_2+N^I+N_3^I))$ which is possible only if prices are not revealing. So $[\underline{y} + N_1^I, \overline{y}] \times [\underline{y} + N_2^I + N_3^I, \overline{y}] \subset \Phi_{11}$.

If $y \in \Phi_{11} / [\underline{y} + N_1^I, \overline{y}] \times [\underline{y} + N_2^I + N_3^I, \overline{y}]$, then $y_1 < \underline{y} + N_1^I$ and $z_1((1,0),(y_1,y_2)) < \underline{y} + N_1^I$ which implies that N_1^I managers haven't demanded asset 1. This only happens when asset is defaulting so $p_1 = \underline{p}$ which is a contradiction.

□

Claim 2. *Suppose $p^e(\chi, y)$ is an equilibrium price mapping. Define*

$$\mathcal{I} = \left\{(p,p) \in (\overline{p}, \frac{1}{R})^2 \mid p^e(\chi, y) = (p,p), \quad \textit{for some} \quad (\chi, y)\right\}$$

Also, define

$$\Phi'_{(\chi_1,\chi_2)} = \{y = (y_1, y_2) | p^e(\chi, y) = (p, p) \quad \textit{for some} \quad (p, p) \in \mathcal{I}\} \tag{2.39}$$

Then,

(i). $\Phi'_{00} = \{y = (y_1, y_2) | \underline{y} \leq y_1 \leq \overline{y} - (N_1^I + \alpha N_3^I), \underline{y} \leq y_2 \leq \overline{y} - N_2^I - (1 - \alpha) N_3^I\}$.

(ii). $\Phi'_{01} = \{(y_1, y_2) | \underline{y} \leq y_1 \leq \overline{y} - N_1^I - N_3^I, \underline{y} + N_2^I + (1 - \alpha) N_3^I \leq y_2 \leq \overline{y}\}$.

(iii). $\Phi'_{10} = \{(y_1, y_2) | \underline{y} + N_1^I + \alpha N_3^I \leq y_1 \leq \overline{y}, \underline{y} \leq y_2 \leq \overline{y} - N_2^I - N_3^I\}$.

(iv). $\Phi'_{11} = \{(y_1, y_2) | \underline{y} + N_1^I + \alpha N_3^I \leq y_1 \leq \overline{y}, \underline{y} + N_2^I + (1 - \alpha) N_3^I \leq y_2 \leq \overline{y}\}$.

where α is the fraction of N_3^I that buy asset 1 when $p^e(\chi, y) = (p, p)$.

Proof. Similar to the proof of Claim 1. □

Lemma 2. Note that by Claim 1

$$\begin{aligned} Pr((p_1^e, p_2^e) \in (\overline{p}, \frac{1}{R})) = (1 - q_1)(1 - q_2) Pr((y_1, y_2) \in \Phi_{00}) + (1 - q_1) q_2 Pr((y_1, y_2) \in \Phi_{01}) \\ + q_1 (1 - q_2) Pr((y_1, y_2) \in \Phi_{10}) + q_1 q_2 Pr((y_1, y_2) \in \Phi_{11}) \end{aligned} \tag{2.40}$$

Therefore,

$$1 - f_1 = \frac{(1 - q_1)(1 - r_1 - q_2 r_3)}{1 - r_1 - (1 - q_1) q_2 r_3} \tag{2.41}$$

$$1 - f_2 = \frac{(1 - q_2)(1 - r_1)}{1 - r_1 - (1 - q_1) q_2 r_3} \tag{2.42}$$

It is clear that $0 < 1 - f_1 < 1$. Since, $\frac{1}{1-q_1} > \frac{r_3}{1-r_1}$, $0 < 1 - f_2 < 1$ as well. Also since $\frac{r_3}{1-r_1} < \frac{q_2-q_1}{(1-q_1)q_2}$, $1 - f_1 > 1 - f_2$.

□

Proof of Lemma 3. Recall that if in equilibrium $\underline{p} < p_i^e < 1/R$, marginal traders are either uninformed managers hired at I_3 funds or uninformed managers at I_j funds. Then $p_1^e = p_2^e = p$ is only possible if posteriors of uninformed managers are also equal. If informed managers of global funds are indifferent between the risky assets when both repay and have the same price, the only strict demand for any asset is coming from the informed managers in specialized funds. But in this case, the posteriors are not equal. Thus, we need to have α^* fraction of N_3^I managers demanding asset 1 and $(1-\alpha^*)$ of them demanding only asset 2 when both assets repay and have the same price where α^* is determined in equilibrium to equate the posteriors on risky assets.

Next, note that

$$1 - f_i = Pr(\chi_i = 0|p_1^e = p, p_2^e = p) = \frac{Pr(p_1^e = p, p_2^e = p, \chi_i = 0)}{Pr(p_1^e = p, p_2^e = p)} \tag{2.43}$$

Using Claim 2,

$$\begin{aligned} Pr(p_1^e = p, p_2^e = p, \chi_1 = 0) &= (1-q_1)(1-q_2)Pr((y_1,y_2) \in \Phi_{00}) + (1-q_1)q_2 Pr((y_1,y_2) \in \Phi_{01}) \\ Pr(p_1^e = p, p_2^e = p) &= (1-q_1)(1-q_2)Pr((y_1,y_2) \in \Phi_{00}) + (1-q_1)q_2 Pr((y_1,y_2) \in \Phi_{01}) \\ &\quad + q_1(1-q_2)Pr((y_1,y_2) \in \Phi_{10}) + q_1 q_2 Pr((y_1,y_2) \in \Phi_{11}) \end{aligned} \tag{2.44}$$

Therefore,

$$1 - f_1 = \frac{(1-q_1)(1-r_2-(1-\alpha^*)r_3)(1-r_1-r_3(1-q_2)\alpha^* - r_3 q_2)}{(1-q_1)G_0(r_1,r_2,r_3,\alpha^*) + q_1 G_1(r_1,r_2,r_3,\alpha^*)} \tag{2.45}$$

$$1 - f_2 = \frac{(1-q_2)(1-r_1-\alpha^* r_3)(1-r_2-r_3+\alpha^*(1-q_1)r_3)}{(1-q_1)G_0(r_1,r_2,r_3,\alpha^*) + q_1 G_1(r_1,r_2,r_3,\alpha^*)} \tag{2.46}$$

where

$$G_0(r_1,r_2,r_3,\alpha^*) = (1-r_2-(1-\alpha^*)r_3)(1-r_1-r_3+(1-q_2)(1-\alpha^*)r_3) \tag{2.47}$$

$$G_1(r_1,r_2,r_3,\alpha^*) = (1-r_1-\alpha^* r_3)(1-r_2-r_3+q_2\alpha^* r_3) \tag{2.48}$$

Since we must have equal posteriors, α^* is the solution to

$$\begin{aligned} H(\alpha) \equiv (1-q_1)(1-r_2-(1-\alpha)r_3)(1-r_1-r_3(1-q_2)\alpha - r_3 q_2) - \\ (1-q_2)(1-r_1-\alpha r_3)(1-r_2-r_3+\alpha(1-q_1)r_3) = 0 \end{aligned} \tag{2.49}$$

Note that, $H(\alpha = 1) > 0$ by $q_2 > q_1$ and $H(\alpha = 0) < 0$ by assumption. Thus, there exists $0 < \alpha^* < 1$ at which $1 - f_1 = 1 - f_2$. □

Proof of Proposition 10. At any simple equilibrium with unequal prices,

$$Pr(p_1^e = p_1, p_2^e = p_2 \mid (\chi_1, \chi_2) = (0,0)) = Pr(\phi_{00}) = (1-r_1)(1-r_2-r_3) \tag{2.50}$$

. But,

$$Pr(p_1^e = p_1 \mid \chi_1 = 0) = (1-r_1)(1-q_2) + (1-r_1-r_3)q_2 \tag{2.51}$$

$$Pr(p_2^e = p_2 \mid \chi_2 = 0) = (1-r_2-r_3)(1-q_1) + (1-r_2-r_3)q_1 = (1-r_2-r_3) \tag{2.52}$$

The probabilities in (2.50)-(2.52) show that at any simple equilibrium with different unrevealing prices, p_1^e and p_2^e are interdependent. The same argument together with the use of Claim 2 shows that at any simple equilibrium with equal prices, p_1^e and p_2^e interdependent.

