Optimal Fees for Sales Online

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Zhen Xu

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Zhen Xu

We, the dissertation committee for the above candidate for the Doctor of Philosophy degree, hereby recommend acceptance of this dissertation.

Yair Tauman – Dissertation Advisor
Leading Professor, Department of Economics

Pradeep Dubey – Chairperson of Defense
Leading Professor, Department of Economics

Sandro Brusco
Professor, Department of Economics

Debapriya Sen
Associate Professor
Department of Economics, Ryerson University

This dissertation is accepted by the Graduate School.

Charles Taber
Interim Dean of the Graduate School

Abstract of the Dissertation

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A typical fee structure of an auction website is a combination of two fees collected from sellers: listing fee and transaction fee. Listing fee is a fixed amount charged when inserting an item onto the website. Transaction fee is a certain percentage of the selling price and is only charged when a success sale is made.

I analyze the optimal fee structure for a profit-maximizing website which provides a platform for many heterogeneous buyers and sellers to transact their products. The results suggest that the optimal listing fee is always positive. The optimal transaction fee is positive only when there is multiply products.

To my loving parents and husband.
Without your support,
I would not pursue Doctor of Philosophy.

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To Past, Present, and Future.

Chapter 1

Introduction

Due to specialization and division of labor, people began to trade with each other since most of them concentrate on a small aspect of production, which raised the level of production efficiency greatly. It soon surmounted boundary limits. Trade between regions can be dated back to 200 BC, when lucrative Chinese silk, as well as many other goods, various technologies, religions and philosophies, traveled along the famous Silk Road, which fueled the development of the civilizations of China, India, Persia, Europe and Arabia significantly.

In modern world, the invention of internet brought a new type of exchanging system, the electronic commerce (e-commerce). Pierre Omidyar, the founder of eBay, did not realize that his AuctionWeb would become a multi-billion dollar business when he tried to sell his broken laser pointer through it in 1994. According to wikinvest, the annul revenue of eBay and Amazon, the largest two online retailing corporations worldwide, grew respectively from \$47.4 million and \$610 million in 1998 to \$11.65 billion and \$48.08 billion in 2011. The trend of online shopping also impacts on the traditional retailers. The online sales of Walmart, the world largest retailer, are estimated to account for 2% of its revenue (\$422 billion) in 2010, or about \$8 billion, according to Deutsche Bank Securities. "There's a lot of value that we get online and a lot of value in the physical store, and at the end of the day we expect the best of both worlds," says Venky Harinarayan, who heads @WalmartLabs. "It's not going to be one channel."

Providing a quick and convenient way of exchanging goods and services disregard of time and geographic limits, e-commerce has boomed. Today, e-commerce has grown into a huge industry of U.S. As shown in Figure 1.1, e-commerce retail is taking an increasingly larger proportion of the total retail, from 0.19% in 1998 to 4.40% in 2010. During this period, retail e-sales increased at an average annual growth rate of 34.82%, while the growth rate of total retail sales was only 3.89%. Although there was a notable recession of the total retail in 2009, the growth rate of e-commerce retail remained positive. According to a preliminary estimation by U.S. Census Bureau of the Department of Commerce, the U.S. retail e-commerce sales for the second quarter of 2012 (adjusted for seasonal variation but not for price changes), was \$54.8 billion, taking a share of 5.1% of total retail sales, an increase of 3.3 percent ($\pm$0.7%) from the first quarter of 2012. Total retail sales for the second quarter of 2012 were estimated at \$1,076.9 billion, a decrease of 0.4

percent ($\pm 0.4\%$) from the first quarter of 2012. The second quarter 2012 e-commerce estimate increased 15.3 percent ($\pm 1.2\%$) from the second quarter of 2011 while total retail sales increased 4.3 percent ($\pm 0.9\%$) in the same period.E-commerce sales in the second quarter of 2012 accounted for 5.1 percent of total sales.

The e-commerce grows with the penetration of Internet. Internet users in U.S. grew from 30.11% in 1998 to 74.25% of population in 2010. Comparing with developed countries, e-commerce in developing countries has a more progressive future due to the relatively low Internet penetration. In South Africa, e-commerce is growing at a rate of around 30 percent a year and the growth is showing no signs of slowing down, while its Internet user grew from 0.89% in 1996 to 12.33% in 2011[1]. Despite its late start, China now has more than twice as many Internet users as that in U.S. According to China Internet Network Information Center, the Internet usage will top 40% in the year 2012. Following the increasing Internet penetration, the e-commerce in China is making rapid progress in recent years. The business-to-customer (B2C) unit of the e-commerce giant Alibaba Group Holding Ltd, Taobao Mall's total sales had hit RMB 100 billion in 2011. Together with the consumer-to-consumer (C2C) unit, Taobao.com, the total transaction in 2011 was RMB 485.6 billion with a daily transaction surge up to RMB 4.38 billion.

The rest of my dissertation is organized as follows. In Chapter 2, I summarize a few previous research papers on the online auction. Chapter 3 contains three models trying to searching for the optimal fee structures of an monopoly auction website. In Model I and Model II, it is assumed that only one type of product is sold in the market. Model I is a fixed-price listing model where the market price is determined by the total supple and demand. Model II is a auction style model simulating the results of Vickrey auction where heterogeneous sellers sell to heterogeneous buyers and sellers obtain the entire willingness to pay from the buyers. However, neither of the two models can explain the coexistence of the two positive fees in practice, listing fee and transaction fee. Thus, in Model III, a model similar to Model II, it is assumed that two type of products are listed at the same time. The results suggest that the optimal listing fee is always positive, while the optimal transaction fee is positive only when there is multiply listings and the differences of buyers' willingness to pay of the products are large enough.

[1]Data source: Internet users World Bank

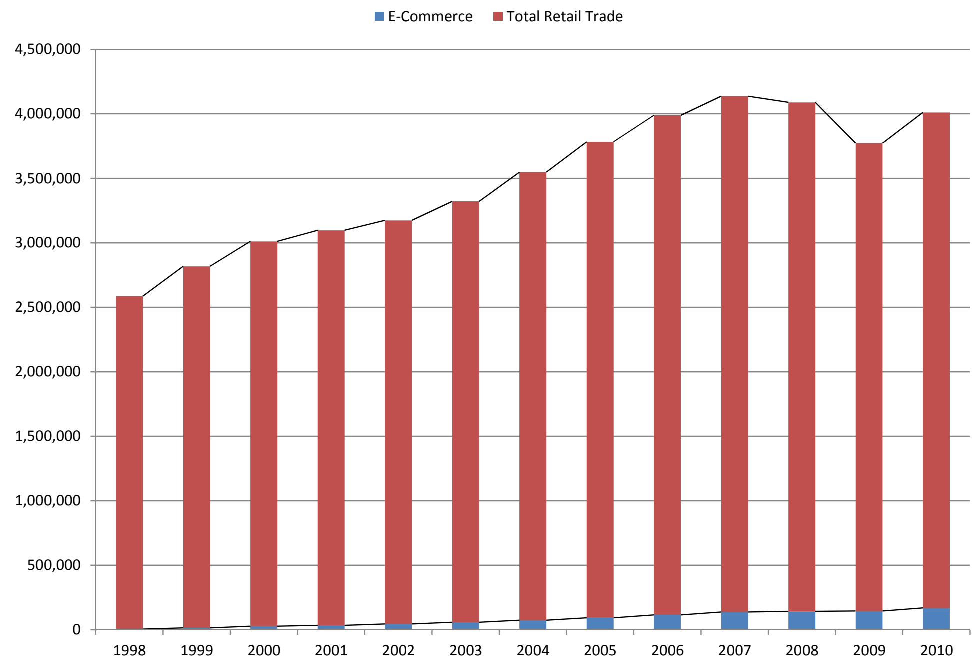


Figure 1.1: U.S. Retail Trade Sales - Total and E-commerce 1998 to 2010 (in millions of dollars)

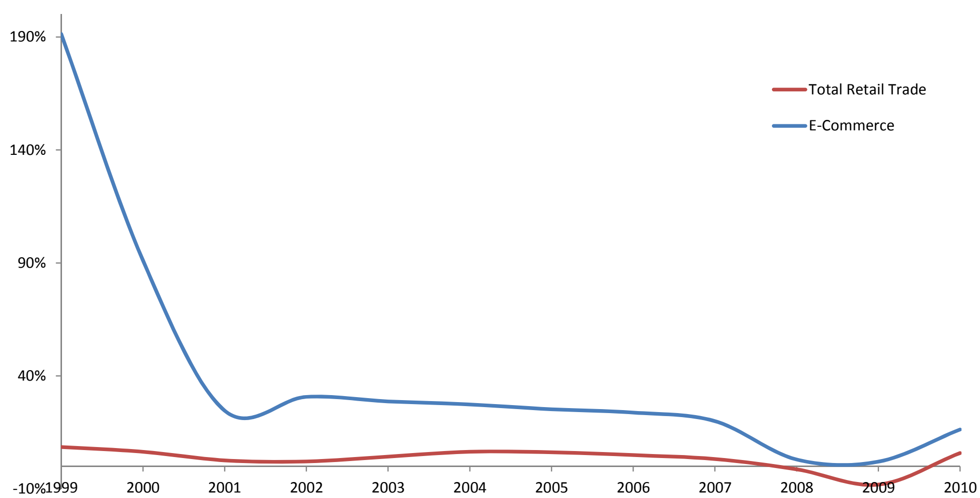


Figure 1.2: The Growth Rate of U.S. Retail Trade Sales - Total and E-commerce

Data Source: U.S. Census Bureau, Annual Retail Trade Survey

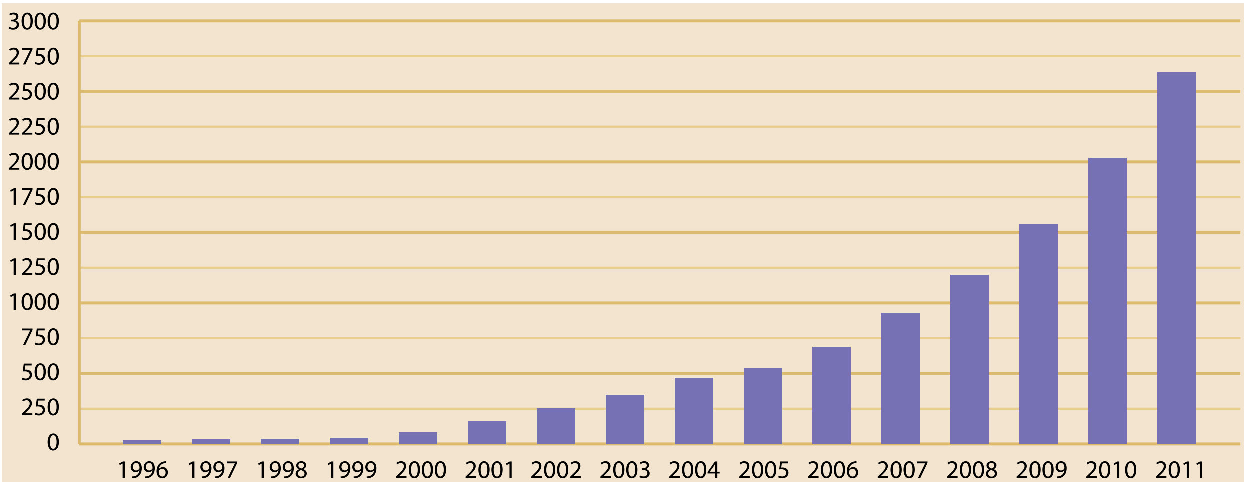


Figure 1.3: Online Retail Revenue in South Africa 1996-2011 (in millions of ZAR)
Data Source: World Wide Worx

Figure 1.4: Online Retail Revenue in China 2007-2014 (in hundred billion of RMB)
Date Source: Iresearch consulting group

Chapter 2

Literature Review

Indubitably, Internet auction, this new type of e-commerce, will play an increasingly significant part of our daily lives and it has become a novelty and exciting area for research. There are several possible explanations that account for its impressive growth. Cohen [2002] chronicles the development of eBay through in-depth interviews of eBay founders, executives, and users. Lucking-Reiley [2000] briefs the history of Internet auctions, and give the results of an extensive survey of 142 different auction websites operating during the fall of 1998. Bajari and Hortacsu [2004] list three factors, which are attributed to the rapid development of Internet auctions: a less-costly marketplace for buyers and sellers on locally thin market, a substitute for traditional market intermediaries, and the fun in participating. Many auction websites create online communities with cheat rooms and forums to allow users communicate with each other, to enhance customer participation and loyal to the site, and thus to increase profits for the sites.

Every day there are millions of sellers list their products onto thousands of auction websites. To help buyers find their desired items, these website organize the listings into different categories. Take eBay for example. There are 35 main categories followed by innumerable numbers of subcategories. Besides, the buyers can also search for listing by designated keywords, such as items sold by which seller, under what titles, or with specific item descriptions provided by the sellers. The searching results can be further refined by categories/subcategories, price range, selling format, shipping options, and et cetera. This allows users to gather considerable information about similar products, which is useful in forming a sale.

There are two typical selling formats, fixed-price listing and auction style listing. The fixed-price listing, also known as "Buy it Now" (BIN) on eBay, allows no bidding, while auction-style listing has several variations, such as auction with reserve price, the auction with BIN option, auction with both reserve price and BIN option. The online auctions usually use proxies simulating Vickrey auction. A buyer can bid the maximum willingness to pay, while the registered bid will only increase with a small increment from the current bid. The maximum bid is kept confidential from the seller and other bidders and it is held by the proxy against other bidders. This buyer will be outbid if someone else submit a bid larger than his maximum bid. Thus, the winner will be the buyer placed the highest bid and he will pay the second highest bid plus a small increment. For example, the current bid for an item is $10.00 and the increment is 50 cents. Bidder A has placed a bid of $18.00 on the item and this amount is sealed from other members. The proxy will put $10.50 as the current high bid. Bidder B views the item and places a bid of $12.00. The proxy then increases the current bid to $12.50 just enough to maintain Bidder A as the highest bidder. Bidder A remains as highest bidder until someone else outbid him. Suppose there is a Bidder C place a bid of $21.00 and wins the auction. Bidder C pays $18.50 as the second highest bid plus 50 cents increment. Therefore, eBay has suggested all bidders should submit their maximum willingness to pay once early in the auction.

2.1 last-minute bidding

The traditional auction theory have difficulties to explain the phenomenon that bids commonly arrive during the last seconds of an internet auction that lasts as long as several days. Vickrey [1961] observes that in a second-price sealed-bid auction with private values, it is a weakly dominant strategy for bidders to bid their true reservation values. The intuition is quite straightforward. If a bidder bids more than his private value, he may win the auction and receive a negative payoff, since the payment will exceed his true private value. If a bidder bids less than his private value, he might loss the auction and receives a zero payoff. However, he could win the auction by bidding his true value and gain a positive payoff.

Despite the attractive theoretical properties of the Vickrey auction format, it is rare in practice before internet auction. Rothkopf et al. [1990] argue that bidders may have concerns about information leakage to relevant parties and auctioneer may cheat based on their true values. In that sense, the online auction bidding proxy bears a nearly perfect second-price sealed-bid auction. The bidders can bid their true willingnesses to pay early in an auction and the bids are confidential to sellers, as well as other bidders. The winner will be the one submitted the highest bid and the payment is the second highest bid plus a small increment. It is obvious that the ”single early bid” strategies suggested by traditional auction theories are best responses to each other.

However, this is clearly not what happens in practice. Roth and Ockenfel [2002] study the second-price auctions run by eBay and Amazon, where 120 eBay Computers with 740 bidders, 120 Amazon Computers with 595 bidders, 120 eBay Antiques with 604 bidders, and 120 Amazon Antiques with 340 bidders are examined. They reveal that in an eBay-style auction with a hard close[1], there is a considerable share of bidders submitted their bids in the last five minutes, 9 percent in Computers auctions and 16 percent in Antiques auctions. Bajari and Hortacsu [2003] notice that more than 50% of final bids are submitted after 90% of the auction duration has passed, and about 32% of the bids are submitted after 97% of the auction has passed (the last two hours of a three-day mint and proof coins auction between September 28 and October 2, 1998.). Winning bids tend to come even later. The median winning bid arrives within the last 73 minutes, and

[1]Amazon, on the other hand, has a soft close, where auctions automatically extend until ten minutes have passed without a bid. Similar, the extensive time is three minutes on uBid.com.

25% of the winning bids arrive in the last eight minutes. However, last-minute bidding may not alway this flurry. Ku et al. [2005], who study 21 live and Internet art auctions throughout North America, where part of the proceeds were donated to charitable causes, observe substantially less late bidding than the previously mentioned studies. Only 1.6 percent of the bids arrive in the last five minutes of the auctions with hard deadlines and 0.5 percent in the auctions with flexible endings.

There is an excessive body of researches on the reason for the popularity of last-minute bidding. The first theory for last-minute bidding is that it may be a form of "tacit collusion" by the bidders against the seller. Roth and Ockenfel [2002] have sent a questionnaire to three hundred and sixty-eight eBay bidders who successfully bid at least once in the last minute of an auction and twenty percent responds to their survey. Most of these bidders unambiguously explains that snipping is a deliberate strategy to avoid a "bidding war" or to keep the price down. The strategic advantages of late bidding are severely attenuated in auctions that apply an automatic extension rule, such as auctions conducted on Amazon. Roth and Ockenfel [2002] and Ockenfels and Roth [2006] find that late bidding appears to be more prevalent in the eBay auctions. On eBay, bids are submitted within the last five minutes in 9 percent of the computer auctions and 16 percent of the antique auctions. On Amazon, about 1 percent of the auctions in these categories receive bids in the last five minutes.

As test of this theory, Gonzalez et al. [2009] use two standard tests[2] for difference of the distribution of the winning bids conditional upon a snipe and the distribution of the winning bids if no snipe occurs, using the data set consists of PC color computer monitors with a size between 14 and 21 inches which were auctioned between February 23, 2000 and June 11, 2000. However, in most of the specifications they examine, they are unable to reject the equality of these two distributions, which is inconsistent with the tacit collusion theory. Similarly, in a data set of bidding for eBay coin auctions, Bajari and Hortacsu [2003] find that reduced form regressions suggests that early bidding activity is not correlated with increased final sales prices.

The second theory for last-minute bidding, also proposed by Roth and Ockenfel [2002], is that it might be a best response to "incremental bidding". For instance, inexperienced bidders might make an analogy with first-price English auctions and be prepared to continually raise their bids to maintain their status as high bidder. Bidding very late would not give those naive bidders sufficient time to respond, and so a sniper competing with such bidders might win the auction at an incremental bidder's initial, low bid. The incremental bidding is not uncommon in internet auctions. Ku et al. [2005] find that most internet auction participants bid more than once in an auction. For example, in Chicago cow auctions, with 466 people (72%) exceeding their pre-set limits a total of 995 times. On average, 13.2 limits were exceeded for each item in the auction. Wilcox [2000][3] argues that more experienced bidders would be more likely to bid during the final

[2] the Kolmogorov-Smirnov and the Rank-Sum test

[3] They collected bidding data from 535 auctions between March 15, 1999 and April 15, 1999 spanning four product categories. The categories for which data were collected included hand-held power drills under the brand name 'DeWalt.', men's neckties under the brand names 'Giorgio Armani' and 'Ermenigildo Zegna' , desk-top staplers, and Rookwood Pottery vases.

moments of the auction. 8.2 percent of the most experienced bidders bid during the last minute, compared to 1.2 percent of the least experienced bidders. Ariely et al. [2005] conduct a controlled laboratory experiment and their experimental results also support that as bidders gain experience, they are more likely to bid late in the eBay conditions.

Late bidding may also be a best response to other incremental bidding behaviors, including that of a seller who uses shill bidders to bid against legitimate bidders in order to unfairly push the price up. Although shilling is strictly forbidden, it is hard to catch because sellers can use anonymous email addresses to bid, or use the assistance of another bidder. Dobrzynski [2000] in the New York Times has talked about this well-publicized examples of shill bidding, illustrating how easy it was for sellers cheating beneath the detective devices eBay used. Despicable as it is, shilling can be evaded by bidding close to the deadline.