Suppose $\mathbf{p^e}(\chi, \mathbf{y})$ is a non-simple equilibrium price vector. Define $\mathcal{P} = \{U^i_{\chi_1\chi_2}\}_{i=1}^n$ as a partition of $\phi_{\chi_1\chi_2}$ and suppose $\mathbf{p^e}(\chi, \mathbf{y}) = (p_1^i, p_2^i) \in (\overline{p}, \frac{1}{R})^2$; $p_1^i \geq p_2^i$, for any $(y_1, y_2) \in U^i_{\chi_1\chi_2}$. Also assume that $p_1^e(\chi, \mathbf{y})$ and $p_2^e(\chi, \mathbf{y})$ are independent. Therefore, for any $i = 1, ..., n$,

$$Pr(\mathbf{p^e}(\chi, \mathbf{y}) = (p_1^i, p_2^i) \mid \chi_1 = 0, \chi_2 = 0) = Pr(p_1^e(\chi, \mathbf{y}) = p_1^i \mid \chi_1 = 0) Pr(p_2^e(\chi, \mathbf{y}) = p_2^i \mid \chi_2 = 0) \tag{2.53}$$

and

$$Pr(\mathbf{p^e}(\chi, \mathbf{y}) = (p_1^i, p_2^i) \mid \chi_1 = 0, \chi_2 = 1) = Pr(p_1^e(\chi, \mathbf{y}) = p_1^i \mid \chi_1 = 0) Pr(p_2^e(\chi, \mathbf{y}) = p_2^i \mid \chi_2 = 1) \tag{2.54}$$

Assume $p_1^i > p_2^i$ or $p_1^i = p_2^i$ for any i. By Claims 1 and 2

$$\sum_{i=1}^n Pr(\mathbf{p^e}(\chi, \mathbf{y}) = (p_1^i, p_2^i) \mid \chi_1 = 0, \chi_2 = 0) > \sum_{i=1}^n Pr(\mathbf{p^e}(\chi, \mathbf{y}) = (p_1^i, p_2^i) \mid \chi_1 = 0, \chi_2 = 1) \tag{2.55}$$

By (2.53) and (2.54),

$$\sum_{i=1}^n Pr(\mathbf{p^e}(\chi, \mathbf{y}) = (p_1^i, p_2^i) \mid \chi_1 = 0, \chi_2 = 0) = \sum_{i=1}^n Pr(\mathbf{p^e}(\chi, \mathbf{y}) = (p_1^i, p_2^i) \mid \chi_1 = 0, \chi_2 = 1) \times \frac{Pr(p_2^i \mid \chi_2 = 1)}{Pr(p_2^i \mid \chi_2 = 0)} \tag{2.56}$$

But by (2.55), we must have $\frac{Pr(p_2^i|\chi_2=1)}{Pr(p_2^i|\chi_2=0)} < 1$ for some i. Since prices are independent for any i,

$$\begin{aligned} Pr(\mathbf{p^e}(\chi,\mathbf{y}) = (p_1^i, p_2^i) \mid \chi_1 = 1, \chi_2 = 1) &= Pr(p_1^i \mid \chi_1 = 1).Pr(p_2^i \mid \chi_2 = 1) \\ &< Pr(p_1^i \mid \chi_1 = 1).Pr(p_2^i \mid \chi_2 = 0) \\ &= Pr(\mathbf{p^e}(\chi,\mathbf{y}) = (p_1^i, p_2^i) \mid \chi_1 = 1, \chi_2 = 0) \end{aligned} \tag{2.57}$$

Therefore,

$$\sum_{i=1}^{n} Pr(\mathbf{p^e}(\chi,\mathbf{y}) = (p_1^i, p_2^i) \mid \chi_1 = 1, \chi_2 = 1) < \sum_{i=1}^{n} Pr(\mathbf{p^e}(\chi,\mathbf{y}) = (p_1^i, p_2^i) \mid \chi_1 = 1, \chi_2 = 0) \tag{2.58}$$

But by Claims 1 and 2,(2.58) is equivalent to $Pr((y_1, y_2) \in \Phi_{11}) < Pr((y_1, y_2) \in \Phi_{10})$. Also, $Pr((y_1, y_2) \in \Phi_{11}) = Pr((y_1, y_2) \in \Phi_{00})$. This means that $Pr((y_1, y_2) \in \Phi_{00}) < Pr((y_1, y_2) \in \Phi_{10})$ but this is a contradiction by Claims 1 and 2.

Now, let $p^e(\chi, y)$ be a non-simple equilibrium price mapping. Suppose also that for some (χ, y), $p^e(\chi, y) = (p_1^i, p_2^i)$ and $p_1^i \geq p_2^i$, and for some (χ, y), $p^e(\chi, y) = (p_1^j, p_2^j)$ and $p_1^j < p_2^j$. Note that symmetric with Φ_{χ_1,χ_2} defined in Claim 1, we can define Φ''_{χ_1,χ_2} as the superset of all $y = (y_1, y_2)$ that don't reveal $\chi = (\chi_1, \chi_2)$ and $p_1^e(\chi, y) = p_1 < p_2 = p_2^e(\chi, y)$.

For any (χ_1, χ_2), the super set of all (y_1, y_2) for which $p^e(\chi, y) \in (\overline{p}, \frac{1}{R})^2$ is

$$\Psi_{\chi_1,\chi_2} = \Phi_{\chi_1,\chi_2} \cup \Phi'_{\chi_1,\chi_2} \cup \Phi''_{\chi_1,\chi_2}$$

Again, let $\{U^i_{\chi_1,\chi_2}\}_i$ be a partition of Ψ_{χ_1,χ_2} where $p^e(\chi, y) = (p_1^i, p_2^i)$ for $y \in U^i_{\chi_1,\chi_2}$. By the same arguments, inequality (2.55) holds and if $p_1^e(\chi, y)$ and $p_2^e(\chi, y)$ are independent, then $Pr((y_1, y_2) \in \Psi_{11}) < Pr((y_1, y_2) \in \Psi_{10})$ which is a contradiction.