A third explanation is that the items sold online might contain common value components whose value have to be estimated by the bidders. One of the common-value elements, discussed by Bajari and Hortacsu [2003], is the bidders' inability to inspect goods in person when placing bids. As pointed out by Milgrom and Weber [1982], the private-values and common-value assumptions can lead to very different bidding patterns and policy prescriptions. Under the assumption of independent private values, the bidder knows the value of the item to themselves with certainty and that value is not related to other bidders' valuations, bidders will be indifferent to the timing of their bids. However, in a common-value auction, the bidders cannot directly observe the true value of the item up for sale. Each bidder receives an imperfect signal about the true value and it is private information. By bidding early, a bidder may signal his information to other bidders and cause them to update their beliefs about the item's true value. Conditional on winning, this may increase the price that a bidder has to pay for the item.

Last-minute biddings, motivated by information about common values, not only allow bidders to incorporate information gathered from the other bidders' earlier bids into their own later bids, but also avoid leaking information to others bidders from their own early bids. Bajari and Hortacsu [2003] report the negative correlation between the bids and the number of bidders. They find that, the "winner's curse" [4] reduces bids by 3.2% for every additional competitor in the auction (with a standard error of 0.4%), which accounts for the late biddings in the coin auctions. Roth and Ockenfel [2002] and Ockenfels and Roth [2006] also notice that there is more last-minute bidding on eBay antiques auctions (16 percent) than in eBay computer auctions (9 percent). They argues that antiques auctions are more likely to possess a common value element than computer auctions. The dealers/experts, who are better able to identify high-value antiques, bid late so as to prevent

[4]Suggested by Milgrom and Weber [1982], the winner's curse test is an empirical test to distinguish between the pure private-values model and the common-value model. In a common-value auction, the Milgrom and Weber model predicts that bidders will rationally lower their bids to prevent a winner's curse from happening. The possibility of a winner's curse is greater in an N-person auction as opposed to an (N-1)-person auction; therefore, the empirical prediction is that the average bid in an N-person second-price or ascending auction is going to be lower than the average bid in an (N -1)-person auction. But in a private-value ascending or second-price auction setting like eBay, the number of bidders should not have an effect on bids since the dominant strategy in these auctions is to bid one's valuation.

other bidders from having time to acquire more precise information on how much they value the object being auctioned. Hence the observed pattern was consistent with the theoretical prediction. Again, in an Amazon-type auction with an automatic extension, the ability to bid without providing information to attentive competitors would be eliminated or substantially attenuated.

A fourth explanation for last-minute bidding is multiplicity of listings. As opposed to the usual assumption that only a single unit of the item is up for sale, Wang [2006], constructs a repeated auction model to explain last minute bidding in online auctions. He find that last-minute bidding is dominant when there is proxy bidding, though there is no "last minute" to bid on for the soft ending rule applied by Amazon. Without repetition, neither proxy bidding nor the fixed ending time per se, could lead to last minute bidding, which to his model is part of the unique equilibrium. In other words, there are multiple units sold which generates last minute bidding behavior Peters and Severinov [2006] build a simultaneous multi-unit auction model similar to eBay and explains that a bidder may bid late since he may consider the possibility that a new seller will enter the market and post a lower reserve price after placing a bid. Ely and Hossain [2009] look at internet auctions of newly released DVD's and find sniping has the higher payoff, which they conclude is because many auctions are run concurrently for the same prize and because some bidders are naive, and behave as if it were a standard English auction.

A fifth explanation for last-minute bidding is that the bidders might have uncertainty about their private valuations. It is not an unreasonable consideration in the online auction. Since the bidders cannot inspect the item up for sale in person when placing a bid, except a few experts or well informed bidders, the others will have a certain level of uncertainty about the authenticity of the item. As in the model of Dan Levin and James Smith (1994), Bennett [2006] assumes that uninformed bidders initially do not know their private information precisely but can pay a fixed fee to learn it. This fixed fee can be considered as the opportunity cost of time required for the learning process. In order to learn their private valuations, bidders will inspect the item and may search for sales prices of previously listed items. He demonstrated that late bidding can occur because bidders wish to economize on the costs of acquiring information.

The empirical finding by Roth and Ockenfel [2002] and Ockenfels and Roth [2006], that last-minute bidding would be less prevalent in flexible-ending rule auctionsis not confirmed in all studies. For example, Ku et al. [2005], Peters and Severinov [2006], and Wang [2006]. Ku et al. [2005] find that for auctions with flexible endings, a greater percentage of the bids arrive in the last hour and the last day than for auctions with hard endings. This is not qualitatively consistent with the prediction proposed by Ockenfels and Roth. However, it is important to note that the authors have limited controls for heterogeneity across auctions that are held in different cities.

2.2 Reputation

Buyers bears more risk when buying from internet. On one hand, the online transaction take place between strangers. Both sellers and buyers are anonymous and their communication are merely on internet, chat rooms or emails. On the other hand, the Buyers need to make payment before receiving the item without inspecting the item in person, which implies that they have to put a considerable amount of money at risk, not mention the time spent in the auction process. Therefore, the websites use several methods to help build a certain level of trust between the unknown sellers and buyers.

The feedback system employed by eBay is a typical method for users to build reputation online. Users on ebay can leave feedback and comments after buying or selling an item. Buyers can leave either positive, negative, or neutral rating to sellers and these ratings are used to determine Feedback scores. The sellers get $+1$ point for each positive rating, 0 points for each neutral rating, and -1 point for each negative rating. In addition to the feedback score, buyers can also rate sellers in 4 additional areas: item as described, communication, shipping time, and shipping and handling charges. These anonymous detailed ratings are based on a one- to five-star scale, where five stars is the highest rating and one star is the lowest rating. Average ratings are computed on a rolling 12-month basis, and will only appear when at least ten ratings have been received. When a buyer clicks a seller's ID, the feedback detail is shown, including the percentage of positive feedback, the feedback score, and the detailed seller rating table plus the number of received ratings.

It is questionable whether the feedback system is a reliable substitution for the traditional trust building system, which allows buyers to inspect the item before payment and the reputation of sellers are built over many years. Resnick and Zeckhauser [2002] examine transactional data from eBay from February 1 to June 30, 1999, including feedback data up to June 30, 1999 and found a high correlation between buyers' and sellers' feedback. In all 36,233 single item auction, sellers received positive feedback 51.3% of the time (18569), neutral or negative feedback 0.4% of the time (173) , no feedback 48.3% of the time (17491), while buyers received positive feedback 59.5% of the time (21560), neutral or negative feedback 1.1% of the time (413) , no feedback 39.4% of the time (14260). It appears that both sides are more likely to receive the same feedback from each other. Sellers receives positive feedback 99.8% of the time when they give positive feedback and buyers receives positive feedback 99.9% of the time when they give positive feedback.

One explanation of the very strong correlation of the positive feedback is that the satisfaction is mutual, the sellers and buyers are both truly happy with their transactions. However, it may also be a result of high courtesy. If one party provided positive feedback, it might create a reciprocal obligation to provide positive feedback in return. There may also be a fear of retaliation for negative feedback. Cabral and Hortacsu [2010] report that a buyer who leaves a negative comment about a seller has a 40-percent chance of getting a negative back from the seller (whereas a neutral comment has a 10-percent chance of being retaliated against). According to the previous eBay spokesman Usher Lieberman, the No. 1 reason buyers cited for decreasing or ceasing their activity on eBay was negative unwarranted retaliatory feedback they received from sellers, even as a bigger problem than not receiving shipment. Thus to obtain more honest feedback from buyers, sellers on eBay can only leave positive feedback (or no feedback at all) about their customers since May 2008.

Despite all the limitations, the internet reputation system has its own advantage. For one thing, any information is nearly costlessly gleaned and can be stored permanently. For another thing, all collected information can be easily transmitted to all potential customers. Thus, any prospective buyer has access to a considerable information about a seller's past performance. Moreover, the scarcity of negative feedback can be a result of consultation. Since the feedback is permanent, eBay encourages buyers to contact sellers to try to resolve problems. Therefore, negative feedback is only as a last resort.

The online reputation matters. Previous research show that buyers react to the sellers' reputation, and thus influence the sellers sales. First, positive feedback increases sale price and negative feedback reduces it. Livingston [2005] collects 861 eBay auctions of Taylor Made Firesole irons, a variety of golf clubs, between October 20, 2000 through August 20, 2001 and his estimated results show that bidder is more likely to place a bid if the seller has more positive reports, though the marginal return is severely diminished. The estimated results of Houser and Wooders [2006], who collect eBay auction data of Pentium III 500 processors during the fall of 1999, suggest that a 10% increase in positive feedback will increase the winning price by about 0.17%, which is smaller in magnitude than the effect of a 10% increase of neutral or negative feedback, which reduct the price by 0.24%. Dewally and Ederington [2006] using Tobit (Heckman) regression, find that the price received by a seller with 100 positive and no negative feedbacks, tends to be about 8.1% (11.0%) higher than that received by a seller with no previous feedbacks and seller with 2.5% negative feedback receives 2.3% (2.7) lower than sellers without any negative feedbacks. Cabral and Hortacsu [2010] construct a panel of eBay seller histories and find that a 1% level increase in the fraction of negative feedback is correlated with a 7.5% decrease in price, however, the estimates have a relatively low level of statistical significance.

The feedback score has also been widely examined and seems to increase revenues quite significantly. Melnik and Alm [2002] study auctions of a 1999 mint condition U.S. $5 gold coin between from May 19 to June 7 of 2000. The estimated results show that the sellers' feedback score have a positive impact on the selling price, which is robust across a variety of specifications though the impact of reputation is generally small[5]. The negative feedback has a negative impact on the selling price and the impact is larger than that of the feedback score. Melnik and Alm [2005] find increases of these reputation effects when there is more uncertainty about the quality of the products, in their case, U.S. silver Morgan dollar coins in "Almost Uncirculated" condition. McDonald and Slawson [2002] collect the dataset of 460 auctions completed between January 1998 and July 1998 of the "collector-quality" first limited-edition Harley-Davidson Barbie doll and find that the difference of selling price between low-reputation seller (less than or equal to the median 21.5) and high-reputation seller (at or above the 90th percentile 140) is $12.09. Dewan and Hsu [2004] find a 10% increase in a seller's feedback score is associated with a average 0.44% increase in eBay's stamp auction price. Also, the field experiment conducted in Resnick et al. [2006] show a highly experienced eBay postcards seller suffer a 8.1% selling price reduction under a newly created, unknown pseudonym. Similar results are also found by Resnick and Zeckhauser [2002], Bajari and Hortacsu [2003], Dewally and Ederington [2006], and Lucking-Reiley et al. [2007].

[5] A seller who doubles his feedback score from 452 to 904, would on average experience an increase in the willingness to pay only by 18 cents, 0.55% of the average current price of the coin at that time.

Second, since the feedback score can be considered as a level of honesty of the sellers, buyers are more willing to buy from sellers with more positive feedback. Livingston [2005] find that comparing with sellers with zero positive feedback, the probability that a bid is placed is 0.034 higher if the seller has from 1 to 25 positive feedback, 0.046 higher for the seller with 26 to 175 positive feedback, 0.051 higher for the seller with 176 to 672 positive reports, and 0.087 higher for the seller with more than 672 positive report. Similar to the impact on the selling price, the returns of positive feedback are severely decreasing, suggesting that after the first couple of feedback have been received, the next several hundred positive feedback have little effect on the chance that at least one bidder places a bid. Cabral and Hortacsu [2010] find that, when a seller first receives negative feedback, his weekly sales rate drops from a positive 5% to a negative 8%; Moreover, subsequent negative feedback arrive 25% more rapidly than the first one and don't have nearly as much impact as the first one. It appears that the sellers with some negative feedbacks are more reliable than the new, unknown sellers. The logistic regression in Resnick and Zeckhauser [2002] show that the probability of a problematic transaction for a newcomer with no previous feedback is 1.91%, comparing with 0.18% for an experienced seller with 100 positive and no negative feedbacks and 0.53% for a seller with 100 positives and three negatives. It is mainly because that the negative feedback would more likely to happen among those active sellers. In the sample of Bajari and Hortacsu [2003], the maximum negative feedback is 21, which corresponds to the seller with the highest feedback score 973.

The negative feedback are supposed to contain more valuable information about the credibility of seller because of its rarity. Resnick et al. [2006] point out that sellers received negative feedback only 1% of the time, and buyers 2% and Bajari and Hortacsu [2003] find that the mean of the negative feedback in their sample is only 0.47. Ba and Pavlou [2002] and Houser and Wooders [2006], as mentioned above, both find that the negative feedback has stronger impact on the selling price than the positive feedback. Using data on eBay coin auctions with different dates and values, Lucking-Reiley et al. [2007] find that a 1% increase in negative feedback causes a 0.11% decrease in auction price on average, while a 1% increase in the seller's positive feedback only yields a 0.03% increase in the auction price on average, and the effect of positive feedback is not statistically significant at the 5% level. Negative feedback also has stronger impact on the probability of sale. The estimation result of Bajari and Hortacsu [2003] show that the negative seller's feedback reduces the number of entrant bidders and the feedback score increases it, and the coefficient of negative feedback is more than twice of that of feedback score.

One of the uncertainty of shopping online is that the buyers need to make payment before receiving the product. Hence, it increases the weight of reputation effect when trading expensive, or ungraded products. Ba and Pavlou [2002] reveal that at a lower level of trust, buyers demand a greater price discount for expensive products than for inexpensive products. When trust reaches a rather high level (7.2 on a 9-point scale), buyers appear to be willing to pay a higher price premium for expensive products. For inexpensive products, however, the relationship between trust and price premiums is not as pronounced. Even at a very high level of trust, buyers still would not be willing to pay a high price premium. Dewan and Hsu [2004] find that comparing with the professional graded stamps sold on Michael Rogers, Inc., the selling price on eBay is 10-15% lower on average, which would be higher without the reputation system. A third party certification can reduce the reputation effects, such as the baseball trading card in Jin and Kato [2006] and collectible comic books in Dewally and Ederington [2006].

2.3 Auction Features

2.3.1 Secret Reserve Price

A reserve price is one of the common auction features of auction-style listings on eBay. A reserve price is the minimum price that a seller is willing to accept for an item. Until the reserve price has been met, the listing shows the message Reserve not met. The reserve price option can allow sellers attract more bidders with a low starting price without worry about selling their items at a price that they feel is too low. The reserve price is hidden from buyers, thus it is often called secret reserve price. However, some sellers include it in the item description or tell buyers when being asked what the reserve price is.

The presence of a reserve price might deter bidder entry and thus affect the selling probability. Therefore, in practice, the sellers use lower minimum bids to attact more bidders. Bajari and Hortacsu [2003] find that the minimum bid, on average 63% of the book value, is negatively correlated with the number of bidders. Sellers who use a secret reserve price turn to keep their minimum bids ever lower. Among auctions with reserve price (14% of all auctions), the average ratio of the minimum bid to the book value is 0.28, with several equal to zero, while the average of this ratio for the auctions with no secret reserve price is 0.69. The presence of reserve price is negatively correlated with the number of bidders and decreases the probability of sale, 49% versus 84%. The presence of a secret reserve price is negatively correlated with revenues, both conditional and unconditional on entry by bidders.

Lucking-Reiley et al. [2007] collect 461 auctions of U.S. Indian Head pennies minted between 1859 and 1909. They find that the presence of a secret reserve price increases the selling price by 15% on average. They explain that the reserve price acts as another competitor, at least until it has been met. Suppose a bidder submits a proxy bid in an auction with a reserve price, if the bid is lower than the reserve price, he will not win the auction; if the bid is higher than the reserve price, the current bid will raise to the amount of the reserve price. In other words, the reserve price might force a bidder bid a larger proportion of his true willingness to pay. However, the overall effect of the reserve price on the expected revenue of the seller is difficult to determine, since the use of reserve price may discourage the bidders' entrance. For their samples, nearly 30% of the auctions had no bids at all.

2.3.2 "Buy it Now" Option

Lucking-Reiley [2000] first notices that online auction sites offer a special feature, such as "Buy it

Now" (BIN) on eBay and "Auction Stop" on LabX, which benefits buyers and sellers by bringing an auction to an end early. Unlike reserve price, the BIN option does not influence the selling probability, but may change the winners of auctions. Take eBay for example. There are several types of listing allowing buyers to purchase an item immediately without waiting for a listing to end. First, auction-style listings with a BIN option, which allows buyers to purchase the items instantly at a fixed price, which is predetermined by the seller and need to be at least 30% higher than the auction starting price. The BIN option on eBay is temporary. It will disappear after the first bid is submitted and the listing will go back to normal auction listing. So, buyers can choose to place a bid and compete in the auction or purchase the item right away at the BIN price [6]. Second, reserve price auctions with a BIN option. The BIN option is shown until the reserve price is met. Third, fixed price listing, which allows no bidding and buyers can simply purchase the item at the BIN price as long as the item is available.

The BIN option is commonly used on eBay. Eaton [2005] finds 44 auctions ended with BIN out of 84 success sale. Also, when browsing the subcategory of "Barbie Contemporary (1973-Now)" on eBay, 154,941 out of 182,173 active listings offer BIN option, which means only less than 15% of the listings use auction style without it[7]. The other auction websites offer such options similar to "BIN" on eBay with some variations, not only on names. For example, "Auction Stop" on LabX, which must be entered 24 hours prior to the auction close or the Auction Stop feature is dropped and the auction continues to the ending date specified. Comparing with BIN on eBay, "Buy Now" on uBid.com is permanent. It is available for the duration of the auction and the auction will close once the entire quantity for a Buy Now auction is sold. For Bid or Buy, the auction style listing do not include a "Buy Now" feature and the Buy Now listing is the same as a fixed price listing allowing no bidding.

A properly set BIN price may increase the expected social welfare and the expected utility of each agent when either buyers or seller are risk-averse. Budish and Takeyama [2001] analyze an English auction augmented with a permanent BIN option. When the two bidders are risk averse, the expected revenue of the seller is increased and it is superior to the first-price sealed-bid and Dutch auctions. Hidvegi et al. [2006] show that this observation holds in an extended model with an arbitrary number of bidders and continuous valuation distributions. Mathews [2003] analyzes an auction model with temporary BIN option mirroring rules of eBay's. When facing risk-neutral buyers, a risk-averse seller choose a BIN price low enough so that the BIN option is exercised with positive probability. Katzman and Mathews [2006] show that this result holds in a similar auction model with reserve price. Reynolds and Wooders [2009] also find that both temporary and permanent BIN options raise the revenue of the seller when facing risk-averse buyers. Moreover, the permanent BIN option raises more revenue than the temporary option.

Time sensitivity is another important reason of introducing the BIN option. Intuitively, an impatient seller facing few patient bidders should set a BIN price low enough so that the BIN

[6] In the following categories, BIN is available until the current bid reaches or exceed 50% of the BIN price: Cell Phones & Accessories, Clothing, Shoes & Accessories, Motors Parts & Accessories, and Tickets.

[7] data is collected on May 27th, 2013

option can exercise with positive probability. The optimal BIN price decreases with the seller's time sensitivity and increases with the bidders' time sensitivity. Wan et al. [2003] report that in the small scale questionnaire (30 respondent out of 266 bidders) they collected, 38.7% respondents agree that the duration of an auction is a consideration in choosing the buyout option and 25% of the respondents replied that an auction with a long duration (7 to 10 days) encourages them to use the BIN option. Mathews [2004] argues that when there is time impatience on either side, the seller will choose a BIN price low enough so that the BIN option is exercised with positive probability. Gallien and Gupta [2007] find when any participant is time sensitive, the seller may significantly increase his utility by offering the BIN option. Furthermore, permanent BIN options yield higher predicted revenue than temporary options.