□

Proof of corollary 1. By assumption (2.9), demands of informed managers and liquidity traders never clear the markets. So in any equilibrium, there must be some amount of risky assets that is allocated to uninformed managers. Since supply is not enough to allocate to all uninformed managers, unrevealing prices must make uninformed managers marginal traders. This means that when marginal traders are N_j^U or N_3^U, unrevealing prices are given by (2.15) or (2.16). But by Proposition 10, there is at least a pair of realizations of $p_1^e(\chi, y)$ and $p_2^e(\chi, y)$ for which equilibrium prices are interdependent. This means that for some values of $p^e(\chi, y) = (p_1, p_2)$, the posteriors of uninformed managers on asset i are not independent of p_j and χ_j. But this implies that $1 - f_i$ must be a function of both q_1 and q_2. But since either uninformed managers of specialized funds or uninformed managers of global funds are marginal traders at $p^e(\chi, y) = (p_1, p_2)$, any shock to q_i changes both p_i and p_j. It only remains to show that $1 - f_i$ is a decreasing function of q_j in any equilibrium. Let $U_{\chi_1\chi_2} = \{(y_1, y_2) \mid p^e(\chi, y) = (p_1, p_2)\}$. Note that,

$$1 - f_1 = \frac{(1 - q_1)[(1 - q_2)Pr(y \in U_{00}) + q_2 Pr(y \in U_{01}))]}{Pr\big(p^e(\chi, y) = (p_1, p_2)\big)} \tag{2.59}$$

where

$$\begin{aligned} Pr\big(p^e(\chi, y) = (p_1, p_2)\big) = {} & (1 - q_1)[(1 - q_2)Pr(y \in U_{00}) + q_2 Pr(y \in U_{01}))] \\ & + q_1[(1 - q_2)Pr(y \in U_{10}) + q_2 Pr(y \in U_{11})] \end{aligned} \tag{2.60}$$

At any (y_1, y_2), the mass of strict demands for asset 1 at $\chi = (1, 0)$ and $\chi = (1, 1)$ are the same. Also $Z_2(p, (1, 0), (y_1, y_2)) = Z_2(p, (1, 1), (y_1, y_2 + N_2^I + N_3^I))$ for $y_2 < \overline{y} - N_2^I - N_3^I$. By belief consistency $p^e((1, 0), (y_1, y_2)) = p^e((1, 1), (y_1, y_2 + N_2^I + N_3^I))$ which means $U_{11} = \{y' \mid y_1' = y_1, y_2' = y_2 + N_2^I + N_3^I, (y_1, y_2) \in U_{10}\} \supseteq U_{10} + (0, N_2^I + N_3^I)$. But again by belief consistency, $U_{10} = \{y' \mid y_1' = y_1, y_2' = y_2 - N_2^I - N_3^I, (y_1, y_2) \in U_{11}\} \supseteq U_{11} - (0, N_2^I + N_3^I)$. Hence, U_{10} and U_{11} are simple shifts of each other and $Pr(y \in U_{11}) = Pr(y \in U_{10})$. Also by belief consistency, U_{00} and U_{10} are simple shifts of each other and occur with the same probability which also implies that $Pr(U_{11}) = Pr(U_{00})$. However, by the same arguments in the proof of Proposition 10, particularly by inequality (2.55),

$$Pr(p^e(\chi, y) = (p_1, p_2) \mid \chi_1 = 0, \chi_2 = 0) > Pr(p^e(\chi, y) = (p_1, p_2) \mid \chi_1 = 0, \chi_2 = 1)$$

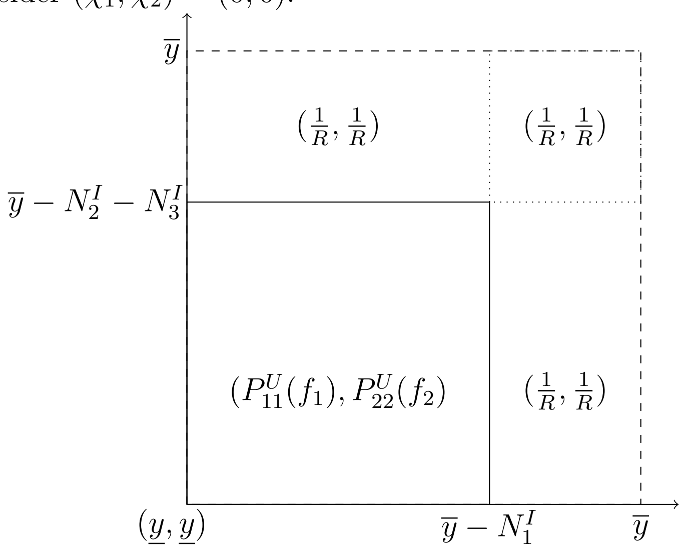
which is equivalent to $Pr(y \in U_{00}) > Pr(y \in U_{01})$. But then,

$$\begin{aligned} \frac{d(1 - f_1)}{dq_2} = & \Big\{(1 - q_1)[Pr(U_{01} - Pr(U_{00}))]D - [(1 - q_1)(Pr(U_{01}) - Pr(U_{00})) \\ & + q_1(-Pr(U_{10} + Pr(U_{11}))N)]\Big\}D^{-2} < 0 \end{aligned} \tag{2.61}$$

where N is the nominator and D is the denominator of (2.59). Therefore, $1 - f_1$ is decreasing in q_2. By the similar arguments, $1 - f_2$ is also decreasing in q_1.

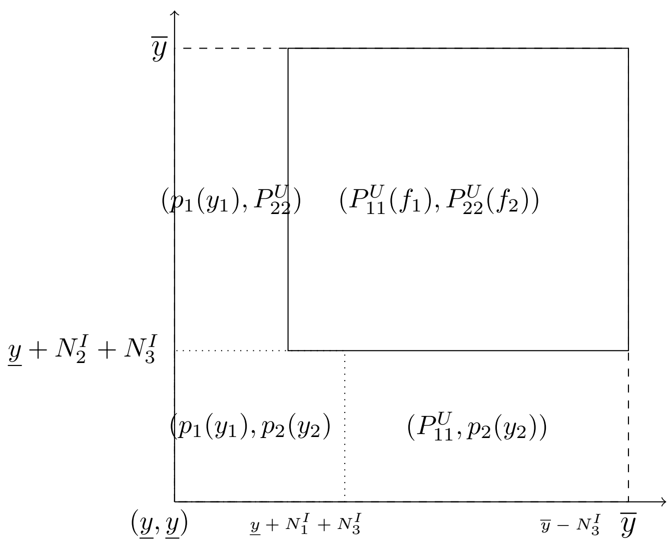
□

Proof of Proposition 11. **Price mapping**
Claim 1 specifies the regions where (χ_1, χ_2) are not revealed in any equilibrium in which unrevealing prices are different. Let us first describe the structure of the equilibrium and the regions where the equilibrium price function determines its revealing, non-revealing and partially revealing values. First consider $(\chi_1, \chi_2) = (0, 0)$.