The multiplicity of listings may also cause the popularity of the BIN option. Kirkegaard and Overgaard [2008] study the BIN option problem under a dynamic environment where two identical items are sold in a sequence of two auctions. Both sellers and bidders are risk-neutral. The bidders has a positive but decreasing valuation for each item. The results suggest that by using the BIN option, the expected revenue of the fist seller increases, but the expected revenue of the second seller decreases, as does the total expected revenue across both auctions. The BIN option hurt the total welfare since the first item may not be awarded to the bidder with highest willingness to pay.

The availability of BIN option also affect the bidders' behavior. Last minute is a common phenomenon in online auctions. By offering BIN option, the bidders can purchase the item immediately and need not to worry about being outbid at the last minute. Though assuming an exogenous Poisson process of bidders' arrivals, Gallien and Gupta [2007] argues that permanent BIN option provide additional incentives for late bidding. Their equilibrium analysis suggests that with a temporary option the first bidder to submit a regular bid will do so immediately upon arrival, while with a permanent option all bidding activities should concentrate near the end of the auction presumably a negative outcome for the seller if bidding activity may be attracting more bidders.

2.4 Fee Structure

Comparing with the traditional auction house, like Sotheby's, who charges 15% of the final bid price from the buyers and 20% of that price from the sellers, the fee charged by the auction websites are much lower. The auction websites usually use fee structures similar to eBay's. At eBay, there is no buyer's premium; all fees are paid by the seller. There are two main components to the seller's fees, insertion fee and final value fee. Insertion fee is a flat rate fee charged when listing an item for sale, depending on the the category of the item and selling format. For auction listing, the insertion fee is calculated by the auction starting price and whether to reserve price or not. For fixed price listing, the insertion fee is only charged once per listing, per category, regardless of the quantity of items. The final value fee is a percentage of the amount of the final bid price and only charged if a success sale is made. Some additional fees are charged for optional promotional services, such as a Gallery Plus (larger the listing picture in search results), Listing Designer (add a theme to the visual appeal of the listings), and etcetera.

Lucking-Reiley [2000] summarizes some statistics of fee structure of auction sites. Among the 107 auction site, 62 charged a seller's commission as a percentage of the final selling price. Of these 62 sites, 28 adopt a percentage of 5% or less, 18 adopt a percentage from 7% to 20%, and the remaining 18 did not give information on the percentage. In addition, 23 sites charge a flat insertion fee to the seller. On the other hand, buyer's premium, common at traditional auction houses, are much less prevalent on the Internet. only 18 of 142 sites charge a buyer's premium, generally between 10% to 15% of the final bid price. Only eight of the sites in the survey charged both a buyer's premium and a seller's commission.

Rochet and Tirole [2003] first notice the significance of the platform. Armstrong [2006] proposes the fee structure should be a two-part tariff, including a lump-sum fee to join the platform and a proportion of the realized market share, so that the revenue of the platform would only partially depends on the its performance. Hagiu [2009] finds the consumer demand for product variety is a key factor determining the optimal platform pricing structures. The stronger demand for variety products makes producers less substitutable and gives them more market power. A monopoly platform is able to extract more profits from producers relative to consumers. However, for competing platforms, this effects, as well as stronger economies of scale in supporting multiple platforms for producers, makes price-cutting strategies on the consumer side is less effective.

Matros and Zapechelnyuk [2008] consider a Vickrey auction model with a seller who has a single item for sale and a large population of bidders mediated by a monopoly auction website. The website charges the seller two fees, a listing fee, a fixed amount regardless of the auction outcome, and a closing fee, a percentage of the final selling price if the item is sold. The seller can re-auction his object until it is sold or consumed by the seller himself. In each of the auction/re-auction, the seller faces a new random drawn from the population of bidders. They find the websites should not charge the sellers any positive lump-sum fee for listing an item to auction. Matros and Zapechelnyuk [2009] generalizes this result to a general class of auction mechanisms. They find an optimal mechanism has a simple implementation as a Vickrey auction with a reserve price and only a fixed percentage from the closing price is charged on the seller. Matros and Zapechelnyuk [2010] show that two different auction websites may coexist in an equilibrium if the population of sellers is sufficiently differentiated in their time preferences. Impatient, "amateur" sellers choose the more popular (with more bidders) but more expensive website, while patient, "professional" sellers choose the less popular and cheaper one. The find that In every equilibrium the listing fees of both auction houses are equal to zero.

In this paper, three parties are taken into concern, a large population of sellers, a large population of buyers, and a monopoly auction websites. The website provides a platform for the interaction between sellers and buyers and adopts the most popular fee structure containing two fees on the sellers. A lump-sum listing fee, L, is charged when sellers list their items for sale, and a transaction fee, δ, is only charged when a successful sale is made as a certain percentage of the final selling price. Each seller has a single object for sale and each buyer wish to buy one unit of the products.

Chapter 3

Model

3.1 Model I

In Model I, we consider the website using classified advertisement with standard sales (as opposed to sales through auctions). It is assumed that the market price p is determined by supply and demand and is not a strategic variable of the sellers (or buyers). Model I simulates a common business-to-consumer / retail model where the market is in equilibrium.

Here, we assume a large population of sellers and each has a single unit item for sale. The sellers are characterized by their reserve prices. The set of sellers is the interval $[0,\infty)$. A seller $s \in [0,\infty)$ is willing to sell his product only if he obtains a price at least s. A large of population of buyers wish to buy one unit of the product each and are characterized by their willingness to pay. The set of buyers is the interval $[0,B]$ and a buyer $t \in [0,B]$ is willing to pay a price at most t. It is assumed that B is a random variable which takes values in the interval $[\underline{B},\overline{B}]$. Let f be the density probability of B and $E(B) = \mu$. Both sellers and buyers are risk-neutral.

Consider the following two-stage game $\mathbb{G}$. In the first stage, the website announces its price scheme (L,δ) and commits to it in later period. Given (L,δ), the sellers can either pay the listing fee L to insert the item for sale or consume it himself. Let $[0,y]$, $y = y(L,\delta)$, be the set of entrant sellers. In the second stage, y becomes fixed and the random variable B is realized. Let b be its realization and $[0,b]$ is the realized set of buyers, $\underline{B} \leq b \leq \overline{B}$. The total demand is then

$$D(b,p) = max\{b-p,0\} \tag{3.1.1}$$

Given (L,δ), an entrant seller s who has already paid the entry fee L is willing to sell his product at a price p iff $(1-\delta)p \geq s$. Hence, the supply at price p is

$$S(p) = min\{y,(1-\delta)p\} \tag{3.1.2}$$

Given (3.1.1) and (3.1.2), the market price p is the one which sets demand equal supply, namely

$$max\{b-p,0\} = min\{y,(1-\delta)p\} \tag{3.1.3}$$

Suppose first that $b \geq p$ and $y \geq (1-\delta)p$, then (3.1.3) is equivalently to

$$b-p = (1-\delta)p$$

then we have

$$p = \frac{b}{2-\delta}$$

The condition $y \geq (1-\delta)p$ is equivalent to

$$b \leq \frac{2-\delta}{1-\delta}y$$

This implies that the market price p is given by

$$p(b,y) = \begin{cases} \frac{b}{2-\delta}, & \text{if } b \leq \frac{2-\delta}{1-\delta}y \\ b-y, & \text{if } b > \frac{2-\delta}{1-\delta}y \end{cases} \tag{3.1.4}$$

The market supply is

$$S(b,y) = \begin{cases} \frac{1-\delta}{2-\delta}b, & \text{if } b \leq \frac{2-\delta}{1-\delta}y \\ y, & \text{if } b > \frac{2-\delta}{1-\delta}y \end{cases}$$

Now consider the payoff of a seller $s \in [0,y]$. When $b \leq \frac{2-\delta}{1-\delta}y$, not all entrant sellers will sell. Assuming the proportional rationing rule, the probability that a seller $s \leq y$ sells his product is $\frac{1-\delta}{2-\delta}\frac{b}{y}$. Given b, the expected revenue of an entrant seller is

$$R(b,y) = \begin{cases} \frac{1-\delta}{2-\delta}\frac{b}{y}(1-\delta)\frac{b}{2-\delta} - L, & \text{if } b \leq \frac{2-\delta}{1-\delta}y \\ (1-\delta)(b-y) - L, & \text{if } b > \frac{2-\delta}{1-\delta}y \end{cases}$$

Integrating over b, the expected payoff of an entrant seller is

$$ER(y) = \int_{\underline{B}}^{\frac{2-\delta}{1-\delta}y} (\frac{1-\delta}{2-\delta})^2 \frac{b^2}{y} f(b)db + \int_{\frac{2-\delta}{1-\delta}y}^{\overline{B}} (1-\delta)(b-y)f(b)db - L$$

provided that $\frac{1-\delta}{2-\delta}\underline{B} \leq y \leq \frac{1-\delta}{2-\delta}\overline{B}$.

We conclude,

$$ER(y) = \begin{cases} \int_{\underline{B}}^{\overline{B}}(1-\delta)(b-y)f(b)db - L, & \text{if } y \leq \frac{1-\delta}{2-\delta}\underline{B} \\ \int_{\underline{B}}^{\overline{B}} \frac{(1-\delta)^2 b^2}{y(2-\delta)^2} f(b)db - L, & \text{if } y \geq \frac{1-\delta}{2-\delta}\overline{B} \\ \int_{\underline{B}}^{\frac{2-\delta}{1-\delta}y}(\frac{1-\delta}{2-\delta})^2 \frac{b^2}{y} f(b)db + \int_{\frac{2-\delta}{1-\delta}y}^{\overline{B}}(1-\delta)(b-y)f(b)db - L, & \text{otherwise} \end{cases} \tag{3.1.5}$$

Given (3.1.5), a seller s will enter the site iff

$$ER(y) \geq s$$

Since the entrant seller with the highest reservation price is y, $ER(y) \geq y$ must hold.

Claim 1

$$ER(y) = y$$

Proof

It is obvious that sellers whose reservation prices exceed $ER(y)$ will not enter the site. Suppose to the contrary $y < ER(y)$. Let $\varepsilon > 0$ be such that $y + \varepsilon < ER(y)$. Then the seller $y + \varepsilon$ does not enter even though the expected payoff exceeds his reservation price, which is a contradiction. ■

Our main result is stated as follows.

Theorem I The game $\mathbb{G}$ has a subgame perfect equilibrium, which is characterized as follows:

(1) Suppose that $\mu \leq 2\underline{B}$. Then listing fee $L^\star = \frac{2-3\delta}{4}\mu$ and transaction fee is any $\delta^\star$ between $[0, \frac{4\underline{B}-2\mu}{4\underline{B}-\mu}]$, the set of sellers is $[0, \frac{\mu}{4}]$ and the expected revenue of the website is $E\pi^\star = \frac{\mu^2}{8}$.

(2) Suppose that $\mu > 2\underline{B}$. Then $\delta^\star = 0$, $0 < L^\star < \mu - \underline{B}$, the set of sellers is $[0, y^\star]$ where $y^\star > \frac{\mu}{4}$ and $E\pi^\star > \frac{1}{2}\underline{B}(\mu - \underline{B})$. The specific value of $y^\star$ and $L^\star$ are the solutions of the two equations:

$$\int_{2y}^{\overline{B}} bf(b)db - 2y\int_{2y}^{\overline{B}} f(b)db - 2y = 0 \qquad \text{(a)}$$
$$\int_{\underline{B}}^{2y} b^2 f(b)db + 2y\int_{2y}^{\overline{B}} bf(b)db - 4Ly = 0 \qquad \text{(b)}$$

(3) The higher μ is, the higher is $y^\star$, the size of the entrant sellers.

The theorem asserts that for $\mu \leq 2\underline{B}$, the outcome does not depend on the specific distribution of B but only on its expected value μ. The optimal transaction fee is not uniquely determined. However, in this paper, we did not take the cost of the website into consideration. If there is a certain level of cost generated in collecting the fees, the website would be better off charging only listing fee. In that case, the optimal fee structure would be $L^\star = \frac{\mu}{2}$ and $\delta^\star = 0$.

When $\mu > 2\underline{B}$, the optimal transaction fee is uniquely determined and $\delta^\star = 0$, the equilibrium outcome does depend on the distribution of B, not only on μ. In this case, the website induce more sellers to enter by charging lower fees.

In every subgame perfect equilibrium, $L^\star > 0$ must hold.

Proof of Theorem I: We consider the following three cases.

Case 1:$y \leq \frac{1-\delta}{2-\delta}\underline{B}$

By (3.1.5) and by Claim 1, the expected revenue of entrant sellers is

$$ER(y) = (1-\delta)(\mu - y) - L = y$$

Thus

$$y = \frac{(1-\delta)\mu - L}{2-\delta} \leq \frac{1-\delta}{2-\delta}\underline{B} \tag{3.1.6}$$

Hence

$$L \geq (1-\delta)(\mu - \underline{B}) \text{ must satisfy}$$

In Case 1, all entrant sellers would sell at price $b-y$. By (3.1.4), the revenue of the website is

$$\pi(b) = Ly + \delta py = Ly + \delta(b-y)y = Ly + \delta by - \delta y^2$$

The expected revenue of the website is then

$$E\pi = E(Ly + \delta by - \delta y^2) = Ly + \delta\mu y - \delta y^2 \tag{3.1.7}$$

The first order condition of $E\pi$ with respect to L is

$$\frac{\partial E\pi}{\partial L} = y + L\frac{\partial y}{\partial L} + \delta\mu\frac{\partial y}{\partial L} - 2\delta y\frac{\partial y}{\partial L}$$

By (3.1.6), $\frac{\partial y}{\partial L} = -\frac{1}{2-\delta}$, this together with (3.1.6) implies that $\frac{\partial E\pi}{\partial L} \geq 0$ iff $L \leq \frac{2-3\delta}{4}\mu$. Consequently,

$$L^{\star} = max\{\frac{2-3\delta}{4}\mu, (1-\delta)(\mu - \underline{B})\}$$

Suppose first $\frac{2-3\delta}{4}\mu \geq (1-\delta)(\mu - \underline{B}), L^{\star} = \frac{2-3\delta}{4}\mu$,

$$(4\underline{B} - \mu)\delta \leq 2(2\underline{B} - \mu) \text{must satisfy}$$

The last inequality determines the following constraints on δ

$$\begin{array}{ll} \text{If } \mu \geq 4\underline{B}, & \delta \geq \frac{2\mu - 4\underline{B}}{\mu - 4\underline{B}} > 1 \\ \text{If } 2\underline{B} < \mu < 4\underline{B}, & \delta \leq \frac{4\underline{B} - 2\mu}{4\underline{B} - \mu} < 0 \\ \text{If } \mu \leq 2\underline{B}, & \delta \leq \frac{4\underline{B} - 2\mu}{4\underline{B} - \mu} \end{array}$$

Thus, the optimal $L^{\star}$ is $\frac{2-3\delta}{4}\mu$ only if $\mu \leq 2\underline{B}$ and $\delta \leq \frac{4\underline{B}-2\mu}{4\underline{B}-\mu}$. Otherwise, $L^{\star} = (1-\delta)(\mu - \underline{B})$.

For $L^\star = \frac{2-3\delta}{4}\mu$, by (3.1.6)

$$y = \frac{(1-\delta)\mu - L^\star}{2-\delta} = \frac{1-\delta-\frac{2-3\delta}{4}}{2-\delta}\mu = \frac{\mu}{4} \tag{3.1.8}$$

By (3.1.7) and (3.1.8)

$$E\pi = Ly + \delta\mu y - \delta y^2 = \frac{2-3\delta}{4}\mu\frac{\mu}{4} + \delta\mu\frac{\mu}{4} - \delta(\frac{\mu}{4})^2 = \frac{\mu^2}{8}$$

irrespective of δ. For $\mu \le 2\underline{B}$, any δ, where $\delta \in [0, \frac{4\underline{B}-2\mu}{4\underline{B}-\mu}]$ together with the related $L^\star = \frac{2-3\delta}{4}\mu$ gives the same revenue level to the website.

Corollary 1.1 Suppose that $y \le \frac{1-\delta}{2-\delta}\underline{B}$ and $\mu \le 2\underline{B}$. The optimal pair $(L^\star, \delta^\star)$ satisfies $L^\star = \frac{2-3\delta^\star}{4}\mu$ and $\delta^\star \in [0, \frac{4\underline{B}-2\mu}{4\underline{B}-\mu}]$. The set of entrant sellers is $[0, \frac{\mu}{4}]$, namely $y^\star = \frac{\mu}{4}$ and the website obtains an expected revenue of $\frac{\mu^2}{8}$, irrespective of the specific choice of $\delta^\star$.

Remark Notice since $\delta^\star \le \frac{4\underline{B}-2\mu}{4\underline{B}-\mu} < \frac{2}{3}$, the optimal listing fee $L^\star$ is always positive.

Suppose next $\mu \le 2\underline{B}$ and $\frac{4\underline{B}-2\mu}{4\underline{B}-\mu} \le \delta \le 1$ or $\mu > 2\underline{B}$, $L^\star = (1-\delta)(\mu - \underline{B})$. By (3.1.6)

$$y = \frac{(1-\delta)\mu - (1-\delta)(\mu-\underline{B})}{2-\delta} = \frac{1-\delta}{2-\delta}\underline{B}$$

Thus by (3.1.7)

$$\begin{aligned} E\pi &= (1-\delta)(\mu-\underline{B})\frac{1-\delta}{2-\delta}\underline{B} + \delta\mu\frac{1-\delta}{2-\delta}\underline{B} - \delta(\frac{1-\delta}{2-\delta}\underline{B})^2 \\ &= \frac{1-\delta}{2-\delta}\underline{B}(\mu - 2\frac{1-\delta}{2-\delta}\underline{B}) \end{aligned}$$

The first order condition of $E\pi$ with respect to δ is

$$\begin{aligned} \frac{\partial E\pi}{\partial\delta} &= -\frac{\underline{B}}{(2-\delta)^2}(\mu - 2\frac{1-\delta}{2-\delta}\underline{B}) + \frac{1-\delta}{2-\delta}\underline{B}\frac{2\underline{B}}{(2-\delta)^2} \\ &= -\frac{\underline{B}}{(2-\delta)^2}(\mu - 4\underline{B}\frac{1-\delta}{2-\delta}) \end{aligned}$$

If the interior solution exists, it satisfies $\frac{\partial E\pi}{\partial\delta} = 0$ and

$$\mu = 4\underline{B}\frac{1-\delta}{2-\delta} \Rightarrow \delta^\star = \frac{4\underline{B}-2\mu}{4\underline{B}-\mu}$$

Since $0 \le \delta \le 1$, $\mu \le 2\underline{B}$ must hold. That is, an interior solution for δ, i.e. $\delta^\star = \frac{4\underline{B}-2\mu}{4\underline{B}-\mu}$, exists only if $\mu \leq 2\underline{B}$, which is included in the optimal solutions derived from the first circumstance, where $L^\star = \frac{2-3\delta}{4}\mu = (1-\delta)(\mu - \underline{B}) = \frac{\mu-\underline{B}}{4\underline{B}-\mu}\mu$.

Next, consider the case where $\mu > 2\underline{B}$. Since $\frac{1-\delta}{2-\delta} \leq \frac{1}{2}$, $\mu > 2\underline{B}$ implies $\mu > 4\underline{B}\frac{1-\delta}{2-\delta}$, where $\frac{\partial E\pi}{\partial \delta} < 0$. Consequently, $\delta^\star = 0$ and $L^\star = \mu - \underline{B}$. In this case, $y^\star = \frac{\underline{B}}{2}$, and by (3.1.7) $E\pi^\star = \frac{\underline{B}(\mu-\underline{B})}{2}$.

Corollary 1.2 Suppose that $y \leq \frac{1-\delta}{2-\delta}\underline{B}$ and $\mu > 2\underline{B}$. The optimal fees are $L^\star = \mu - \underline{B}$ and $\delta^\star = 0$. In this case, the set of entrant sellers is $[0, \frac{\underline{B}}{2}]$ and the expected revenue of the website is $\frac{\underline{B}(\mu-\underline{B})}{2}$.