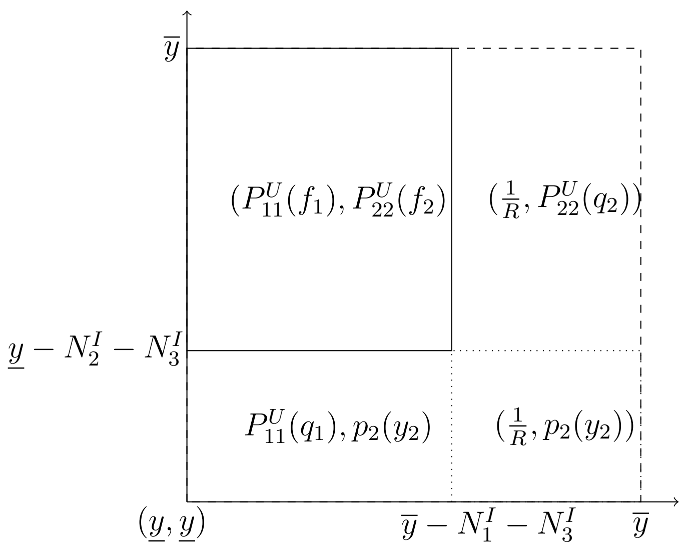
When $(y_1, y_2) \in (\overline{y} - N_1^I, \overline{y}] \times (\overline{y} - N_2^I - N_3^I, \overline{y}]$, $Z_j(y_j, \chi) \geq \overline{y}$. Clearly, this reveals $(p_1, p_2) = (\frac{1}{R}, \frac{1}{R})$. When $(y_1, y_2) \in (\overline{y} - N_1^I, \overline{y}] \times [\underline{y}, \overline{y} - N_2^I - N_3^I)$, $Z_1 \geq \overline{y}$. Again, it's clear that asset 1 is repaying and it's price is $\frac{1}{R}$. However, $Z_2(y_2, (0, 0)) = Z_2(y_2 + N_2^I + N_3^I, (0, 1))$ so it's not revealed that asset 2 repays. Since we now have only one risky asset, the model is identical to the baseline model of Guerrieri and Kondor (2012) and for $(y_1, y_2) \in (\overline{y} - N_1^I, \overline{y}] \times [\underline{y}, \overline{y} - N_2^I - N_3^I)$ the posteriors on risky asset 2 are the same as priors,i.e., $1 - f_2 = 1 - q_2$. Thus, $p_2^e((0, 0), (y_1, y_2)) = P_{22}^U(q_2)$. By the symmetric argument, $p_1^e((0, 0), (y_1, y_2)) = P_{11}^U(q_1)$ for $(y_1, y_2) \in [\underline{y}, \overline{y} - N_1^I) \times (\overline{y} - N_2^I - N_3^I, \overline{y}]$.

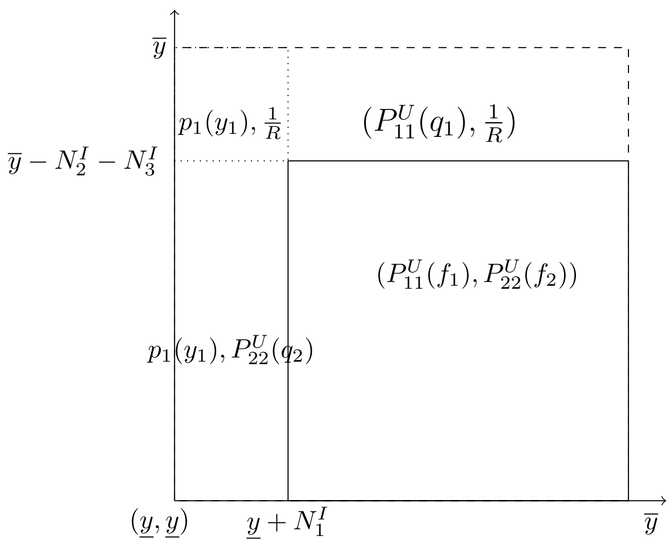
Consider now the case of $(\chi_1, \chi_2) = (1, 1)$.



Again, the solid rectangle shows Φ_{11}. Since $P^U_{22} < P^U_{11}$, N^I_3 managers strictly demand asset 2 irrespective of the default or repay of asset 1 when $\chi_2 = 0$. So $\chi_2 = 1$ is immediately revealed whenever $Z_2 < \underline{y} + N^I_2 + N^I_3$. Moreover, when $\chi_2 = 1$ all N^I_3 managers buy asset 1 if it repays. So when the default of asset 2 is revealed, $\chi_1 = 1$ is revealed if the mass of strict demands of asset 1 is less than $\underline{y} - N^I_1 - N^I_3$. Therefore, $(\chi_1, \chi_2) = (1, 1)$ is revealed when $(y_1, y_2) \in [\underline{y}, \underline{y} + N^I_1 + N^I_3) \times [\underline{y}, \underline{y} + N^I_2 + N^I_3)$ and $p^e((1, 1), (y_1, y_2)) = (p_1(y_1), p_2(y_2))$. Moreover, when asset 2 is revealed defaulting and $y_1 \in [\underline{y} + N^I_1 + N^I_3, \overline{y}]$, $Z_1((1, 1), (y_1, y_2)) = Z_1((0, 1), (y_1 - N^I_1 - N^I_3, y_2))$ so the default of asset 1 is not revealed. Again, since we are only left with one unrevealed risky asset we are back to Guerrieri and Kondor (2012) and $p^e_1 = P^U_{11}(q_1)$. The symmetric argument applies to the region where $(y_1, y_2) \in [\underline{y}, \underline{y} + N^I_1) \times [\underline{y}, \underline{y} + N^I_2 + N^I_3)$, hence, $p^e_2 = P^U_{22}(q_2)$. Note that since $P^U_{11} > P^U_{22}$, the default of asset 1 is only revealed when $Z_1 < \underline{y} + N^I_1$ if $\chi_2 = 1$ is not revealed. This is because, N^I_3 managers always buy asset 2 when it repays, irrespective of the default or repay of asset 1. So when $y_2 \in [\underline{y}, \underline{y} + N^I_2 + N^I_3)$ and it's not clear if N^I_3 have bought asset 2 or not, the default of asset 1 is only revealed for $y_1 \in [\underline{y}, \underline{y} + N^I_1)$.

Consider $(\chi_1, \chi_2) = (0, 1)$ or $(1, 0)$. The following figures show the revelation , partial revelation and non-revelation regions for these cases.





Like the two other cases, when one asset is revealed we are back to Guerrieri and Kondor (2012) and $p^e_j = P_{jj}(q_j)$ when the price is j is not revealing.

Demands

The demands of N^U_1 and N^U_2 managers are given as

$$d^U_1 = \begin{cases} (1,1,0) & \text{if } p_1 \in \{\frac{1}{R}, P^U_{11}(f_1), P^U_{11}(q_1)\} \\ ((1,0,0) & \text{otherwise} \end{cases} \tag{2.62}$$

$$d_2^U = \begin{cases} (1,0,1) & \text{if } p_2 \in \{\frac{1}{R}, P_{22}^U(f_2), P_{22}^U(q_2)\} \\ ((1,0,0) & \text{otherwise} \end{cases} \tag{2.63}$$

where P_{jj}^U, $j = 1,2$ are solved from (2.10). At $p_j^e = P_{jj}^U$, N_j^U managers are indifferent between risky asset j and bond. So their payoff of buying asset j is $\gamma R + \delta\omega f_j W_j^U$. When $p_1^e > p_2^e$, the maximum prices that N_3^U managers pay are solved from (2.11) and (2.12). But at P_{j3}^U the payoff to an N_3^U manager of receiving asset j is $\gamma R + \delta\omega f_1 f_2 W_3^U$. Recall that N_3^U managers are only retained when they buy the less expensive risky asset when both assets repay. This means that N_3^U managers are fired with a higher probability than N_j^U managers and therefore, $W_3^U < W_j^U$. But then, $\gamma R + \delta\omega f_j W_j^U > \gamma R + \delta\omega f_1 f_2 W_3^U$ and clearly $P_{jj}^U > P_{j3}^U$. Also, recall that $1 - f_1 > 1 - f_2$ so $P_{22}^U < P_{11}^U$. But this implies that for any (χ_1, χ_2) all the uninformed managers at global funds are strictly demanding asset 2. That is, at (P_{11}^U, P_{22}^U) for any (χ_1, χ_2) and $y_2 \in [\underline{y}, \overline{y}]$, $Z_2(\chi, y) \geq \underline{y} + N_3^U$. The demands of N_3^U contain no information and are constant at any $(\chi, \mathbf{y})$ for which $\mathbf{p^e}(\chi, \mathbf{y}) = (P_{11}^U, P_{22}^U)$. But this is equivalent to assuming that at (P_{11}^U, P_{22}^U)- and only at this price pair- the amount of noise demands have increased by the mass of N_3^U managers. Hence, the nonrevelation regions when the nonrevealing prices are equal to (P_{11}^U, P_{22}^U) are identical to the regions specified in Claim 1. The demands of N_3^U managers are given as

$$d_3^U = \begin{cases} (1,1,1) & \text{if } (p_1,p_2) = (\frac{1}{R}, \frac{1}{R}) \\ (0,0,1) & \text{if } p_2 = P_{22}^U(f_2) \quad \text{or} \quad p_2 = P_{22}^U(q_2) \\ (0,1,0) & \text{if } (p_1,p_2) = (P_{11}^U(q_1), \frac{1}{R}) \quad \text{or} \quad (p_1,p_2) = (P_{11}^U(q_1), p_2(y_2)) \\ (1,0,0) & \text{otherwise} \end{cases} \tag{2.64}$$

Note that since $W_3^U \leq W_2^U$, N_3^U managers are willing to pay higher than $P_{jj}^U(q_j, W_j^U)$ for asset j and they strictly demand asset j.

Allocations

It only remains to show that the allocations are market clearing and feasible. Recall that $D_k(p)$ is the set of all the traders with strict demands for asset k at price p, $Z_k(\chi_k, y_k)$ is the mass of all d^q, $q \in D_k(p)$ at $(\chi, \mathbf{y})$. Let $A(p) = (A_1(p), A_2(p))$ be the aggregate demand vector at price p. Let $\bar{D}_k = \{q | q \notin D_k(p), d_k^q = 1, d_l^q = 0; l \neq k\}$. Then, at each price p the auctioneer allocates the asset $k = 1,2$ according to the following rule.

- $x_k(d^q) = 1$ if $q \in D_k(p)$.
- $x_k(d^q) = \max\{\frac{b_k p_k - Z_k(p)}{A_k - Z_k}, 0\}$ if $q \in \bar{D}_k$
- $x_k(d^q) = 0$ if $d_k^q = 0$.

The probability of receiving risk-less bond is equal to

- $x_0(d^q) = \max\{1 - x_1(d^q) - x_2(d^q), 0\}$.

So at $(P_{11}^U(f_1), P_{22}^U(f_2)$ liquidity traders are assigned asset j with probability 1. If any of the risky assets repay, N_j^I managers have strictly demanded it and should be assigned as well. N_3^I managers always demand asset 2 when it repays and only demand asset 1 if it repays and asset 2 defaults. Note that N_3^U managers always demand asset 2 for any χ_2 so like noise traders they must be allocated asset 2 for sure. N_1^U and N_2^U managers are marginal traders at $(P_{11}^U(f_1), P_{22}^U(f_2))$, the allocation probabilities to N_1^U and N_2^U managers are

$$x_1(d_1^U) = \frac{b_1 P_{11}^U - (1-\chi_1)(N_1^I + \chi_2 N_3^I) - y_1}{N_1^U}$$
$$x_2(d_2^U) = \frac{b_2 P_{22}^U - (1-\chi_2)(N_2^I + N_3^I) - N_3^U - y_2}{N_2^U} \quad (2.65)$$

Clearly by assumptions (i) and (ii) of Proposition 11 and (2.37),

$$x_1(d_1^U) > \frac{b_1 P_{11}^U - M^I - \overline{y}}{N_1^U} > \frac{\underline{C} - M^I - \overline{y}}{N_1^U} > 0 \quad (2.66)$$
$$x_1(d_1^U) < \frac{b_1 P_{11}^U}{N_1^U} < \frac{b_1 P_{11}^U}{(I_1 - M^I)} < \frac{\overline{C}}{I_1 - M^I} < 1 \quad (2.67)$$

and

$$x_2(d_2^U) > \frac{b_2 P_{22}^U - (M^I + I_3) - \overline{y}}{N_2^U} > \frac{\underline{C} - M^I - I_3 - \overline{y}}{N_2^U} > 0 \quad (2.68)$$
$$x_2(d_2^U) < \frac{b_2 P_{22}^U - N_3^U}{N_2^U} < \frac{\overline{C}}{I_2 - M^I} < 1 \quad (2.69)$$

So allocations are feasible at $(P_{11}^U(f_1), P_{22}^U(f_2))$.

At $(p_1^e = \frac{1}{R}, p_2^e = \frac{1}{R})$, everyone in I_k and I_3 funds has nonzero demand for asset k also noise traders are the only agents with strict demands for risky asset. Managers of global funds are indifferent between all assets while managers of specialized funds are indifferent between asset 1(2) and risk less bond.

Hence, $A_k = N_k^I + N_k^U + N_3^I + N_3^U + y_k$, $Z_k = y_k$. This means that the auctioneer must first allocate the risky assets to the managers of specialized funds and if anything remained to global funds. This means that,

$$x_k(d^q(\frac{1}{R}, \frac{1}{R})) = \frac{\frac{b_k}{R} - y_k}{N_k^I + N_k^U + N_3^I + N_3^U} \tag{2.70}$$

But by assumption $b_k < R\min\{I_1, I_2\} + y_k$ which implies that $x_k(d^q(\frac{1}{R}, \frac{1}{R})) < 1$

At $\mathbf{p} = (p_1(y_1), p_2(y_2))$, only liquidity traders demand risky assets and since $p_k(y_k) = \frac{y_k}{b_k}$ market is certainly cleared. At $(\frac{1}{R}, \overline{p}_2)$,

$$x_1(d^q(\frac{1}{R}, p_2(y_2))) = \frac{\frac{b_1}{R} - y_1}{N_1^I + N_1^U + N_3^I + N_3^U} \tag{2.71}$$

which is again feasible by supply assumptions.

It remains to show that allocations are feasible at $(P_{11}^U(q_1, W_1^U), \frac{1}{R})$ and $(\frac{1}{R}, P_{22}^U(q_2, W_2^U))$. At $p_j^e = P_{jj}^U(q_j, W_j^U)$, and $p_k^e = \frac{1}{R}$, N_j^I and N_3^I managers are strictly demanding asset j when it repays. Since $W_3^U \leq W_2^U$, N_3^U managers are willing to pay higher than $P_{jj}^U(q_j, W_j^U)$ for asset j and they strictly demand asset j. N_j^U managers are indifferent between asset j and bond. Thus,

$$x_j(d_j^U) = \frac{b_j P_{jj}^U(q_j) - (1 - \chi_j)(N_j^I + N_3^I) - N_3^U - y_j}{N_j^U} \tag{2.72}$$

Again by assumptions (i) and (ii), allocations are feasible.