Remark Note that when $\mu > 2\underline{B}$, Case 1 ($y \leq \frac{1-\delta}{2-\delta}\underline{B}$) does not yield the optimal solution to the website, which will be demonstrated in Case 3.

Case 2: $y \geq \frac{1-\delta}{2-\delta}\overline{B}$

In this case, by Claim 1 and by (3.1.5)

$$ER(y) = \int_{\underline{B}}^{\overline{B}} \frac{(1-\delta)^2 b^2}{y(2-\delta)^2} f(b)db - L = y$$

Equivalently,

$$y^2 + Ly - (\frac{1-\delta}{2-\delta})^2 \int_{\underline{B}}^{\overline{B}} b^2 f(b)db = 0$$

We have

$$y = \frac{\sqrt{L^2 + 4(\frac{1-\delta}{2-\delta})^2 \int_{\underline{B}}^{\overline{B}} b^2 f(b)db} - L}{2}$$

and it needs to satisfy

$$\frac{\sqrt{L^2 + 4(\frac{1-\delta}{2-\delta})^2 \int_{\underline{B}}^{\overline{B}} b^2 f(b)db} - L}{2} \geq \frac{1-\delta}{2-\delta}\overline{B}$$

This is equivalent to

$$L^2 + 4(\frac{1-\delta}{2-\delta})^2 \int_{\underline{B}}^{\overline{B}} b^2 f(b)db \geq (2\frac{1-\delta}{2-\delta}\overline{B} + L)^2 = 4(\frac{1-\delta}{2-\delta})^2\overline{B}^2 + 4\frac{1-\delta}{2-\delta}\overline{B}L + L^2$$

rearranging terms, we have

$$(\frac{1-\delta}{2-\delta})^2 \int_{\underline{B}}^{\overline{B}} b^2 f(b)db \geq (\frac{1-\delta}{2-\delta})^2\overline{B}^2 + \frac{1-\delta}{2-\delta}\overline{B}L \tag{3.1.9}$$

Since $b \in [\underline{B}, \overline{B}]$,

$$\int_{\underline{B}}^{\overline{B}} b^2 f(b)db < \overline{B}^2$$

which contradicts (3.1.9).

Corollary 2 There is no solution to the case $y \geq \frac{1-\delta}{2-\delta}\overline{B}$.

Case 3: $\frac{1-\delta}{2-\delta}\underline{B} \leq y \leq \frac{1-\delta}{2-\delta}\overline{B}$

In this case, by Claim 1 and again (3.1.5)

$$ER(y) = \frac{(1-\delta)^2}{y(2-\delta)^2}\int_{\underline{B}}^{\frac{2-\delta}{1-\delta}y} b^2 f(b)db + (1-\delta)\int_{\frac{2-\delta}{1-\delta}y}^{\overline{B}} (b-y)f(b)db - L = y \tag{3.1.10}$$

The measure y of the entrant sellers is the solution of (3.1.10) or equivalently the solution of

$$\frac{(1-\delta)^2}{(2-\delta)^2}\int_{\underline{B}}^{\frac{2-\delta}{1-\delta}y} b^2 f(b)db + (1-\delta)y\int_{\frac{2-\delta}{1-\delta}y}^{\overline{B}} (b-y)f(b)db - Ly - y^2 = 0 \tag{3.1.11}$$

Denote $x = \frac{2-\delta}{1-\delta}y$. Then $\underline{B} \leq x \leq \overline{B}$ and $y = \frac{1-\delta}{2-\delta}x$. Now (3.1.11) can be rewritten as

$$\frac{(1-\delta)^2}{(2-\delta)^2}\int_{\underline{B}}^{x} b^2 f(b)db + \frac{(1-\delta)^2}{2-\delta}x\int_{x}^{\overline{B}} bf(b)db - \frac{(1-\delta)^3}{(2-\delta)^2}x^2\int_{x}^{\overline{B}} f(b)db - \frac{1-\delta}{2-\delta}Lx - (\frac{1-\delta}{2-\delta}x)^2 = 0$$

Let

$$F(x) \equiv \frac{1-\delta}{2-\delta}\int_{\underline{B}}^{x} b^2 f(b)db + (1-\delta)x\int_{x}^{\overline{B}} bf(b)db - \frac{(1-\delta)^2}{2-\delta}x^2\int_{x}^{\overline{B}} f(b)db - Lx - \frac{1-\delta}{2-\delta}x^2 \tag{3.1.12}$$

Lemma 1 Suppose $L \leq (1-\delta)(\mu - \underline{B})$. There exists a unique $x \in [\underline{B}, \overline{B})$ such that $F(x) = 0$.
The proof is deferred to Appendix A.

Lemma 2 $L = (1-\delta)(\mu - \underline{B})$ iff $x = \underline{B}$.

Proof By (3.1.12),

$$\begin{aligned} F(\underline{B}) &= (1-\delta)\underline{B}\int_{\underline{B}}^{\overline{B}} bf(b)db - \frac{(1-\delta)^2}{2-\delta}\underline{B}^2\int_{\underline{B}}^{\overline{B}} f(b)db - L\underline{B} - \frac{1-\delta}{2-\delta}\underline{B}^2 \\ &= (1-\delta)\underline{B}\mu - \frac{(1-\delta)^2}{2-\delta}\underline{B}^2 - L\underline{B} - \frac{1-\delta}{2-\delta}\underline{B}^2 \\ &= \underline{B}[(1-\delta)(\mu - \underline{B}) - L] \end{aligned} \tag{3.1.13}$$

Suppose first $L = (1-\delta)(\mu - \underline{B})$. By (3.1.13), $F(\underline{B}) = 0$. By lemma 1, $x = \underline{B}$.
Suppose next $x = \underline{B}$, that is $F(\underline{B}) = 0$. By (3.1.13), $L = (1-\delta)(\mu - \underline{B})$. ■

Corollary 3 For every $\{(L,\delta)|L \leq (1-\delta)(\mu - \underline{B})\}$, there exists a unique $y = y(L,\delta)$, $y \in [\frac{1-\delta}{2-\delta}\underline{B}, \frac{1-\delta}{2-\delta}\overline{B})$ such that $ER(y) = y$.

Let us next find the optimal (L,δ) in the set $\{(L,\delta)|L \leq (1-\delta)(\mu-\underline{B})\}$ that maximize the expected revenue of the website. First note that when $b \leq \frac{2-\delta}{1-\delta}y$ ($y \geq \frac{1-\delta}{2-\delta}b$), the market price is $\frac{b}{2-\delta}$ and only sellers with reservation prices no higher than $\frac{1-\delta}{2-\delta}b$ are willing to sell. The transaction fee collected by the website in this case is $\delta\frac{b}{2-\delta}\frac{1-\delta}{2-\delta}b$. When $b > \frac{2-\delta}{1-\delta}y$, the realized demand exceeds the supply provided by the entrant sellers, and all entrant sellers sell at the market price $b-y$. The transaction fee collected by the website in this case is $\delta(b-y)y$. Consequently, the expected revenue of the website is

$$\begin{aligned} E\pi(y) &= Ly+\delta[\int_{\underline{B}}^{\frac{2-\delta}{1-\delta}y}\frac{b}{2-\delta}\frac{1-\delta}{2-\delta}bf(b)db+\int_{\frac{2-\delta}{1-\delta}y}^{\overline{B}}(b-y)yf(b)db] \\ &= Ly+\frac{\delta(1-\delta)}{(2-\delta)^2}\int_{\underline{B}}^{\frac{2-\delta}{1-\delta}y}b^2f(b)db+\delta y\int_{\frac{2-\delta}{1-\delta}y}^{\overline{B}}bf(b)db-\delta y^2\int_{\frac{2-\delta}{1-\delta}y}^{\overline{B}}f(b)db \end{aligned}$$

Substituting for $x=\frac{2-\delta}{1-\delta}y$, we have

$$\begin{aligned} E\pi(x) &= \frac{1-\delta}{2-\delta}Lx+\frac{\delta(1-\delta)}{(2-\delta)^2}\int_{\underline{B}}^{x}b^2f(b)db+\frac{\delta(1-\delta)}{2-\delta}x\int_{x}^{\overline{B}}bf(b)db-\delta(\frac{1-\delta}{2-\delta}x)^2\int_{x}^{\overline{B}}f(b)db \\ &= \frac{1-\delta}{2-\delta}Lx+\frac{\delta}{2-\delta}[\frac{1-\delta}{2-\delta}\int_{\underline{B}}^{x}b^2f(b)db+(1-\delta)x\int_{x}^{\overline{B}}bf(b)db-\frac{(1-\delta)^2}{2-\delta}x^2\int_{x}^{\overline{B}}f(b)db] \\ &= \frac{1-\delta}{2-\delta}Lx+\frac{\delta}{2-\delta}[F(x)+Lx+\frac{1-\delta}{2-\delta}x^2] \end{aligned}$$

By Lemma 1, there exists $x=x(L,\delta)$ such that $F(x)=0$ when $L \leq (1-\delta)(\mu-\underline{B})$. Then for $\{(L,\delta)|L \leq (1-\delta)(\mu-\underline{B})\}$,

$$E\pi(x)=\frac{L}{2-\delta}x+\frac{\delta(1-\delta)}{(2-\delta)^2}x^2 \tag{3.1.14}$$

Lemma 3 For $y \in [\frac{1-\delta}{2-\delta}\underline{B}, \frac{1-\delta}{2-\delta}\overline{B})$, the optimal listing fee $L^\star=(1-\delta)(\mu-\underline{B})$ if $\mu \leq 2\underline{B}$ and $0 \leq \delta \leq \frac{4\underline{B}-2\mu}{4\underline{B}-\mu}$. Otherwise, $L^\star$ is an interior solution and $L^\star < (1-\delta)(\mu-\underline{B})$.

The proof is deferred to Appendix B.

Claim 2 For $y \in [\frac{1-\delta}{2-\delta}\underline{B}, \frac{1-\delta}{2-\delta}\overline{B})$, $L^\star=(1-\delta)(\mu-\underline{B})$ is not optimal for the website.

Proof By Lemma 3, the optimal listing fee $L^\star=(1-\delta)(\mu-\underline{B})$ when $\mu \leq 2\underline{B}$ and $0 \leq \delta \leq \frac{4\underline{B}-2\mu}{4\underline{B}-\mu}$. In this case, by Lemma 2, $x^\star=\underline{B}$. By (3.1.14), the expected revenue of the website is

$$E\pi(\underline{B})=\frac{(1-\delta)(\mu-\underline{B})}{2-\delta}\underline{B}+\frac{\delta(1-\delta)}{(2-\delta)^2}\underline{B}^2=\frac{1-\delta}{2-\delta}\mu\underline{B}-2(\frac{1-\delta}{2-\delta})^2\underline{B}^2$$

which is exceeded by $\frac{\mu^2}{8}$, the expected revenue of the website in Case 1 when $\mu \leq 2\underline{B}$,

$$\begin{aligned}\frac{\mu^2}{8} - E\pi(\underline{B}) &= \frac{\mu^2}{8} - \frac{1-\delta}{2-\delta}\mu\underline{B} + 2(\frac{1-\delta}{2-\delta})^2\underline{B}^2 \\ &= \frac{1}{8}(\mu - \frac{1-\delta}{2-\delta}4\underline{B})^2 \geq 0\end{aligned}$$

Hence $L^\star = (1-\delta)(\mu - \underline{B})$ is weakly dominated by $(L^\star, \delta^\star)$ shown in Corollary 1.1. ■

It is left to check the case that $L^\star$ is an interior solution. Notice it is the case when $\mu > 2\underline{B}$ or $\mu \leq 2\underline{B}$ and $\delta \geq \frac{2(2\underline{B}-\mu)}{4\underline{B}-\mu}$. The interior solution $L^\star < (1-\delta)(\mu - \underline{B})$.

Lemma 4 For $y \in [\frac{1-\delta}{2-\delta}\underline{B}, \frac{1-\delta}{2-\delta}\overline{B})$, the optimal transaction fee $\delta^\star = 0$.

The proof is deferred to Appendix C.

Recall for the interior solution case, the inequalities

$$L^\star < (1-\delta^\star)(\mu - \underline{B}) \text{ and } g(\underline{B}) = \mu - \frac{1-\delta}{2-\delta}4\underline{B} > 0$$

must hold. Therefore, when $\mu > 2\underline{B}$, the optimal fee scheme $(L^\star, \delta^\star)$ such that $0 < L^\star < \mu - \underline{B}$ and $\delta^\star = 0$.

Our next step is to find the optimal $(L^\star, \delta^\star)$ when $\mu > 2\underline{B}$. We need to compare the expected revenue of the website with $\frac{\underline{B}(\mu-\underline{B})}{2}$, the expected revenue of the website in Case 1 when by choosing $\{(L^\star, \delta^\star) | L^\star = \mu - \underline{B}, \delta^\star = 0\}$.

By (3.1.14), for any $L < \mu - \underline{B}$, at $x = x(L, 0)$,

$$E\pi(x) = \frac{1}{2}Lx = \frac{1}{2}Lx(L, 0) \tag{3.1.15}$$

and by (B.0.3), for $x = x(L, 0)$,

$$\frac{\partial E\pi}{\partial L} = \frac{1}{2}\frac{\partial x}{\partial L}[\int_x^{\overline{B}} bf(b)db - x\int_x^{\overline{B}} f(b)db - x]$$

Since $L^\star > 0$ (as $\delta^\star = 0$), $\frac{\partial E\pi(x(L^\star,0))}{\partial L} = 0$ holds and therefore for $x^\star = x(L^\star, 0)$

$$g(x^\star) = \int_{x^\star}^{\overline{B}} bf(b)db - x^\star\int_{x^\star}^{\overline{B}} f(b)db - x^\star = 0 \tag{3.1.16}$$

This proves formula (a) of part (2) of the theorem.

We can rewrite $g(x^\star)$ as

$$g(x^\star) = \mu - \int_{\underline{B}}^{x^\star} bf(b)db - x^\star[1 - J(x^\star)] - x^\star = \mu - \int_{\underline{B}}^{x^\star} bf(b)db + x^\star J(x^\star) - 2x^\star$$

suppose $J'(x) = f(x)$. Integrating by part, we have

$$\begin{aligned} g(x^\star) &= \mu - bJ(b)|_{\underline{B}}^{x^\star} + \int_{\underline{B}}^{x^\star} J(b)db + x^\star J(x^\star) - 2x^\star \\ &= \mu + \int_{\underline{B}}^{x^\star} J(b)db - 2x^\star = 0 \end{aligned} \tag{3.1.17}$$

since $J(\overline{B}) = 1$ and $J(\underline{B}) = 0$. Notice that by (3.1.17),

$$g(\frac{\mu}{2}) = \mu + \int_{\underline{B}}^{\frac{\mu}{2}} J(b)db - \mu > 0$$

Since $g(x)$ is strictly decreasing in $[\underline{B}, \overline{B}]$, for $x^\star = x(L^\star, 0)$ such that $g(x^\star) = 0$, we have $x^\star > \frac{\mu}{2}$. Since $x^\star = 2y^\star$, we have $y^\star > \frac{\mu}{4}$ as claimed.

Notice that by (3.1.12) and $F(x^\star) = 0$, we have

$$\frac{1}{2}\int_{\underline{B}}^{x^\star} b^2 f(b)db + x^\star \int_{x^\star}^{\overline{B}} bf(b)db - \frac{1}{2}(x^\star)^2 \int_{x^\star}^{\overline{B}} f(b)db - Lx^\star - \frac{1}{2}(x^\star)^2 = 0 \tag{3.1.18}$$

By (3.1.16) and (3.1.18),

$$\frac{\int_{\underline{B}}^{x^\star} b^2 f(b)db + x^\star \int_{x^\star}^{\overline{B}} bf(b)db}{2} = L^\star x^\star \tag{3.1.19}$$

This proves formula (b) of Part (2) of the Theorem. By (3.1.19), we have

$$E\pi^\star = \frac{\int_{\underline{B}}^{x^\star} b^2 f(b)db + x^\star \int_{x^\star}^{\overline{B}} bf(b)db}{4}$$

Note that $x = \underline{B}$ (the equilibrium outcome of Case 1) is not a solution to (3.1.16). Since $x = \underline{B}$ belongs to both Case 1 and Case 3, by Claim 2, we conclude that the expected payoff of the website is larger than $\frac{\underline{B}(\mu - \underline{B})}{2}$, the expected revenue of the website in Case 1 when $\mu > 2\underline{B}$.

Thus when $\mu > 2\underline{B}$, the optimal transaction fee is $\delta^\star = 0$. In this case, $x^\star > \frac{\mu}{2}$ and $x^\star$ is determined by the equation (3.1.16). The optimal listing fee $0 < L^\star < \mu - \underline{B}$ and $L^\star$ is determined by the equation (3.1.18) (or (3.1.19)). The values of $x^\star$ and $L^\star$ depend on the specific distribution of B and not only on its expected value μ. By formula (b) applied to the case where $\mu > 2\underline{B}$,

$$\mu = 4y^\star - \int_{\underline{B}}^{2y^\star} J(b)db$$

The right-hand side of the equality is increasing in $y^\star$. Hence the higher μ is, the higher is $y^\star$. This is also true for the case where $\mu \leq 2\underline{B}$ since $y^\star = \frac{\mu}{4}$. This proves part (3) of the Theorem I and the proof of Theorem I is complete.■

3.2 Model II

The framework of Model II is similar to Model I. The main difference is that in Model II, the prices differ across sellers and these prices resemble those generated by second-price auctions, i.e., the buyers pay their entire willingness to pay. Thus a seller s sells his product to a random buyer t, $t \geq s$, who pays her entire willingness to pay, namely t.

For simplicity, it is assumed that the set of buyers $[0,B]$ and the set of sellers $[0,S]$ are commonly known. The buyers and the sellers are uniformly distributed on $[0,B]$ and $[0,S]$, respectively. The density functions are therefore

$$f_B(t) = \begin{cases} \frac{1}{B} & 0 \leq t \leq B \\ 0 & \text{otherwise} \end{cases}$$

and

$$f_s(s) = \begin{cases} \frac{1}{S} & 0 \leq s \leq S \\ 0 & \text{otherwise} \end{cases}$$

3.2.1 Proportional Rationing Rule (PRR)

Under the proportional Rationing Rule (PRR), it is assumed that all buyers whose willingness to pay are higher than the price have equal probabilities in getting the item for sale. Together with the uniform distribution assumption, a seller s sells his product at expected price of $\frac{B+s}{2}$, given $B \geq s$.

The expected revenue of an entrant seller $s \in [0,S]$ is

$$R(s) = -L + \frac{(1-\delta)(B+s)}{2} \tag{3.2.1}$$

A seller s enters the website iff $R(s) \geq s$. The size of entrant sellers is therefore

$$y(L,\delta) = \max[\min\{\frac{(1-\delta)B-2L}{1+\delta}, S\}, 0]$$

Namely, the set of sellers that are willing to pay the listing fee is $[0, y(L,\delta)]$. It is clear that any (L,δ) such that $\frac{(1-\delta)B-2L}{1+\delta} \leq 0$ is not optimal for the website. Thus, without loss of generality, we can assume that $L < \frac{1-\delta}{2}B$. Thus, the size of entrant seller is

$$y(L,\delta) = \begin{cases} S, & \text{if } L \leq \frac{(1-\delta)B-(1+\delta)S}{2} \\ \frac{(1-\delta)B-2L}{1+\delta}, & \text{if } \frac{(1-\delta)B-(1+\delta)S}{2} < L < \frac{1-\delta}{2}B \end{cases} \tag{3.2.2}$$

The expected revenue the website extracts from a seller s is the listing fee L plus δ times the expected price s received, i.e.,

$$\pi(s) = L + \frac{\delta(B+s)}{2}$$

Thus the expected revenue of the website is

$$\begin{aligned} E\pi &= Ly + \delta \int_0^y \frac{B+s}{2} ds \\ &= Ly + \frac{\delta}{2}(By + \frac{1}{2}y^2) \\ &= Ly + \frac{\delta}{2}By + \frac{\delta}{4}y^2 \end{aligned} \tag{3.2.3}$$

<u>Case 1</u> Suppose $L \leq \frac{(1-\delta)B-(1+\delta)S}{2}$

By (3.2.2) and (3.2.3), the expected revenue of the website is

$$E\pi = LS + \frac{\delta}{2}BS + \frac{\delta}{4}S^2$$

Since $E\pi$ is increasing with the listing fee L,

$$L^\star = \frac{(1-\delta)B-(1+\delta)S}{2}$$

The expected revenue of the website is then

$$E\pi = \frac{(1-\delta)B-(1+\delta)S}{2}S + \frac{\delta}{2}BS + \frac{\delta}{4}S^2 = \frac{S(B-S)}{2} - \frac{\delta}{4}S^2$$

Since the expected revenue of the website is decreasing with the transaction fee δ,

$$\delta^\star = 0 \text{ and } L^\star = \frac{B-S}{2}$$

In this case, the set of entrant seller is $[0,S]$ and the website obtains an expected revenue $\frac{S(B-S)}{2}$.