At $p_j^e = P_{jj}^U$ and $p_i^e = p_i(y_i)$, no one demands asset i. N_j^U managers are indifferent between asset j and riskless bond but N_3^U strictly demand asset j. Thus, for $k = 3, j$

$$x_j(d_k^U) = \frac{b_j P_{jj}^U(q_j) - (1 - \chi_j)(N_j^I + N_3^I) - N_3^U - y_j}{N_j^U} \tag{2.73}$$

Again, this allocation is feasible by assumption. □

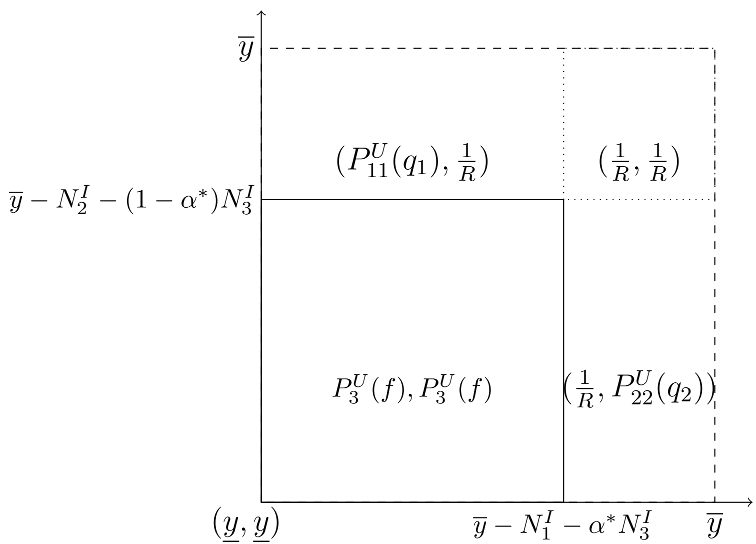


Figure 2.1: $(\chi_1, \chi_2) = (0, 0)$

Proof of Proposition 12. **Price mapping**
Like the proof of Proposition 11, Figures 2.1-2.4 show the values that price mapping takes at any $(\chi, \mathbf{y})$.

Recall that Claim 2 gives the regions of non-revelation for any (χ_1, χ_2) and the solid rectangle in any figure shows Φ_{χ_1,χ_2}. The main difference between the structure of this equilibrium and the equilibrium in Proposition 11 is the fixed fractions of N^I_3 managers that demand each risky asset when both of them repay. It is worth noting that when $(\chi_1, \chi_2) = (1, 1)$ and $\chi_1 = 1$ is revealed, all N^I_3 managers are buying asset 2 if it repays and $Z_2 = y_2 + N^I_2 + N^I_3 \geq \underline{y} + N^I_2 + N^I_3$. So for $y_2 < \underline{y} + N^I_2 + N^I_3$ asset 2 is also revealed defaulting. However, for $y_2 \geq \underline{y} + N^I_2 + N^I_3$ the default is not revealed. But then we are left with only one risky asset and the model is identical to Guerrieri and Kondor (2012). Hence, the $1 - f_2 = 1 - q_2$ for $y_2 \geq \underline{y} + N^I_2 + N^I_3$ The same argument applies to asset 1 when asset 2 is revealed defaulting.

Demands

We explained in the proof of Proposition 11 that given any pair of posteriors uninformed managers of specialized funds pay at most $P^U_{jj}(f_j)$ that is less than $P^U_{j3}(f_j)$, the maximum that N^U_3 managers pay for asset j. So when $p^e_j = P^U_{j3}$ for $j = 1, 2$, N^U_j managers are out of risky asset markets and only demand risk free bond . Since $P^U_{13} = P^U_{23}$ and $1 - f_1 = 1 - f_2$, from this point

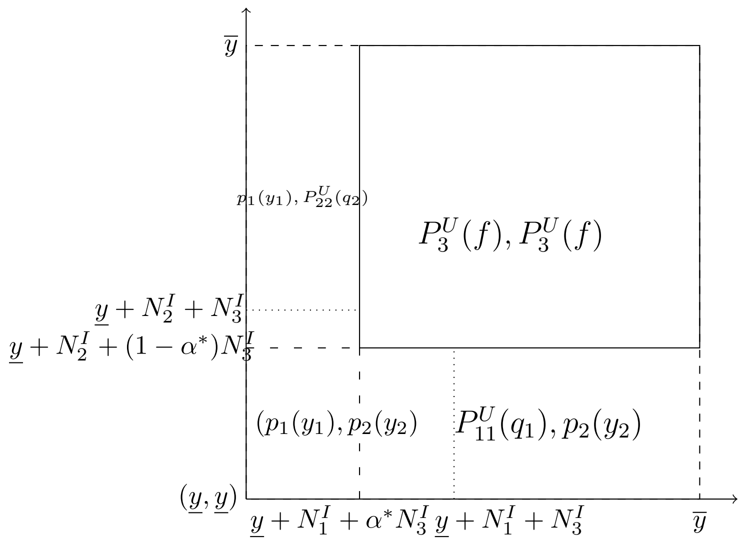


Figure 2.2: $(\chi_1, \chi_2) = (1, 1)$

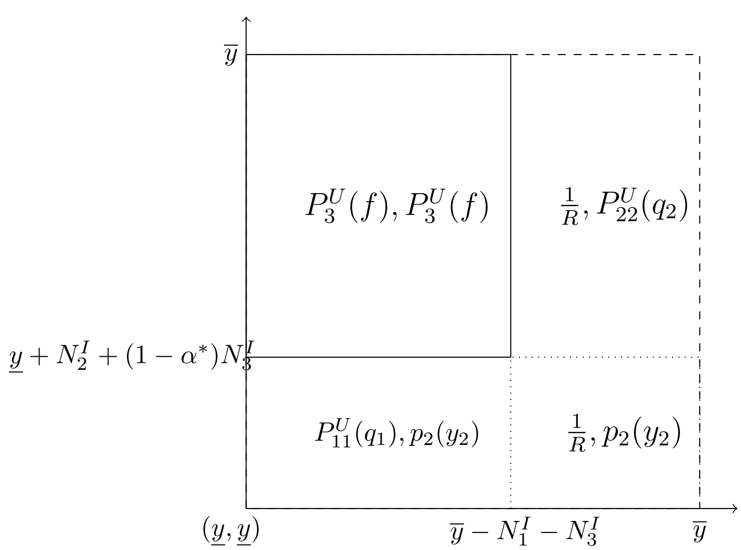


Figure 2.3: $(\chi_1, \chi_2) = (0, 1)$

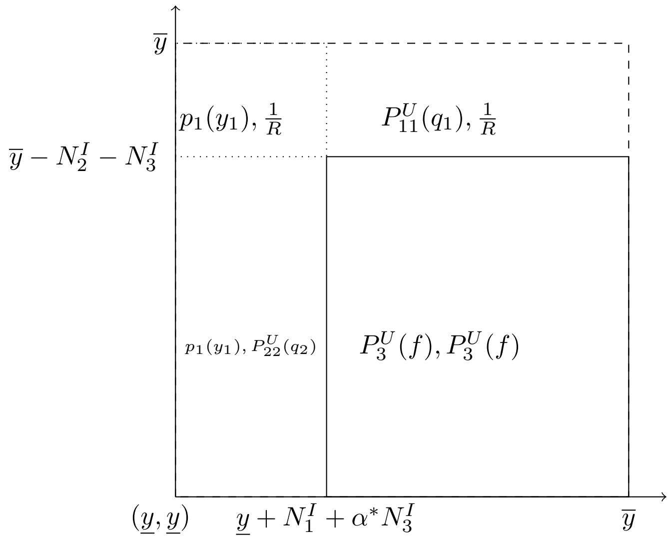


Figure 2.4: $(\chi_1, \chi_2) = (1, 0)$

on we refer to both by P^U_3 and $1 - f$. The demands of N^U_j managers are the same as (2.62) and (2.63). The demands of N^U_3 managers are given as

$$d^U_3 = \begin{cases} (1,1,1) & \text{if } (p_1, p_2) \in \{(\frac{1}{R}, \frac{1}{R}), (P^U_3(f), P^U_3(f))\} \\ (0,1,0) & \text{if } (p_1, p_2) \in \{(P^U_{11}(q_1), p_2(y_2), (P^U_{11}(q_1), \frac{1}{R})\} \\ (0,0,1) & \text{if } (p_1, p_2) \in \{(p_1(y_1), P^U_{22}(q_2)), (\frac{1}{R}, P^U_{22}(q_2))\} \\ (1,0,0) & \text{otherwise} \end{cases} \tag{2.74}$$

Note that since $W^U_3 \leq W^U_j$, N^U_3 strictly demand asset j when asset i is revealed and $p_j = P^U_{jj}(q_j)$. The reason is at $p_j = P^U_{jj}(q_j)$, marginal traders are uninformed managers of I_j funds but then N^U_3 managers can't be marginal and strictly demand asset j.

Allocations

To prove the existence, it only remains to show that the markets are cleared at $\mathbf{p^e}(\chi, \mathbf{y})$ for any $(\chi, \mathbf{y})$ and the allocation probabilities are feasible. Recall that by allocation rule given in the proof of Proposition 11 at each $p = (p_1, p_2)$ the auctioneer assigns asset k to anyone with strict demand for asset k and then to anyone indifferent between assets and the bond. At $p^e = (P^U_3, P^U_3)$, noise traders strictly demand risky assets. In addition, informed managers of specialized funds and α^*-$1 - \alpha^*$- fraction of informed managers of global funds strictly demand risky asset 1-2- when both assets repay. However, all N_3^I managers strictly demand the repaying asset when either asset 1 or asset 2 defaults while N_j^I managers only demand riskless bond when risky asset j defaults. Consequently,

$$x_1(d_3^U) = \frac{b_1 P_3^U - y_1 - (1-\chi_1)\big(N_1^I + (1-\chi_2)\alpha^* N_3^I + \chi_2 N_3^I\big)}{N_3^U} \tag{2.75}$$

$$x_2(d_3^U) = \frac{b_2 P_3^U - y_2 - (1-\chi_2)\big(N_2^I + (1-\chi_1)(1-\alpha^*) N_3^I + \chi_1 N_3^I\big)}{N_3^U} \tag{2.76}$$

It is clear that (2.75) and (2.76) are market clearing. We only need to show that they are feasible, that is $x_j(d_3^U) \in [0,1]$. Notice that,

$$\frac{b_1 P_3^U - y_1 - N_1^I - N_3^I}{N_3^U} \leq x_1(d_3^U) \leq \frac{b_1 P_{13}^U - y_1}{N_3^U} \tag{2.77}$$

$$\frac{b_2 P_3^U - y_2 - N_2^I - N_3^I}{N_3^U} \leq x_2(d_3^U) \leq \frac{b_2 P_3^U - y_2}{N_3^U} \tag{2.78}$$

$$\tag{2.79}$$

Besides,

$$1 - f_1 > (1-q_1)(1-\frac{M^I}{\Delta y})^2 > (1-q_2)(1-\frac{M^I}{\Delta y})^2. \tag{2.80}$$

This implies that

$$P_3^U = \frac{\gamma(1-f_1)}{\gamma R - (1-2f_1)\delta\omega\beta W_3^U} > \frac{\gamma(1-f_1)}{\gamma R + \delta\omega\beta W_3^U} > \frac{(1-q_2)(1-\frac{M^I}{\Delta y})^2}{\gamma R + \delta\omega\beta W_3^U} \tag{2.81}$$

Using (2.81),

$$b_j P_3^U > b_j \frac{(1-q_2)(1-\frac{M^I}{\Delta y})^2}{\gamma R + \delta\omega\beta W_3^U} > \underline{C} > \overline{y} + M^I \tag{2.82}$$

where the last inequality follows from the assumption on supply. But by (2.82),

$$x_j(d_3^U) > \frac{b_j P_3^U - \overline{y} - M^I}{N_j^U} > 0 \tag{2.83}$$

Additionally,

$$\frac{b_j P_3^U - y_j}{N_3^U} < \frac{b_j}{RN_3^U} < \frac{b_j}{R(I_3 - M^I)} < \frac{b_1 + b_2}{R(I_3 - M^I)} < 1$$

Hence, $x_1(d_3^U)$ and $x_2(d_3^U)$ are feasible. Note also that assumptions guarantee that $x_1(d_3^U) + x_2(d_3^U) \leq 1$.

When $\mathbf{p^e}(\chi, \mathbf{y})$ is partially revealing, the allocations are the same as the ones in the proof of Proposition 11 and are feasible by the assumption.

□

Claim 3. *Suppose (p_1, p_2) is a price pair such that $\mathbf{p}(\chi, \mathbf{y}) = (p_1, p_2) \in (\overline{p}, \frac{1}{R})^2$ for some $(\chi, \mathbf{y})$. Also, suppose that $Pr(\chi_j = 0 \mid \mathbf{p}(\chi, \mathbf{y}) = (p_1, p_2)) = 1 - q_j$; $j = 1, 2$. Define $\widetilde{\phi}_{\chi_1\chi_2} = \{y \in [\underline{y}, \overline{y}]^2 \mid \mathbf{p}(\chi, \mathbf{y}) = (p_1, p_2))\}$. Then,*

1. $\widetilde{\phi}_{00} = [\underline{y} + N_1^I + N_3^I, \overline{y} - N_1^I - N_3^I] \times [\underline{y} + N_2^I + N_3^I, \overline{y} - N_2^I - N_3^I]$
2. $\widetilde{\phi}_{01} = [\underline{y} + N_1^I, \overline{y} - N_1^I - 2N_3^I] \times [\underline{y} + 2(N_2^I + N_3^I), \overline{y}]$
3. $\widetilde{\phi}_{10} = [\underline{y} + 2N_1^I + N_3^I, \overline{y} - N_3^I] \times [\underline{y} + N_2^I + N_3^I, \overline{y} - N_2^I - N_3^I]$
4. $\widetilde{\phi}_{11} = [\underline{y} + 2N_1^I + N_3^I, \overline{y} - N_3^I] \times [\underline{y} + 2(N_2^I + N_3^I), \overline{y}]$

Proof. First note that $\widetilde{\phi}_{\chi_1\chi_2} \subset \Phi_{\chi_1\chi_2}$ for any (χ_1, χ_2). Thus, (p_1, p_2) are not revealing (χ_1, χ_2). Moreover, posteriors of managers given (p_1, p_2) are the same as priors. This is because the size of $\widetilde{\phi}_{\chi_1\chi_2}$ is constant for any (χ_1, χ_2) so the probability of $\widetilde{\phi}_{\chi_1\chi_2}$ is the same at any (χ_1, χ_2). Hence, $Pr(\mathbf{p}(\chi, \mathbf{y}) = (p_1, p_2)) = Pr(y \in \widetilde{\phi}_{\chi_1\chi_2})$. But then,

$$\begin{aligned} Pr(\chi_1 = 0 \mid \mathbf{p}(\chi, \mathbf{y}) = (p_1, p_2)) =& \frac{(1 - q_1)[Pr(y \in \widetilde{\phi}_{00})(1 - q_2) + Pr(y \in \widetilde{\phi}_{01})q_2]}{Pr(y \in \widetilde{\phi}_{\chi_1\chi_2})} \\ =& (1 - q_1) \end{aligned} \tag{2.84}$$