<u>Case 2</u> Suppose $\frac{(1-\delta)B-(1+\delta)S}{2} < L < \frac{1-\delta}{2}B$

By (3.2.3), the first order condition with respect to L is

$$\frac{\partial E\pi}{\partial L} = y + (L + \frac{\delta B}{2})\frac{\partial y}{\partial L} + \frac{\delta}{2}y\frac{\partial y}{\partial L} \tag{3.2.4}$$

By (3.2.2), the first order derivative of y with respect to L is

$$\frac{\partial y}{\partial L} = -\frac{2}{1+\delta}$$

Thus (3.2.4) can be written as

$$\frac{\partial E\pi}{\partial L} = \frac{(1-\delta)B - 2L}{1+\delta} - (L + \frac{\delta B}{2})\frac{2}{1+\delta} - \frac{\delta}{2}\frac{(1-\delta)B-2L}{1+\delta}\frac{2}{1+\delta}$$

and

$$\frac{\partial E\pi}{\partial \delta} \geq 0 \text{ iff } L \leq \frac{1-2\delta-\delta^2}{2(2+\delta)}B$$

It is easy to verify that $\frac{1-2\delta-\delta^2}{2(2+\delta)}B < \frac{(1-\delta)B}{2}$. Thus

$$L^\star = \max\{\frac{1-2\delta-\delta^2}{2(2+\delta)}B, \frac{(1-\delta)B-(1+\delta)S}{2}\}$$

The condition $\frac{1-2\delta-\delta^2}{2(2+\delta)}B > \frac{(1-\delta)B-(1+\delta)S}{2}$ is equivalent to $B < (2+\delta)S$. Hence

$$L^\star = \begin{cases} \frac{1-2\delta-\delta^2}{2(2+\delta)}B, & \text{if } \delta > \frac{B}{S} - 2 \\ \frac{(1-\delta)B-(1+\delta)S}{2}, & \text{if } \delta \leq \frac{B}{S} - 2 \end{cases} \tag{3.2.5}$$

Notice here we allow the optimal listing fee to be negative.

<u>**Subcase 2.1**</u> $\delta > \frac{B}{S} - 2$

By (3.2.5) and (3.2.2),

$$L^\star = \frac{1-2\delta-\delta^2}{2(2+\delta)}B \text{ and } y(L^\star, \delta) = \frac{B}{2+\delta} < S$$

By (3.2.3), the expected revenue of the website is

$$E\pi = \frac{1-2\delta-\delta^2}{2(2+\delta)}B\frac{B}{2+\delta} + \frac{\delta}{2}B\frac{B}{2+\delta} + \frac{\delta}{4}(\frac{B}{2+\delta})^2 = \frac{B^2}{4(2+\delta)}$$

Since $E\pi$ is decreasing with δ, the website would choose

$$\delta^\star = 0 \text{ and } L^\star = \frac{B}{4}$$

The condition $\delta > \frac{B}{S} - 2$ is equivalent to $B < 2S$. The set of sellers that enter the website is $[0, \frac{B}{2}]$, half the size of the buyers. The expected revenue of the website is $\frac{B^2}{8}$, which increases with B, the size of the buyers.

Recall in Case 1, the website obtains an expected revenue $\frac{S(B-S)}{2}$ when choosing $\delta^\star = 0$ and

$L^{\star} = \frac{B-S}{2}$. Since

$$\frac{B^2}{8} \geq \frac{S(B-S)}{2} \text{ holds for all } B \text{ and } S$$

Thus when $B < 2S$, the website is better off by charging

$$L^{\star} = \frac{B}{4} \text{ and } \delta^{\star} = 0$$

and receives an expected revenue of $\frac{B^2}{8}$. In this case, the set of entrant seller is $[0, \frac{B}{2}]$.

<u>**Subcase 2.2**</u> $\delta \leq \frac{B}{S} - 2$

By (3.2.5),

$$L^{\star} = \frac{(1-\delta)B - (1+\delta)S}{2}$$

By (3.2.2), the size of entrant seller is

$$y(L^{\star}, \delta) = \frac{(1-\delta)B - 2\frac{(1-\delta)B-(1+\delta)S}{2}}{1+\delta} = S$$

By (3.2.3), the expected revenue of the website is

$$E\pi = \frac{(1-\delta)B - (1+\delta)S}{2}S + \frac{\delta B}{2}S + \frac{\delta}{4}S^2 = \frac{B-S}{2}S - \frac{\delta}{4}S^2$$

Since $E\pi$ is decreasing with δ, the optimal transaction fee $\delta^{\star} = 0$, and $L^{\star} = \frac{B-S}{2}$. The condition $\delta \leq \frac{B}{S} - 2$ is equivalent to $B \geq 2S$. In this case, all sellers enter the website and the expected revenue of the website is $E\pi = \frac{S(B-S)}{2}$. The equilibrium outcomes are the same as that in Case 1.

We summarize our result as follows:

<u>Theorem 2.1</u> Consider Model II with PRR. It has a unique subgame perfect equilibrium. Irrespective of B, $\delta^{\star} = 0$ and $L^{\star} > 0$. More precisely,

i If $B \leq 2S$, $\delta^{\star} = 0$, $L^{\star} = \frac{B}{4}$. The set of entrant sellers is $[0, \frac{B}{2}]$, half of the buyers. The expected revenue of the website is $\frac{B^2}{8}$. An entrant seller s, $0 \leq s \leq \frac{B}{2}$, obtains a net payoff of $\frac{B-2s}{4}$, which is increasing in the buyers' size B.

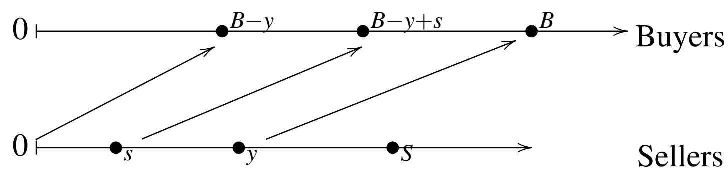
ii If $B \geq 2S$, $\delta^{\star} = 0$, $L^{\star} = \frac{B-S}{2}$. The set of entrant sellers is $[0, S]$, less than half of the buyers. The expected revenue of the website is $\frac{S(B-S)}{2}$. An entrant seller s, $0 \leq s \leq S$, obtains a net payoff of $\frac{S-s}{2}$, irrespective of B.

iii The size of the set of entrant sellers is at most half the size of buyers.

The higher the size of the buyers is, the higher is the expected price a seller receives and the higher is the expected revenue of the website. When $B \leq 2S$, the size of the set of entrant sellers increases with B and when $B \geq 2S$, all sellers enter. The listing fee increases with B and the optimal transaction fee is zero.

3.2.2 Efficient Rationing Rule

Under the efficient rationing rule (ERR), it is assumed that buyers with higher willingness to pay buy from sellers with higher reservation prices.



Hence the revenue of a seller s if enters is

$$R(s) = -L + (1-\delta)(B - y + s) \tag{3.2.6}$$

A seller s is willing to sell his product iff $R(s) \geq s$ and the last entrant seller, y, satisfies $R(y) = y$. Namely

$$y = \max\{\min[-L + (1-\delta)B, S], 0\}$$

Since for $L \geq (1-\delta)B$, there is no sellers are willing to enter the website. Thus without loss of generality, we assume $L < (1-\delta)B$. Thus, the size of entrant seller is

$$y(L,\delta) = \begin{cases} S, & \text{if } L \leq (1-\delta)B - S \\ -L + (1-\delta)B, & \text{if } (1-\delta)B - S < L < (1-\delta)B - S \end{cases} \tag{3.2.7}$$

<u>**Case 1**</u> **Suppose** $L \leq (1-\delta)B - S$

In this case, all sellers enter the website. The website obtains from an entrant seller s

$$\pi(s) = L + \delta(B - y + s) = L + \delta(B - S + s)$$

Thus the expected revenue of the website is

$$E\pi = [L + \delta(B-S)]S + \int_0^S sds = LS + \delta(B-S)S + \frac{\delta}{2}S^2$$

Since $E\pi$ is increasing with L, $L^\star = (1-\delta)B - S$. Thus

$$E\pi = [(1-\delta)B - S + \delta(B-S)]S + \frac{\delta}{2}S^2 = S(B-S) - \frac{\delta}{2}S^2$$

Since the expected revenue of the website is decreasing with δ, the website would charge

$$\delta^{\star} = 0 \text{ and } L^{\star} = B - S$$

The set of entrant seller is $[0, S]$. The expected revenue of the website $E\pi = S(B-S)$.

<u>Case 2</u> Suppose $(1-\delta)B - S \le L < (1-\delta)B$

In this case, the size of the entrant sellers $y = -L + (1-\delta)B \in (0, S]$. The website obtains from an entrant seller s

$$\pi(s) = L + \delta(B - y + s) = \delta^2 B + (1+\delta)L + \delta s$$

Hence, the expected revenue of the website is

$$\begin{aligned} E\pi &= [\delta^2 B + (1+\delta)L]y + \delta \int_0^y s ds \\ &= \delta^2 By + (1+\delta)Ly + \frac{\delta}{2}y^2 \end{aligned} \tag{3.2.8}$$

The first order condition of $E\pi$ with respect to L is

$$\frac{\partial E\pi}{\partial L} = \delta^2 B \frac{\partial y}{\partial L} + (1+\delta)y + (1+\delta)L\frac{\partial y}{\partial L} + \delta y \frac{\partial y}{\partial L}$$

Since $\frac{\partial y}{\partial L} = -1$, we have

$$\begin{aligned} \tfrac{\partial E\pi}{\partial L} &= -\delta^2 B + (1+\delta)y - (1+\delta)L - \delta y \\ &= -\delta^2 B - (1+\delta)L - L + (1-\delta)B \\ &= (1-\delta-\delta^2)B - (2+\delta)L \end{aligned}$$

Therefore

$$\frac{\partial E\pi}{\partial L} \ge 0, \text{ iff } L \le \frac{1-\delta-\delta^2}{2+\delta}B$$

It is easy to verify that $\frac{1-\delta-\delta^2}{2+\delta}B < (1-\delta)B$, the optimal listing fee is

$$L^{\star} = \max\{\frac{1-\delta-\delta^2}{2+\delta}B, (1-\delta)B - S\}$$

or more precisely,

$$L^{\star} = \begin{cases} \frac{1-\delta-\delta^2}{2+\delta}B, & \text{if } \delta > \frac{B}{S} - 2 \\ (1-\delta)B - S, & \text{if } \delta \le \frac{B}{S} - 2 \end{cases}$$

<u>Subcase 2.1</u> Suppose $\delta > \frac{B}{S} - 2$

In the interior solution case,

$$y(L^\star, \delta) = -\frac{1-\delta-\delta^2}{2+\delta}B + (1-\delta)B = \frac{B}{2+\delta}$$

By (3.2.8),

$$E\pi(L^\star, \delta) = \delta^2 B \frac{B}{2+\delta} + (1+\delta)\frac{1-\delta-\delta^2}{2+\delta}B\frac{B}{2+\delta} + \frac{\delta}{2}\frac{B^2}{(2+\delta)^2} = \frac{B^2}{2(2+\delta)}$$

Since $E\pi(L^\star, \delta)$ is decreasing in δ, the website would charge

$$\delta^\star = 0 \text{ and } L^\star = \frac{B}{2}$$

The size of entrant seller is $y^\star = \frac{B}{2}$ and each entrant seller receives zero net payoff. The website extracts all the surplus and receives an expected revenue $\frac{B^2}{4}$. The condition $\delta > \frac{B}{S} - 2$ is equivalent to $B < 2S$.

Recall in Case 1, the website obtains an expected revenue $S(B-S)$ by charging $\delta^\star = 0$ and $L^\star = B - S$. It is easy to verify that

$$\frac{B^2}{4} \geq S(B-S)$$

Thus when $B < 2S$, The optimal transaction fee is $\delta^\star = 0$ and the optimal listing fee is $L^\star = \frac{B}{2}$. The set of entrant sellers is $[0, \frac{B}{2}]$. The the expected revenue of the website is $E\pi = \frac{B^2}{4}$.

<u>Subcase 2.2</u> Suppose $\delta \leq \frac{B}{S} - 2$

In this case,

$$L^\star = (1-\delta)B - S$$

By (3.2.7), the size of entrant sellers is

$$y^\star = S$$

By (3.2.8), the expected revenue of the website is

$$E\pi = \delta^2 BS + (1+\delta)[(1-\delta)B - S]S + \frac{\delta}{2}S^2 = BS - S^2 - \frac{\delta}{2}S^2$$

Therefore, the optimal transaction fee is $\delta^\star = 0$ and the optimal listing fee is $L^\star = B - S$. The size of entrant sellers is $y^\star = S$. The website extracts all the surplus and receives an expected revenue $S(B-S)$, which is the same as the equilibrium in Case 1. The condition $\delta \leq \frac{B}{S} - 2$ is equivalent to $B \geq 2S$.

Theorem 2.2 Consider Model II with ERR. There exists a unique subgame perfect equilibrium outcome.

i Suppose $B \leq 2S$, the optimal listing fee is $L^{\star} = \frac{B}{2}$ and the optimal transaction fee is $\delta^{\star} = 0$. The size of entrant sellers is $[0, \frac{B}{2}]$ and the expected revenue of the website is $\frac{B^2}{4}$.

ii Suppose $B > 2S$, the optimal listing fee is $L^{\star} = B - S$ and the optimal transaction fee is $\delta^{\star} = 0$. The size of entrant sellers is $[0, S]$ and the expected revenue of the website is $E\pi = S(B-S)$.

iii The size of the set of entrant sellers is at most half the size of buyers.

iv Each entrant seller receives zero net payoff.

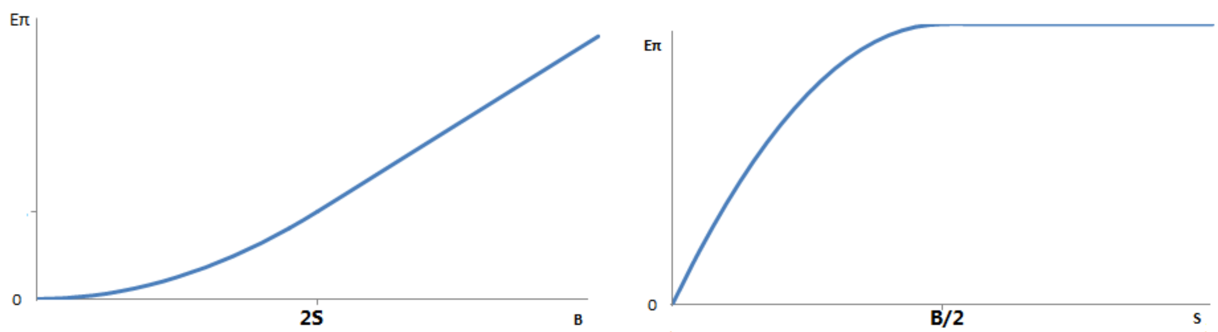


Figure 3.1: The expected revenue of the website $E\pi$

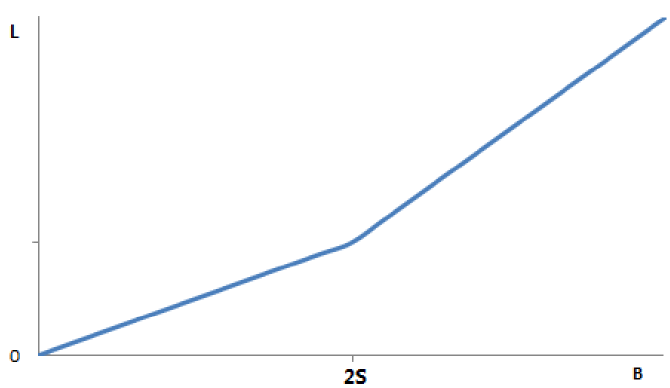


Figure 3.2: The optimal listing fee $L^{\star}$

As we can see in Figure 3.1, the expected revenue of the website is strictly increasing with B, the size of the buyers. It is also strictly increasing with S, the size of the set of sellers, provided that the size of sellers is at most half the size of the buyers.

Let us compare the equilibrium outcomes of the two rationing rules. The expected price a seller s receives in PRR model is $\frac{B+s}{2}$. The same seller obtains a higher price in the model with ERR, to be more specific, $\frac{B}{2}+s$ for $B \leq 2S$ and $B-S+s$ for $B \geq 2S$. However, in the ERR model, the website extracts all the surplus and the net payoff of each entrant seller is zero. In the PRR model the net payoff of a seller s is $\frac{B-2s}{4}$ when $B \leq 2S$ and $\frac{S-s}{2}$ when $B \geq 2S$. Though the set of entrant sellers is the same in the two models, the website is able to charge a listing fee twice as large as in the PRR model. The website is better off with the ERR model.

Remark Consider the case where the buyers only pay part of their willingness to pay, namely αt, $0 \leq \alpha \leq 1$, instead of their entire willingness to pay. If α is a personal choice decided by the buyers themselves, the equilibrium is the same as above. Suppose α is a exogenous parameter. The equilibrium is as following:

Denote $\overline{\alpha}$ such that $\overline{\alpha} = \frac{2S}{B}$

- The equilibrium under PRR

  i When $B < 2S$ or $B \geq 2S$ and $\alpha < \overline{\alpha}$, the website would charge $\delta^\star = 0$ and $L^\star = \frac{\alpha B}{4}$. The set of entrant seller is $[0, \frac{\alpha B}{2}]$ and the expected revenue of the website is $\frac{\alpha^2 B^2}{8}$.

  ii When $B \geq 2S$ and $\alpha \geq \overline{\alpha}$, the website would charge $\delta^\star = 0$ and $L^\star = \frac{\alpha B - S}{2}$. The set of entrant seller is $[0, S]$ and the expected revenue of the website is $\frac{(\alpha B - S)S}{2}$.

- The equilibrium under ERR

  i When $B < 2S$ or $B \geq 2S$ and $\alpha < \overline{\alpha}$, the website would charge $\delta^\star = 0$ and $L^\star = \frac{\alpha}{2}B$. The set of entrant seller is $[0, \frac{\alpha}{2}B]$ and he expected revenue of the website is $\frac{\alpha^2 B^2}{4}$.

  ii When $B \geq 2S$ and $\alpha \geq \overline{\alpha}$, the website would charge $\delta^\star = 0$ and $L^\star = \alpha B - S$. The set of entrant sellers is $[0, S]$ and the website would obtain a expected revenue $E\pi = (\alpha B - S)S$.

The detailed proof can be found in Appendix D.

3.3 Model III

Since the previous two models cannot explain the coexistence of the two fees, we then consider a model with multiplicity of listing. The setup is the same as the ERR of Model II. Now there are two types of goods selling by two different group of sellers and both sets of sellers are $[0,\infty)$. The buyers of the ith good are represented by the interval $[0,B_i]$, $i=1,2$. Assume $B_2 \geq B_1$. The revenue of seller s_i obtained from a unit of Good i is

$$R_i(s) = -L + (1-\delta)(B_i - y_i + s),\; i = 1,2$$

A seller s_i would enter the website iff $R_i(s_i) \geq s_i$. The last seller y_i satisfies $R_i(y_i) = y_i$. Thus the sizes of the entrant sellers are

$$\begin{aligned} y_1 &= -L + (1-\delta)B_1 \\ y_2 &= -L + (1-\delta)B_2 \end{aligned} \tag{3.3.1}$$

Since $B_2 \geq B_1$, $L \leq (1-\delta)B_1$ must satisfy. Otherwise, only Good 2 is sold in the market and the result is the same as in Model II with the ERR. We present the equilibrium outcome in the next theorem.

Theorem III Consider Model III with the efficient rationing rule. There exists a unique subgame perfect equilibrium. Let $t = \frac{B_2}{B_1} \geq 1$. There exists t_0, $1 < t_0 < 3$ such that

(1) If $1 \leq t \leq t_0$, $\delta^\star = 0$ and $L^\star = \frac{B_1+B_2}{4}$

(2) If $t > t_0$, $\delta^\star > 0$ and $L^\star > 0$.

Theorem III asserts that if the size of the set of buyers of one good is sufficiently larger than that of the other good, the website is better off charging positive listing fee as well as positive transaction fee. This result is empirically validated.

Proof

The expected revenue of the website is

$$\begin{aligned} E\pi &= E\pi_1 + E\pi_2 \\ &= \delta^2 B_1 y_1 + (1+\delta)Ly_1 + \delta\int_0^{y_1} s ds + \delta^2 B_2 y_2 + (1+\delta)Ly_2 + \delta\int_0^{y_2} s ds \\ &= \delta^2(B_1y_1 + B_2y_2) + (1+\delta)L(y_1+y_2) + \frac{\delta}{2}(y_1^2 + y_2^2) \end{aligned} \tag{3.3.2}$$

The first order condition of the expected revenue of the website with respect to L is

$$\frac{\partial E\pi}{\partial L} = \delta^2(B_1\frac{\partial y_1}{\partial L} + B_2\frac{\partial y_2}{\partial L}) + (1+\delta)(y_1+y_2) + (1+\delta)L(\frac{\partial y_1}{\partial L} + \frac{\partial y_2}{\partial L}) + \delta(y_1\frac{\partial y_1}{\partial L} + y_2\frac{\partial y_2}{\partial L})$$

By (3.3.1), $\frac{\partial y_i}{\partial L} = -1$ for $i = 1,2$. Hence,

$$\begin{aligned}
\frac{\partial E\pi}{\partial L} &= -\delta^2(B_1+B_2)+(1+\delta)(y_1+y_2)-2(1+\delta)L-\delta(y_1+y_2) \\
&= y_1+y_2-\delta^2(B_1+B_2)-2(1+\delta)L \\
&= (1-\delta-\delta^2)(B_1+B_2)-2(2+\delta)L
\end{aligned}$$

Since $L \leq (1-\delta)B_1$, we have

$$L^\star = \min[(1-\delta)B_1,\ \frac{1-\delta-\delta^2}{2(2+\delta)}(B_1+B_2)]$$

Note that

$$\frac{1-\delta-\delta^2}{2(2+\delta)}(B_1+B_2) \leq (1-\delta)B_1, \text{ iff } \frac{1-\delta-\delta^2}{4-2\delta-2\delta^2} \leq \frac{B_1}{B_1+B_2} \tag{3.3.3}$$

Let $B_2 = tB_1$, where $t \geq 1$. Denote

$$h(\delta) \equiv \frac{1-\delta-\delta^2}{4-2\delta-2\delta^2}$$

Then (3.3.3) can be written as

$$h(\delta) \leq \frac{1}{1+t}$$

Since

$$h'(\delta) = \frac{(-1-2\delta)(2-\delta-\delta^2)-(1-\delta-\delta^2)(-1-2\delta)}{2(2-\delta-\delta^2)^2} = -\frac{1+2\delta}{2(2-\delta-\delta^2)^2} < 0,$$

$h(\delta)$ is strictly decreasing with δ and $h(0) = \frac{1}{4}$. If $\frac{1}{1+t} \geq \frac{1}{4}$, or equivalently $t \leq 3$, the inequality $h(\delta) \leq \frac{1}{1+t}$ holds for all $\delta \in [0,1]$. If $t > 3$, there exists a unique $\overline{\delta}$, such that

$$h(\overline{\delta}) = \frac{1-\overline{\delta}-\overline{\delta}^2}{4-2\overline{\delta}-2\overline{\delta}^2} = \frac{1}{1+t}$$

We have

$$\overline{\delta} = \sqrt{\frac{5}{4}-\frac{2}{t-1}}-\frac{1}{2} \in (0, \frac{\sqrt{5}-1}{2}) \tag{3.3.4}$$

When $t > 3$,

- if $0 \leq \delta \leq \overline{\delta}$, $h(\delta) \geq \frac{1}{1+t}$, $L^\star = (1-\delta)B_1$
- if $\delta \geq \overline{\delta}$, $h(\delta) \leq \frac{1}{1+t}$, $L^\star = \frac{1-\delta-\delta^2}{2(2+\delta)}(B_1+B_2)$

Corollary 4

(1) Suppose $B_2 \leq 3B_1$ ($1 \leq t \leq 3$), the optimal listing fee

$$L^\star = \frac{1-\delta-\delta^2}{2(2+\delta)}(B_1+B_2) = \frac{1-\delta-\delta^2}{2(2+\delta)}(1+t)B_1$$

(2) Suppose $B_2 \geq 3B_1$ ($t \geq 3$). Let $\overline{\delta} = \sqrt{\frac{5}{4}-\frac{2}{t-1}} - \frac{1}{2} \in [0, \frac{\sqrt{5}-1}{2})$. Then

$$L^\star = \begin{cases} (1-\delta)B_1, & \text{if } 0 \leq \delta \leq \overline{\delta} \\ \frac{1-\delta-\delta^2}{2(2+\delta)}(B_1+B_2) = \frac{1-\delta-\delta^2}{2(2+\delta)}(1+t)B_1, & \text{if } \delta \geq \overline{\delta} \end{cases}$$

Next let us first find the optimal transaction fee δ in the region $[0, \frac{\sqrt{5}-1}{2}]$.

Case 1 Suppose $1 \leq t \leq 3$

By Corollary 4, the optimal listing fee is

$$L^\star = \frac{1-\delta-\delta^2}{2(2+\delta)}(1+t)B_1 \tag{3.3.5}$$

By (3.3.1), the sizes of the entrant sellers are

$$\begin{aligned} y_1 &= \frac{(3-\delta-\delta^2)B_1-(1-\delta-\delta^2)B_2}{2(2+\delta)} = \frac{3-\delta-\delta^2-(1-\delta-\delta^2)t}{2(2+\delta)}B_1 \\ y_2 &= \frac{(3-\delta-\delta^2)B_2-(1-\delta-\delta^2)B_1}{2(2+\delta)} = \frac{(3-\delta-\delta^2)t-(1-\delta-\delta^2)}{2(2+\delta)}B_1 \end{aligned}$$

By (3.3.2), the first order condition of $E\pi$ with respect to δ is

$$\begin{aligned} \frac{\partial E\pi}{\partial \delta} &= 2\delta(B_1y_1+B_2y_2)+\delta^2(B_1\frac{\partial y_1}{\partial \delta}+B_2\frac{\partial y_2}{\partial \delta})+L(y_1+y_2) \\ &\quad +(1+\delta)L(\frac{\partial y_1}{\partial \delta}+\frac{\partial y_2}{\partial \delta})+\frac{y_1^2+y_2^2}{2}+\delta(y_1\frac{\partial y_1}{\partial \delta}+y_2\frac{\partial y_2}{\partial \delta}) \end{aligned}$$

By (3.3.1), $\frac{\partial y_i}{\partial \delta} = -B_i$ for $i = 1,2$. Thus

$$\begin{aligned} \frac{\partial E\pi}{\partial \delta} &= 2\delta(B_1y_1+B_2y_2)-\delta^2(B_1^2+B_2^2)+L(y_1+y_2)-(1+\delta)L(B_1+B_2)+\frac{y_1^2+y_2^2}{2}-\delta(B_1y_1+B_2y_2) \\ &= \delta(B_1y_1+B_2y_2)-\delta^2(B_1^2+B_2^2)+L(y_1+y_2)-(1+\delta)L(B_1+B_2)+\frac{y_1^2+y_2^2}{2} \\ &= \delta[(1-\delta)(B_1^2+B_2^2)-(B_1+B_2)L]-\delta^2(B_1^2+B_2^2)+L[(1-\delta)(B_1+B_2)-2L] \\ &\quad -(1+\delta)L(B_1+B_2)+\frac{(1-\delta)^2(B_1^2+B_2^2)-2(1-\delta)(B_1+B_2)L+2L^2}{2} \\ &= \frac{1-3\delta^2}{2}(B_1^2+B_2^2)-(1+2\delta)(B_1+B_2)L-L^2 \\ &= \frac{1-3\delta^2}{2}(1+t^2)B_1^2-(1+2\delta)(1+t)B_1L-L^2 \end{aligned}$$

By (3.3.5),

$$\frac{\partial E\pi}{\partial \delta}|_{L^\star} = \frac{1-3\delta^2}{2}(1+t^2)B_1^2 - \frac{(1+2\delta)(1-\delta-\delta^2)}{2(2+\delta)}(1+t)^2B_1^2 - \frac{(1-\delta-\delta^2)^2}{4(2+\delta)^2}(1+t)^2B_1^2$$

Rearranging terms, we have

$$\frac{\partial E\pi}{\partial \delta} \geq 0 \text{ iff } \frac{1-3\delta^2}{2}(1+t^2) - \frac{(5+9\delta+3\delta^2)(1-\delta-\delta^2)}{4(2+\delta)^2}(1+t)^2 \geq 0 \tag{3.3.6}$$

Notice $\frac{\partial E\pi}{\partial \delta} < 0$ when $\delta = \frac{\sqrt{5}-1}{2}$, $\delta^\star < \frac{\sqrt{5}-1}{2}$. Thus for $0 \leq \delta < \frac{\sqrt{5}-1}{2}$, $\frac{\partial E\pi}{\partial \delta} \geq 0$ iff

$$\frac{1}{5+4\delta-11\delta^2-12\delta^3-3\delta^4} \leq 1 - \frac{(1+t)^2}{2(1+t^2)} = \frac{(t-1)^2}{2(1+t^2)}$$

Suppose $t=1$, $\frac{\partial E\pi}{\partial \delta} < 0$, the optimal listing fee is $\delta^\star = 0$. Then for $1 < t \leq 3$, $\frac{\partial E\pi}{\partial \delta} \geq 0$, iff

$$5+4\delta-11\delta^2-12\delta^3-3\delta^4 \geq \frac{2(1+t^2)}{(t-1)^2}$$

rearranging terms, we have

$$f(\delta) \equiv \delta(4-11\delta-12\delta^2-3\delta^3) \geq \frac{(3-t)(3t-1)}{(t-1)^2} \equiv g(t) \tag{3.3.7}$$

As we can see in Figure 3.3, $g(t)$ is continuous and decreasing in $t \in (1,\infty)$ and $\lim_{t\to\infty} g(t) = -3$. For $1 < t \leq 3$, the image of $g(t)$ is $[0,\infty)$.

It is clear to see in Figure 3.4, the function $f(\delta)$ is maximized at $\hat{\delta} = 0.1455$ and $f(\hat{\delta}) = 0.3108$. Let t_0 to be the solution of $g(t) = f(\hat{\delta})$. It can be verified that $t_0 = 2.8583$.

For $t \in (1,t_0)$, $g(t) > f(\delta)$ holds for all $\delta \in [0, \frac{\sqrt{5}-1}{2})$ and $\frac{\partial E\pi}{\partial \delta} < 0$. In this case, $\delta^\star = 0$ and by (3.3.5), $L^\star = \frac{1}{4}(1+t)B_1$.

For $t \in [t_0,3]$, the equation $f(\delta) = g(t)$ has two solution $\delta_1(t)$ and $\delta_2(t)$ (see Figure 3.4). Since $\frac{\partial E\pi}{\partial \delta} \geq 0$ iff $\delta_1(t) \leq \delta \leq \delta_2(t)$, $\delta^\star = \delta_2(t)$, where $0.1455 \leq \delta_2(t) \leq 0.2753$.

Corollary 5 When $t \in [1,3]$, there exists $t_0 = 2.8583$.

(1) If $1 \leq t < t_0$, the optimal transaction fee is $\delta^\star = 0$.

(2) If $t_0 \leq t \leq 3$, the optimal transaction is $\delta^\star = \delta_2(t)$, where $\delta_2(t)$ is the higher solution of $f(\delta) = g(t)$ and $0.1455 \leq \delta_2(t) \leq 0.2753$.

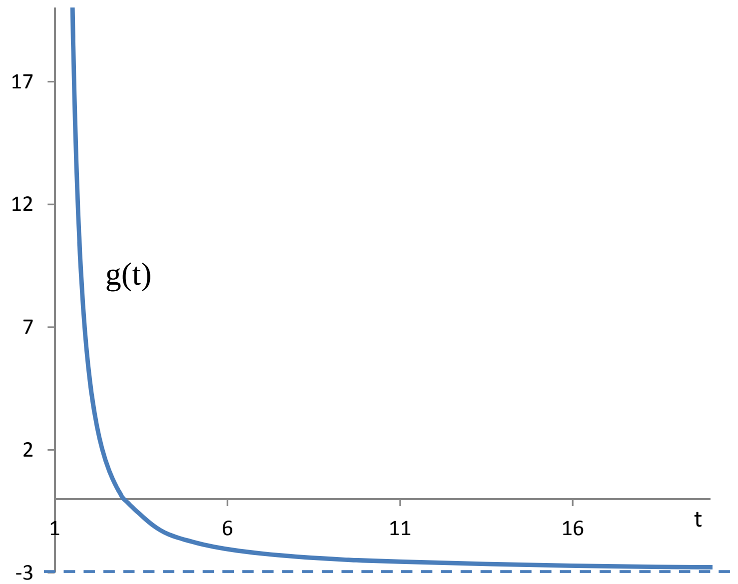


Figure 3.3: $g(t)$

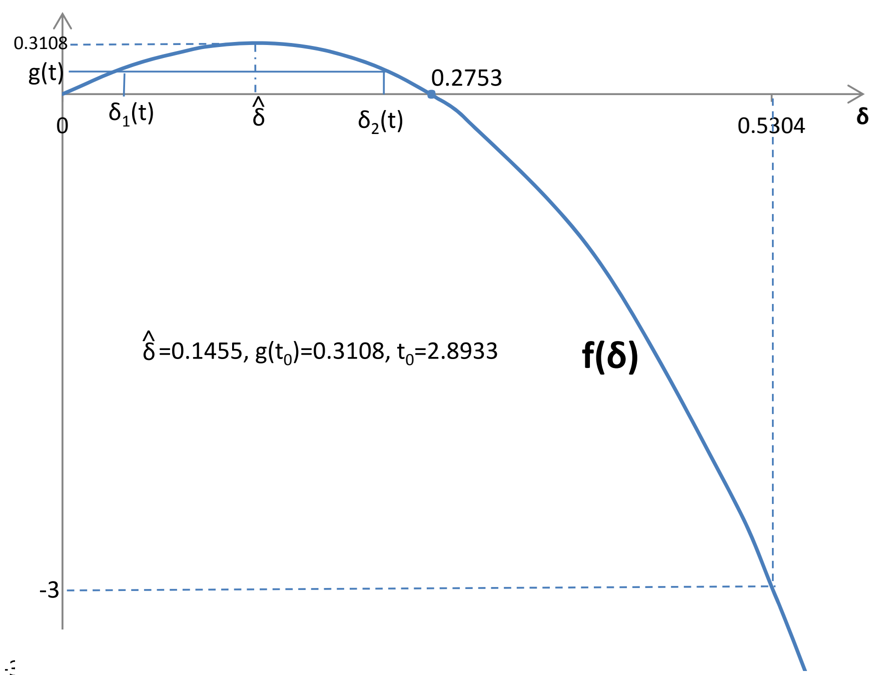


Figure 3.4: $f(\delta)$

<u>Case 2</u> $B_2 \geq 3B_1$ i.e. $t \geq 3$.

In this case, the optimal listing fee $L^\star$ depends on whether $\delta \geq \overline{\delta}$ or not, where $\overline{\delta} = \sqrt{\frac{5}{4} - \frac{2}{t-1}} - \frac{1}{2} \in [0, \frac{\sqrt{5}-1}{2})$.

<u>Subcase 2.1</u> Suppose $0 \leq \delta \leq \overline{\delta}$

By Corollary 4, the optimal listing fee is $L^\star = (1-\delta)B_1$.

In this case, by (3.3.1), the sizes of the entrant sellers are

$$y_1^\star = 0 \text{ and } y_2^\star = (1-\delta)(B_2 - B_1) = (1-\delta)(t-1)B_1$$

By (3.3.2), the expected revenue of the website

$$E\pi = B_2(B_2 - B_1)\delta^2(1-\delta) + B_1(B_2 - B_1)(1+\delta)(1-\delta)^2 + \frac{1}{2}(B_2 - B_1)^2\delta(1-\delta)^2$$

Rearranging terms, we have

$$\begin{aligned} E\pi = &\ \tfrac{1}{2}(B_2 - B_1)(1-\delta)[\delta(1+\delta)B_2 + (1-\delta)(2+\delta)B_1] \\ = &\ \tfrac{1}{2}(t-1)B_1(1-\delta)[\delta(1+\delta)t + (1-\delta)(2+\delta)]B_1 \\ = &\ \tfrac{t-1}{2}B_1^2(1-\delta)[\delta(1+\delta)t + (1-\delta)(2+\delta)] \end{aligned}$$

Hence the first order condition of $E\pi$ with respect to δ

$$\begin{aligned} \tfrac{\partial E\pi}{\partial \delta} = &\ \tfrac{t-1}{2}B_1^2\{-\delta(1+\delta)t - (1-\delta)(2+\delta) + (1-\delta)[(1+2\delta)t - 1 - 2\delta]\} \\ = &\ \tfrac{t-1}{2}B_1^2[-\delta(1+\delta)t - (1-\delta)(2+\delta) + (1-\delta)(1+2\delta)t - (1-\delta)(1+2\delta)] \\ = &\ \tfrac{t-1}{2}B_1^2[(1-3\delta^2)t - 3(1-\delta^2)] \\ = &\ \tfrac{t-1}{2}B_1^2[t - 3 - 3(t-1)\delta^2] \end{aligned}$$

and

$$\frac{\partial E\pi}{\partial \delta} \geq 0 \text{ iff } \delta \leq \sqrt{\frac{t-3}{3(t-1)}}$$

Consequently

$$\delta_{21}^\star = \min[\overline{\delta}, \sqrt{\frac{t-3}{3(t-1)}}\,] = \min[\sqrt{\frac{5}{4} - \frac{2}{t-1}} - \frac{1}{2}, \sqrt{\frac{t-3}{3(t-1)}}\,] > 0$$

and

$$L_{21}^\star = (1-\delta_{21}^\star)B_1 > 0$$

Namely both the listing fee and the transaction fee are positive.