By the same argument $Pr(\chi_1 = 0 \mid \mathbf{p}(\chi, \mathbf{y}) = (p_1, p_2)) = (1 - q_2)$ □

Proof of proposition 13. **Price mapping**
Claim 3 specifies the regions where prices are unrevealing. Figures 2.5-2.8

show these regions and the regions where prices are partially and fully revealing. Note that except for the no-revelation regions, the rest of the regions are the same as the ones in Proposition 11. Furthermore, the partial revelation regions where prices equal $P^U_{jj}(f_j)$ are not fully revealing (χ_1, χ_2) but are not entirely unrevealing either. As Figures 2.5-2.8 show, $\mathbf{p^e}(\chi, \mathbf{y}) = (P^U_{11}(f_1), P^U_{22}(f_2))$ for $y \in \Phi_{\chi_1\chi_2} - \widetilde{\Phi}_{\chi_1\chi_2}$. Therefore, the posteriors of uninformed managers at $(P^U_{11}(f_1), P^U_{22}(f_2))$ are given as

$$1 - f_1 = \frac{(1-q_1)\big((1-q_2)A + q_2 B\big)}{\big[(1-q_2) + q_1 q_2\big]A + (1-q_1)q_2 B} \tag{2.85}$$

$$1 - f_2 = \frac{(1-q_2)A}{\big[(1-q_2) + q_1 q_2\big]A + (1-q_1)q_2 B} \tag{2.86}$$

where,

$$A = (r_1 + r_2 + 3r_3) + (r_2 + r_3)[r_1 - 4(r_1 + r_3)]$$
$$B = (r_1 + r_2 + 2r_3) - 3(r_1 + r_3)(r_2 + r_3)$$

Note that since

$$\frac{(r_1 + r_2 + 3r_3) + (r_2 + r_3)[r_1 - 4(r_1 + r_3)]}{(r_1 + r_2 + 2r_3) - 3(r_1 + r_3)(r_2 + r_3)} < \frac{(1-q_1)q_2}{q_1(1-q_2)}$$

$1 - f_1 > 1 - f_2$ Demands are the same as the demands given in Proposition 11 and as we showed in the proof of Proposition 11 the allocations are feasible as well.

□

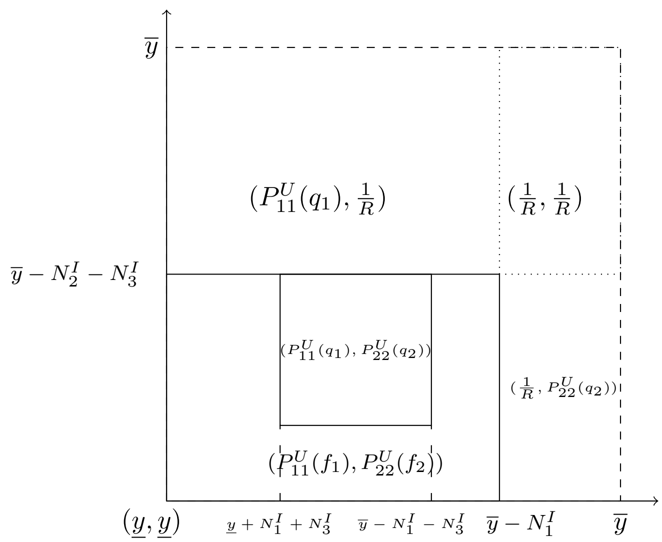


Figure 2.5: $(\chi_1, \chi_2) = (0, 0)$

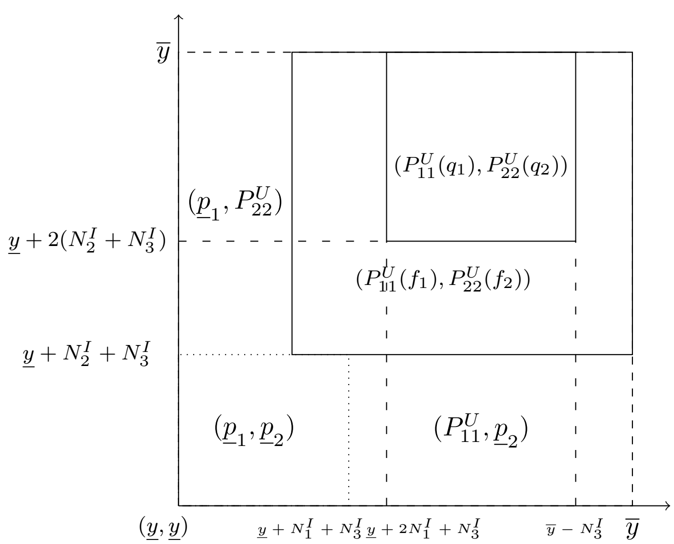


Figure 2.6: $(\chi_1, \chi_2) = (1, 1)$

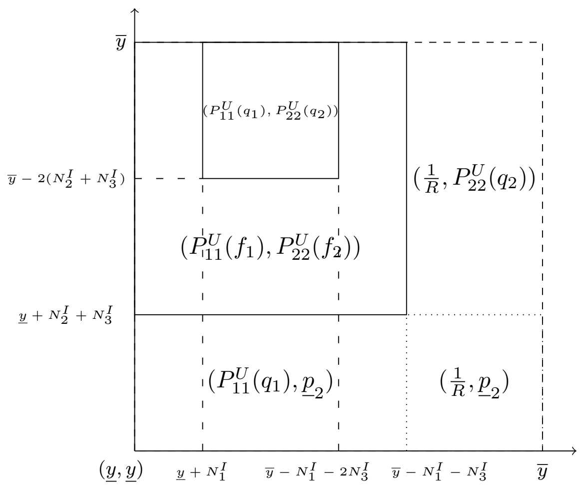


Figure 2.7: $(\chi_1, \chi_2) = (0, 1)$

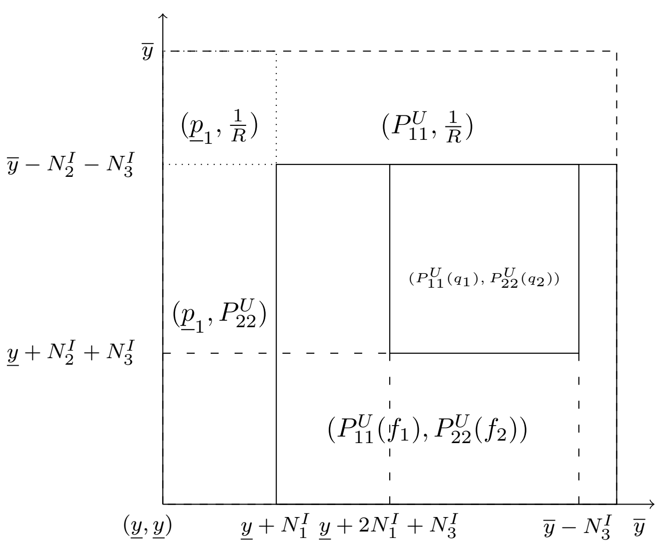


Figure 2.8: $(\chi_1, \chi_2) = (1, 0)$

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