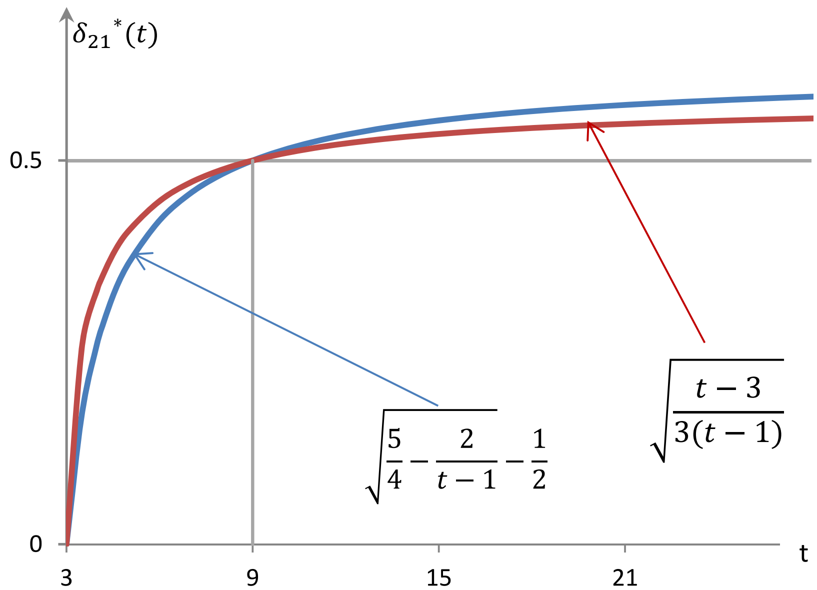


Figure 3.5: $\delta_{21}^{\star}$

Corollary 6 For $t \geq 3$ and $0 \leq \delta \leq \overline{\delta}$, where $\overline{\delta} = \sqrt{\frac{5}{4} - \frac{2}{t-1}} - \frac{1}{2}$,

(1) when $3 \leq t \leq 9$, $\delta_{21}^{\star} = \overline{\delta} = \sqrt{\frac{5}{4} - \frac{2}{t-1}} - \frac{1}{2}$,

(2) when $t \geq 9$, $\delta_{21}^{\star} = \sqrt{\frac{t-3}{3(t-1)}}$.

which can be easily verified (see Figure 3.5).

Hence, for the case $0 \leq \delta \leq \overline{\delta}$,

- when $3 \leq t \leq 9$, the website would charge

$$\delta_{21}^{\star} = \sqrt{\frac{5}{4} - \frac{2}{t-1}} - \frac{1}{2} \text{ and } L_{21}^{\star} = (\frac{3}{2} - \sqrt{\frac{5}{4} - \frac{2}{t-1}})B_1$$

and obtain an expected revenue $E\pi = \frac{(t-1)^2}{2} B_1^2 (\frac{3}{2} - \sqrt{\frac{5}{4} - \frac{2}{t-1}})$

- when $t \geq 9$, the website would charge

$$\delta_{21}^{\star} = \sqrt{\frac{t-3}{3(t-1)}} \text{ and } L_{21}^{\star} = (1 - \sqrt{\frac{t-3}{3(t-1)}})B_1$$

and obtain an expected revenue $E\pi = (t-1)B_1^2[1 + \frac{t-3}{3}\sqrt{\frac{t-3}{3(t-1)}}]$.

<u>Subcase 2.2</u> Suppose $\overline{\delta} \leq \delta \leq \frac{\sqrt{5}-1}{2}$

By Corollary 4, the optimal listing fee

$$L^\star = \frac{1-\delta-\delta^2}{2(2+\delta)}(B_1+B_2) = \frac{1-\delta-\delta^2}{2(2+\delta)}(1+t)B_1$$

Similar to Case 1,

$$\frac{\partial E\pi}{\partial \delta} \geq 0, \text{ iff } 2(1-3\delta^2)(2+\delta)^2(1+t^2) \geq (5+9\delta+3\delta^2)(1-\delta-\delta^2)(1+t)^2 \tag{3.3.8}$$

When $3 \leq t < 13+6\sqrt{3}$, $\overline{\delta} = \sqrt{\frac{5}{4}-\frac{2}{t-1}} - \frac{1}{2} < \frac{\sqrt{3}}{3}$. For $\delta \geq \frac{\sqrt{3}}{3}$, $1-3\delta^2 \leq 0$ suggests $\frac{\partial E\pi}{\partial \delta} \leq 0$. Thus the optimal transaction fee $\delta^\star \in [\overline{\delta}, \frac{\sqrt{3}}{3})$. Same as Subcase 1, $\frac{\partial E\pi}{\partial \delta} \geq 0$ iff

$$f(\delta) = \delta(4-11\delta-12\delta^2-3\delta^3) \geq \frac{(3-t)(3t-1)}{(t-1)^2} = g(t)$$

For $3 \leq t < 13+6\sqrt{3}$, $-2.8134 < g(t) \leq 0$. There is a unique solution to $f(\delta) = g(t)$ (see Figure 3.6). Denote this unique solution as $\delta(t)$ and $0.2753 \leq \delta(t) < 0.5207$. The optimal transaction fee $\delta^\star = \delta(t)$.

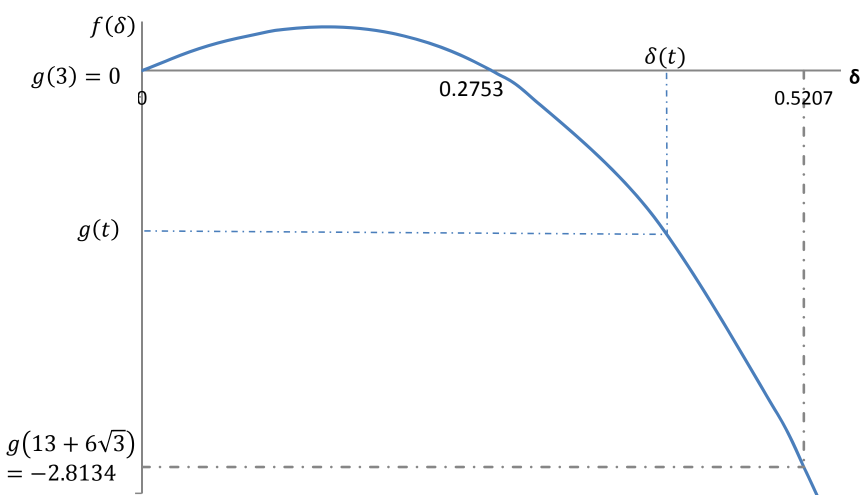


Figure 3.6: $\delta^\star_{22}$

When $t \geq 13+6\sqrt{3}$, $\overline{\delta} \geq \frac{\sqrt{3}}{3}$. For all $\delta \in [\overline{\delta}, \frac{\sqrt{5}-1}{2}]$, $\frac{\partial E\pi}{\partial \delta} \leq 0$. The optimal transaction fee $\delta^\star = \overline{\delta}$ and $\frac{\sqrt{3}}{3} \leq \overline{\delta} \leq \frac{\sqrt{5}-1}{2}$.

Corollary 7 For $t \geq 3$ and $\overline{\delta} \leq \delta \leq \frac{\sqrt{5}-1}{2}$,

(1) when $3 \leq t < 13+6\sqrt{3}$, the optimal transaction fee $\delta_{22}^{\star} = \delta(t)$, where $\delta(t)$ is the unique solution to $f(\delta) = g(t)$ and $0.2753 \leq \delta(t) < 0.5207$.

(2) when $t \geq 13+6\sqrt{3}$, the optimal transaction fee $\delta_{22}^{\star} = \overline{\delta} = \sqrt{\frac{5}{4} - \frac{2}{t-1}} - \frac{1}{2}$.

The optimal fee structure is determined by which one generates a higher expected revenue of the website. For the case $\overline{\delta} \leq \delta \leq \frac{\sqrt{5}-1}{2}$, the expected revenue of the website is

$$
\begin{aligned}
E\pi = & \ \tfrac{\delta}{2}(1-\delta^2)(B_1^2+B_2^2) + (1-\delta-\delta^2)(B_1+B_2)L - (2+\delta)L^2 \\
= & \ \tfrac{\delta}{2}(1-\delta^2)(B_1^2+B_2^2) + (1-\delta-\delta^2)(B_1+B_2)\tfrac{1-\delta-\delta^2}{2(2+\delta)}(B_1+B_2) - (2+\delta)[\tfrac{1-\delta-\delta^2}{2(2+\delta)}(B_1+B_2)]^2 \\
= & \ \tfrac{\delta}{2}(1-\delta^2)(B_1^2+B_2^2) + \tfrac{(1-\delta-\delta^2)^2}{2(2+\delta)}(B_1+B_2)^2 - \tfrac{(1-\delta-\delta^2)^2}{4(2+\delta)}(B_1+B_2)^2 \\
= & \ \tfrac{\delta}{2}(1-\delta^2)(B_1^2+B_2^2) + \tfrac{(1-\delta-\delta^2)^2}{4(2+\delta)}(B_1+B_2)^2 \\
= & \ [\tfrac{\delta}{2}(1-\delta^2)(1+t^2) + \tfrac{(1-\delta-\delta^2)^2}{4(2+\delta)}(1+t)^2]B_1^2
\end{aligned}
$$

Hence,

- when $3 \leq t < 13+6\sqrt{3}$, the website would charge

$$\delta_{22}^{\star} = \delta(t) \text{ and } L_{22}^{\star} = \frac{1-\delta(t)-\delta(t)^2}{2(2+\delta(t))}(1+t)B_1$$

  and obtain an expected revenue $E\pi = [\frac{\delta(t)}{2}(1-\delta(t)^2)(1+t^2) + \frac{(1-\delta(t)-\delta(t)^2)^2}{4(2+\delta(t))}(1+t)^2]B_1^2$

- when $t \geq 13+6\sqrt{3}$, the website would charge

$$\delta_{22}^{\star} = \sqrt{\frac{5}{4} - \frac{2}{t-1}} - \frac{1}{2} \text{ and } L_{22}^{\star} = \frac{1-\delta(t)-\delta(t)^2}{2(2+\delta(t))}(1+t)B_1 = (\frac{3}{2} + \sqrt{\frac{5}{4} - \frac{2}{t-1}})(1+t)B_1$$

  and obtain an expected revenue $E\pi = \frac{(t-1)^2}{2}(\frac{3}{2} - \sqrt{\frac{5}{4} - \frac{2}{t-1}})B_1^2$.

Compare the expected revenue of the website of Subcase 2.1 and Subcase 2.2, we conclude, for $0 \leq \delta \leq \frac{\sqrt{5}-1}{2}$, there exists $t_0 = 2.8583$,

(1) when $1 \leq t < t_0$, the website would charge

$$\delta^{\star} = 0 \text{ and } L^{\star} = \frac{B_1+B_2}{4}$$

and obtains $E\pi = L^{\star}(y_1^{\star} + y_2^{\star}) = \frac{(B_1+B_2)^2}{8} = \frac{(t+1)^2}{8}B_1^2$. In this case, $y_1^{\star} = B_1 - L = \frac{3B_1-B_2}{4}$ and $y_2^{\star} = B_2 - L = \frac{3B_2-B_1}{4}$.

(2) when $t_0 \leq t \leq 3$, the website would charge

$$\delta^\star = \delta_2(t) \text{ and } L^\star = \frac{1-\delta_2(t)-\delta_2(t)^2}{2(2+\delta_2(t))}(1+t)B_1$$

where $\delta_2(t) \in [0.1455, 0.2753]$ is the solution of

$$f(\delta) = \delta(4-11\delta-12\delta^2-3\delta^3) = \frac{(3-t)(3t-1)}{(t-1)^2} = g(t)$$

and obtains $E\pi = [\frac{\delta_2(t)}{2}(1-\delta_2(t)^2)(1+t^2) + \frac{(1-\delta_2(t)-\delta_2(t)^2)^2}{4(2+\delta_2(t))}(1+t)^2]B_1^2$. In this case,
$y_1^\star = \frac{3-\delta_2(t)-\delta_2(t)^2-(1-\delta_2(t)-\delta_2(t)^2)t}{2(2+\delta_2(t))}B_1$ and
$y_2^\star = \frac{(3-\delta_2(t)-\delta_2(t)^2)t-(1-\delta_2(t)-\delta_2(t)^2)}{2(2+\delta_2(t))}B_1$.

(3) when $3 \leq t < 13+6\sqrt{3}$, the website would charge

$$\delta^\star = \delta(t) \text{ and } L^\star = \frac{1-\delta(t)-\delta(t)^2}{2(2+\delta(t))}(1+t)B_1$$

where $\delta(t)$ is the unique solution to $f(\delta) = g(t)$ and $0.2753 \leq \delta(t) < 0.5207$,
and obtains $E\pi = \frac{\delta(t)(1-\delta(t)^2)(B_2-B_1)^2}{4} + \frac{(B_1+B_2)^2}{4(2+\delta(t))}$. In this case,
$y_1^\star = \frac{3-\delta(t)-\delta(t)^2-(1-\delta(t)-\delta(t)^2)t}{2(2+\delta(t))}B_1$ and
$y_2^\star = \frac{(3-\delta(t)-\delta(t)^2)t-(1-\delta(t)-\delta(t)^2)}{2(2+\delta(t))}B_1$

(4) when $t \geq 13+6\sqrt{3}$, the website would charge

$$\delta^\star = \sqrt{\frac{t-3}{3(t-1)}} \text{ and } L^\star = (1-\sqrt{\frac{t-3}{3(t-1)}})B_1$$

and obtains $E\pi = (t-1)B_1^2[1+\frac{t-3}{3}\sqrt{\frac{t-3}{3(t-1)}}]$. In this case,
$y_1^\star = 0$ and $y_2^\star = (1-\delta^\star)(t-1)B_1 = (1-\sqrt{\frac{t-3}{3(t-1)}})(t-1)B_1$

Lemma 5 Transaction fee $\delta > \frac{\sqrt{5}-1}{2}$ is not optimal for the website.
The proof is deferred to Appendix E.
By Corollaries 4-7 and Lemma 5, for $t > t_0$, we have $\delta^\star > 0$ and $L^\star > 0$. The proof of Theorem III is complete.■

Chapter 4

Conclusion

In this paper, I show that the optimal listing fee for a monopoly auction website is alway positive. The transaction fee is positive only when there is multiple productions.

There are several limitations in the assumptions my models.

- Auction website is monopoly
  When there is competition, the websites may lower the listing fee in order to attract more sellers to enter.

- No off-site trades
  It is assumed that all trading are made through the auction website. In practice, the sellers might use the website as advertising and make the transaction privately.

- Both sellers and buyers are risk-neutral.
  A risk-averse seller would prefer to pay a lower listing fee and higher listing fee.

- No cost of the website.
  The cost of the website should have positive relationship with the size of the set of entrant seller and the number of bidders. In order to lower its cost, the website may raise the listing fee to control the size of entrant sellers.

- No searching cost in buyers
  In practice, buyers are overwhelmed by the large number of similar products and they need to look though the items for sale to pick the favorable one. It is natural to consider there is a cost generated in the searching process, which depends on the level of time impatience.

- The fee structure
  Though the use of listing and transaction fee is quite common setup for the websites, I have not compared the fee structure (L, δ) with other possible fee structures.

The above limitations need to be explored in future research.

Appendix A

Proof of Lemma 1

By (3.1.12),

$$\begin{aligned}
F'(x) &= \frac{1-\delta}{2-\delta}x^2 f(x) + (1-\delta)[\int_x^{\overline{B}} bf(b)db - x^2 f(x)] \\
&\quad -\frac{(1-\delta)^2}{2-\delta}[2x\int_x^{\overline{B}} f(b)db - x^2 f(x)] - L - 2\frac{1-\delta}{2-\delta}x \\
&= (1-\delta)\int_x^{\overline{B}} bf(b)db - 2\frac{(1-\delta)^2}{2-\delta}x\int_x^{\overline{B}} f(b)db - L - 2\frac{1-\delta}{2-\delta}x
\end{aligned}$$

Note that $F'(x)$ is decreasing for $\underline{B} \le x \le \overline{B}$ since

$$\begin{aligned}
F''(x) &= -(1-\delta)xf(x) - \frac{2(1-\delta)^2}{2-\delta}(\int_x^{\overline{B}} f(b)db - xf(x)) - 2\frac{1-\delta}{2-\delta} \\
&= -\frac{\delta(1-\delta)}{2-\delta}xf(x) - 2\frac{(1-\delta)^2}{2-\delta}\int_x^{\overline{B}} f(b)db - 2\frac{1-\delta}{2-\delta} < 0
\end{aligned}$$

Next observe

$$\begin{aligned}
F'(\underline{B}) &= (1-\delta)\int_{\underline{B}}^{\overline{B}} bf(b)db - 2\frac{(1-\delta)^2}{2-\delta}\underline{B}\int_{\underline{B}}^{\overline{B}} f(b)db - L - 2\frac{1-\delta}{2-\delta}\underline{B} \\
&= (1-\delta)\mu - 2\frac{(1-\delta)^2}{2-\delta}\underline{B} - L - 2\frac{1-\delta}{2-\delta}\underline{B} \\
&= (1-\delta)(\mu - 2\underline{B}) - L \qquad \text{(A.0.1)}
\end{aligned}$$

and recall (3.1.13),

$$F(\underline{B}) = \underline{B}[(1-\delta)(\mu-\underline{B}) - L]$$

Suppose $L > (1-\delta)(\mu-\underline{B})$. By (3.1.13), $F(\underline{B}) < 0$. In this case, $L > (1-\delta)(\mu-2\underline{B})$ satisfied, by (A.0.1), $F'(\underline{B}) < 0$. Since $F'(x)$ is decreasing for all x, $\underline{B} \leq x \leq \overline{B}$, $F'(\underline{B}) < 0$ implies $F'(x) < 0$ for all x and $F(x)$ is decreasing for all $x \in [\underline{B}, \overline{B}]$. Since $F(\underline{B}) < 0$, $F(x) < 0$ for all $\underline{B} \leq x \leq \overline{B}$ and thus such (L, δ) is not an equilibrium outcome. Therefore, without loss of generality, we can assume that the website chooses (L, δ) such that $L \leq (1-\delta)(\mu - \underline{B})$.

By (3.1.12)

$$F(\overline{B}) = \frac{1-\delta}{2-\delta}\int_{\underline{B}}^{\overline{B}} b^2 f(b)db - L\overline{B} - \frac{1-\delta}{2-\delta}\overline{B}^2$$

Since $\int_{\underline{B}}^{\overline{B}} b^2 f(b)db < \overline{B}^2 \int_{\underline{B}}^{\overline{B}} f(b)db = \overline{B}^2$, we have $F(\overline{B}) < 0$.

Suppose $(1-\delta)(\mu - 2\underline{B}) \leq L \leq (1-\delta)(\mu - \underline{B})$, by (A.0.1), $F'(\underline{B}) \leq 0$, together with $F'(x)$ is decreasing, $F(x)$ is decreasing in $[\underline{B}, \overline{B}]$. Since $F(\underline{B}) \geq 0$, $F(\overline{B}) < 0$, there is a unique x, $\underline{B} \leq x < \overline{B}$ such that $F(x) = 0$.

Suppose next $L < (1-\delta)(\mu - 2\underline{B})$, $F'(\underline{B}) > 0$ and $F(\underline{B}) > 0$. Since $F'(x)$ is decreasing, there is $\bar{x}$ such that $F'(x) > 0$ for $\underline{B} \leq x < \bar{x}$ and $F'(x) < 0$ for $\bar{x} < x \leq \overline{B}$. Thus $F(x)$ is increasing on $[\underline{B}, \bar{x}]$ and decreasing on $[\bar{x}, \overline{B}]$. This implies that there is a unique x, $\bar{x} < x < \overline{B}$ such that $F(x) = 0$. (see Figure 1)■

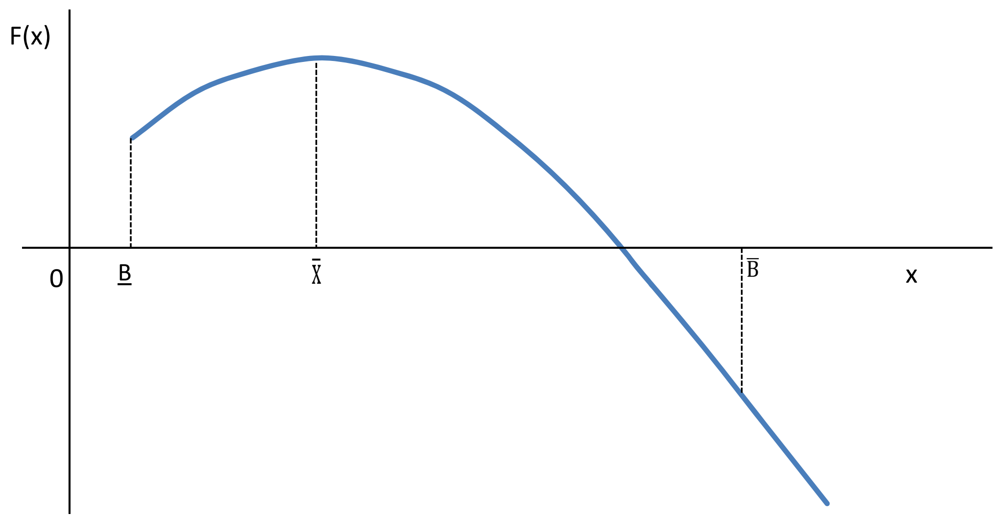


Figure A.1: $F(x)$

Appendix B

Proof of Lemma 3

In order to find the optimal fee scheme $(L^\star, \delta^\star)$, we need to calculate the partial derivative of $E\pi(x)$ with respective to L

$$\frac{\partial E\pi(x)}{\partial L} = \frac{1}{2-\delta}(x + L\frac{\partial x}{\partial L}) + \frac{\delta(1-\delta)}{(2-\delta)^2}2x\frac{\partial x}{\partial L} \tag{B.0.1}$$

Since there exists $x(L,\delta)$ such that $F(x(L,\delta)) \equiv 0$ for all $\{(L,\delta)|L \le (1-\delta)(\mu - \underline{B})\}$,

$$\frac{\partial F(x(L,\delta))}{\partial L} = 0 \text{ and } \frac{\partial F(x(L,\delta))}{\partial \delta} = 0$$

Then

$$\begin{aligned}
\frac{\partial F(x)}{\partial L} &= \frac{(1-\delta)x^2}{2-\delta}f(x)\frac{\partial x}{\partial L} + (1-\delta)\int_x^{\overline{B}} bf(b)db\frac{\partial x}{\partial L} - (1-\delta)x^2 f(x)\frac{\partial x}{\partial L} \\
&\quad - \frac{2(1-\delta)^2 x}{2-\delta}\int_x^{\overline{B}} f(b)db\frac{\partial x}{\partial L} + \frac{(1-\delta)^2x^2}{2-\delta}f(x)\frac{\partial x}{\partial L} - x - L\frac{\partial x}{\partial L} - 2\frac{(1-\delta)x}{2-\delta}\frac{\partial x}{\partial L} \\
&= (1-\delta)\int_x^{\overline{B}} bf(b)db\frac{\partial x}{\partial L} - \frac{2(1-\delta)^2 x}{2-\delta}\int_x^{\overline{B}} f(b)db\frac{\partial x}{\partial L} - x - L\frac{\partial x}{\partial L} - \frac{2(1-\delta)}{2-\delta}x\frac{\partial x}{\partial L} = 0
\end{aligned}$$

rearranging terms, we have,

$$(1-\delta)[\int_x^{\overline{B}} bf(b)db - \frac{1-\delta}{2-\delta}2x\int_x^{\overline{B}} f(b)db - \frac{2x}{2-\delta}]\frac{\partial x}{\partial L} = x + L\frac{\partial x}{\partial L} \tag{B.0.2}$$

Thus, for $x = x(L,\delta)$, (B.0.1) can be rewritten as

$$\begin{aligned}
\frac{\partial E\pi(x)}{\partial L} &= \frac{1-\delta}{2-\delta}\frac{\partial x}{\partial L}[\int_x^{\overline{B}} bf(b)db - \frac{2(1-\delta)}{2-\delta}x\int_x^{\overline{B}} f(b)db - \frac{2}{2-\delta}x] + \frac{\delta(1-\delta)}{(2-\delta)^2}2x\frac{\partial x}{\partial L} \\
&= \frac{1-\delta}{2-\delta}\frac{\partial x}{\partial L}[\int_x^{\overline{B}} bf(b)db - \frac{2(1-\delta)}{2-\delta}x\int_x^{\overline{B}} f(b)db - \frac{2(1-\delta)}{2-\delta}x]
\end{aligned} \tag{B.0.3}$$

By (B.0.2),

$$
\begin{aligned}
\frac{\partial x}{\partial L} &= \frac{x}{(1-\delta)\int_x^{\overline{B}} bf(b)db - \frac{2(1-\delta)^2 x}{2-\delta}\int_x^{\overline{B}} f(b)db - L - \frac{2(1-\delta)}{2-\delta}x} \\
&= \frac{x^2}{(1-\delta)x\int_x^{\overline{B}} bf(b)db - \frac{2(1-\delta)^2}{2-\delta}x^2\int_x^{\overline{B}} f(b)db - Lx - \frac{2(1-\delta)}{2-\delta}x^2} \\
&= \frac{x^2}{\frac{1-\delta}{2-\delta}\int_{\underline{B}}^x b^2 f(b)db + (1-\delta)x\int_x^{\overline{B}} bf(b)db - \frac{2(1-\delta)^2}{2-\delta}x^2\int_x^{\overline{B}} f(b)db - Lx - \frac{2(1-\delta)}{(2-\delta)}x^2} \\
&= \frac{x^2}{F(x) - \frac{1-\delta}{2-\delta}\int_{\underline{B}}^x b^2 f(b)db - \frac{(1-\delta)^2}{2-\delta}x^2\int_x^{\overline{B}} f(b)db - \frac{1-\delta}{(2-\delta)}x^2}
\end{aligned}
$$

Since $F(x) = 0$, we have

$$
\frac{\partial x}{\partial L} = \frac{x^2}{-\frac{1-\delta}{2-\delta}\int_{\underline{B}}^x b^2 f(b)db - \frac{(1-\delta)^2}{2-\delta}x^2\int_x^{\overline{B}} f(b)db - \frac{1-\delta}{2-\delta}x^2} < 0 \tag{B.0.4}
$$

Denote for all $z \in [\underline{B}, \overline{B}]$

$$
g(z) = \int_z^{\overline{B}} bf(b)db - \frac{2(1-\delta)}{2-\delta}z\int_z^{\overline{B}} f(b)db - \frac{2(1-\delta)}{2-\delta}z \tag{B.0.5}
$$

The first order condition of $E\pi(x)$ with respect to L can be rewritten as

$$
\frac{\partial E\pi(x)}{\partial L} = \frac{1-\delta}{2-\delta}\frac{\partial x}{\partial L}g(x)
$$

Thus $\frac{\partial E\pi(x)}{\partial L} \geq 0$ iff $g(x) \leq 0$.

Notice $g(z)$ is strictly decreasing,

$$
\begin{aligned}
g'(z) &= -zf(z) - \frac{2(1-\delta)}{2-\delta}\int_z^{\overline{B}} f(b)db + \frac{2(1-\delta)}{2-\delta}zf(z) - \frac{2(1-\delta)}{2-\delta} \\
&= -\frac{\delta}{2-\delta}zf(z) - \frac{2(1-\delta)}{2-\delta}\int_z^{\overline{B}} f(b)db - \frac{2(1-\delta)}{2-\delta} < 0
\end{aligned}
$$

and

$$
g(\overline{B}) = -\frac{2(1-\delta)}{2-\delta}z < 0
$$

Thus,

- If $g(\underline{B}) < 0$, $g(z) < 0$ for all $z \in [\underline{B}, \overline{B}]$. In this case, $\frac{\partial E\pi(x)}{\partial L} > 0$ always holds. The optimal listing fee $L^\star$ is a corner solution, i.e., $L^\star = (1-\delta)(\mu - \underline{B})$. By Lemma 2, $x^\star = \underline{B}$.

- If $g(\underline{B}) = 0$, we have $x^\star = \underline{B}$. By Lemma 2, $L^\star = (1-\delta)(\mu - \underline{B})$.

- If $g(\underline{B}) > 0$, there is a unique interior solution and $L^\star < (1-\delta)(\mu - \underline{B})$.

By (B.0.5),

$$g(\underline{B}) = \mu - \frac{1-\delta}{2-\delta} 4\underline{B}$$

Since $\delta \in [0,1]$,

$$g(\underline{B}) \leq 0, \text{ iff } \mu \leq 2\underline{B} \text{ and } 0 \leq \delta \leq \frac{4\underline{B} - 2\mu}{4\underline{B} - \mu}$$

Thus for $\mu \leq 2\underline{B}$ and $0 \leq \delta \leq \frac{4\underline{B}-2\mu}{4\underline{B}-\mu}$, the optimal listing fee $L^\star = (1-\delta)(\mu - \underline{B})$ and $x^\star = \underline{B}$. Otherwise, the optimal listing fee $L^\star$ is an interior solution.■

Appendix C

Proof of Lemma 4

When $\mu > 2\underline{B}$ or $\mu \leq 2\underline{B}$ and $\delta \geq \frac{2(2\underline{B}-\mu)}{4\underline{B}-\mu}$, the optimal $L^\star$ is an interior solution and $L^\star < (1-\delta)(\mu-\underline{B})$. At $x = x(L^\star,\delta)$, we have $F(x) = 0$ and $g(x) = 0$.

By (3.1.12) and (B.0.5),

$$\begin{aligned}
F(x(L^\star,\delta)) &= \frac{1-\delta}{2-\delta}\int_{\underline{B}}^{x} b^2 f(b)db + (1-\delta)x[\int_{x}^{\overline{B}} bf(b)db - \frac{1-\delta}{2-\delta}x\int_{x}^{\overline{B}} f(b)db] - L^\star x - \frac{1-\delta}{2-\delta}x^2 \\
&= \frac{1-\delta}{2-\delta}\int_{\underline{B}}^{x} b^2 f(b)db + (1-\delta)x[\frac{1-\delta}{2-\delta}x\int_{x}^{\overline{B}} f(b)db + \frac{2(1-\delta)}{2-\delta}x] - L^\star x - \frac{1-\delta}{2-\delta}x^2 \\
&= \frac{1-\delta}{2-\delta}\int_{\underline{B}}^{x} b^2 f(b)db + \frac{(1-\delta)^2}{2-\delta}x^2\int_{x}^{\overline{B}} f(b)db + \frac{(1-\delta)(1-2\delta)}{2-\delta}x^2 - L^\star x \\
&= 0
\end{aligned} \tag{C.0.1}$$

Thus

$$L^\star x = \frac{1-\delta}{2-\delta}[\int_{\underline{B}}^{x} b^2 f(b)db + (1-\delta)x^2\int_{x}^{\overline{B}} f(b)db + (1-2\delta)x^2] \tag{C.0.2}$$

By (3.1.14) and (C.0.2)

$$\begin{aligned}
E\pi(x(L^\star,\delta)) &= \frac{1-\delta}{(2-\delta)^2}[\int_{\underline{B}}^{x} b^2 f(b)db + (1-\delta)x^2\int_{x}^{\overline{B}} f(b)db + (1-2\delta)x^2] + \frac{\delta(1-\delta)}{(2-\delta)^2}x^2 \\
&= \frac{1-\delta}{(2-\delta)^2}[\int_{\underline{B}}^{x} b^2 f(b)db + (1-\delta)x^2\int_{x}^{\overline{B}} f(b)db + (1-\delta)x^2]
\end{aligned} \tag{C.0.3}$$

Thus

$$
\begin{aligned}
\frac{\partial E(\pi(L^\star,\delta))}{\partial\delta} &= -\frac{\delta}{(2-\delta)^3}[\int_{\underline{B}}^{x} b^2 f(b)db+(1-\delta)x^2\int_x^{\overline{B}} f(b)db+(1-\delta)x^2]+\frac{1-\delta}{(2-\delta)^2} \\
&\quad \{[\delta x f(x)+2(1-\delta)\int_x^{\overline{B}} f(b)db+2(1-\delta)]x\frac{\partial x}{\partial\delta}-x^2\int_x^{\overline{B}} f(b)db-x^2\} \\
&= -\frac{1}{(2-\delta)^3}[\delta\int_{\underline{B}}^{x} b^2 f(b)db+2(1-\delta)x^2\int_x^{\overline{B}} f(b)db+2(1-\delta)x^2] \\
&\quad +\frac{(1-\delta)x}{(2-\delta)^2}\frac{\partial x}{\partial\delta}[\delta x f(x)+2(1-\delta)\int_x^{\overline{B}} f(b)db+2(1-\delta)] \qquad \text{(C.0.4)}
\end{aligned}
$$

Since $g(x(L^\star,\delta))=0$, we have

$$
\begin{aligned}
\frac{\partial g(x(L^\star,\delta))}{\partial\delta} &= -xf(x)\frac{\partial x}{\partial\delta}+\frac{2x}{(2-\delta)^2}\int_x^{\overline{B}} f(b)db-\frac{2(1-\delta)}{2-\delta}\int_x^{\overline{B}} f(b)db\frac{\partial x}{\partial\delta} \\
&\quad +\frac{2(1-\delta)}{2-\delta}xf(x)\frac{\partial x}{\partial\delta}+\frac{2x}{(2-\delta)^2}-\frac{2(1-\delta)}{2-\delta}\frac{\partial x}{\partial\delta} \\
&= -\frac{1}{2-\delta}\frac{\partial x}{\partial\delta}[\delta x f(x)+2(1-\delta)\int_x^{\overline{B}} f(b)db+2(1-\delta)]+\frac{2x\int_x^{\overline{B}} f(b)db+2x}{(2-\delta)^2} \\
&= 0 \qquad \text{(C.0.5)}
\end{aligned}
$$

Suppose first the optimal transaction fee $\delta^\star$ is an interior solution, i.e., $\delta^\star>0$. Thus $\delta^\star$ solves $\frac{\partial E\pi(L^\star,\delta^\star)}{\partial\delta}=0$. By (C.0.4) and (C.0.5), $\delta^\star$ is the solution of

$$
\delta\int_{\underline{B}}^{x} b^2 f(b)db+2(1-\delta)x^2\int_x^{\overline{B}} f(b)db+2(1-\delta)x^2=(1-\delta)x[2x\int_x^{\overline{B}} f(b)db+2x]
$$

which is equivalent (at $\delta=\delta^\star$) to

$$
\delta^\star\int_{\underline{B}}^{x^\star} b^2 f(b)db=0
$$

If $\delta^\star>0$, we have $x^\star=\underline{B}$. By Lemma 2,

$$
L^\star=(1-\delta^\star)(\mu-\underline{B})
$$

which is not an optimal solution by Claim 2.

Therefore, $\delta^\star=0$.■

Appendix D

when buyer t pays αt

D.1 Proportional Rationing Rule (PRR)

Under PRR, a seller s sells his product at expected price of $\frac{\alpha B+s}{2}$, given $\alpha B \geq s$. The expected revenue $R(s)$ of an entrant seller $s \in [0,S]$ is

$$R(s) = -L + \frac{(1-\delta)(\alpha B+s)}{2}$$

A seller s enters the website iff $R(s) \geq s$. The size of sellers entering the website is therefore

$$y(L,\delta) = \max[\min\{\frac{(1-\delta)\alpha B - 2L}{1+\delta}, S\}, 0]$$

It is clear that any (L,δ) such that $\frac{(1-\delta)B-2L}{1+\delta} \leq 0$ is not optimal for the website. Thus, without loss of generality, we can assume that $L < \frac{1-\delta}{2}\alpha B$. Thus

$$y(L,\delta) = \begin{cases} S, & \text{if } L \leq \frac{(1-\delta)\alpha B-(1+\delta)S}{2} \\ \frac{(1-\delta)\alpha B-2L}{1+\delta}, & \text{if } \frac{(1-\delta)\alpha B-(1+\delta)S}{2} \leq L < \frac{1-\delta}{2}\alpha B \end{cases}$$

The expected revenue the website extracts from a seller s is

$$\pi(s) = L + \frac{\delta(\alpha B+s)}{2}$$

The expected revenue of the website is

$$\begin{aligned} E\pi &= Ly + \delta \int_0^y \frac{\alpha B+s}{2} ds \\ &= Ly + \frac{\delta}{2}(\alpha By + \frac{1}{2}y^2) \\ &= Ly + \frac{\delta}{2}\alpha By + \frac{\delta}{4}y^2 \end{aligned}$$

Case 1 $L \leq \frac{(1-\delta)\alpha B-(1+\delta)S}{2}$

In this case, the set of sellers is $y = S$. The expected revenue of the website is

$$E\pi = LS + \frac{\delta}{2}\alpha BS + \frac{\delta}{4}S^2$$

Since $E\pi$ is increasing with L,

$$L^\star = \frac{(1-\delta)\alpha B-(1+\delta)S}{2}$$

and

$$E\pi = \frac{(1-\delta)\alpha B-(1+\delta)S}{2}S + \frac{\delta}{2}\alpha BS + \frac{\delta}{4}S^2 = \frac{\alpha B - S}{2}S - \frac{\delta}{4}S^2$$

Since the expected revenue of the website is decreasing with δ,

$$\delta^\star = 0 \text{ and } L^\star = \frac{\alpha B - S}{2}$$

The website obtains an expected revenue of $\frac{S(\alpha B-S)}{2}$.

Case 2 $\frac{(1-\delta)\alpha B-(1+\delta)S}{2} \leq L < \frac{1-\delta}{2}\alpha B$

Recall the expected revenue of the website is

$$E\pi = Ly + \frac{\delta}{2}\alpha By + \frac{\delta}{4}y^2$$

The first order condition with respect to L is

$$\frac{\partial E\pi}{\partial L} = y + L\frac{\partial y}{\partial L} + \frac{\delta}{2}\alpha B\frac{\partial y}{\partial L} + \frac{\delta}{2}y\frac{\partial y}{\partial L}$$

and

$$y = \frac{(1-\delta)\alpha B - 2L}{1+\delta} \in (0,S) \Rightarrow \frac{\partial y}{\partial L} = -\frac{2}{1+\delta}$$

Thus

$$\begin{aligned}
\frac{\partial E\pi}{\partial L} &= \frac{(1-\delta)\alpha B-2L}{1+\delta} - \frac{2}{1+\delta}L - \frac{\delta}{2}\alpha B\frac{2}{1+\delta} - \frac{\delta}{2}\frac{(1-\delta)\alpha B-2L}{1+\delta}\frac{2}{1+\delta} \\
&= \frac{(1-\delta)\alpha B}{1+\delta} - \frac{4}{1+\delta}L - \frac{\delta}{1+\delta}\alpha B - \frac{\delta(1-\delta)\alpha B-2\delta L}{(1+\delta)^2} \\
&= \frac{1-2\delta}{1+\delta}\alpha B - \frac{\delta(1-\delta)}{(1+\delta)^2}\alpha B - \frac{4}{1+\delta}L + \frac{2\delta}{(1+\delta)^2}L \\
&= \frac{1-2\delta-\delta^2}{(1+\delta)^2}\alpha B - \frac{2(2+\delta)}{(1+\delta)^2}L
\end{aligned}$$

Thus

$$\frac{\partial E\pi}{\partial \delta} \geq 0 \text{ iff } L \leq \frac{1-2\delta-\delta^2}{2(2+\delta)}\alpha B$$

It is easy to verify that $\frac{1-2\delta-\delta^2}{2(2+\delta)}\alpha B < \frac{1-\delta}{2}\alpha B$. Thus

$$L^\star = \max\{\frac{1-2\delta-\delta^2}{2(2+\delta)}\alpha B, \frac{(1-\delta)\alpha B-(1+\delta)S}{2}\}$$

The condition $\frac{1-2\delta-\delta^2}{2(2+\delta)}B > \frac{(1-\delta)B-(1+\delta)S}{2}$ is equivalent to $\alpha B < (2+\delta)S$. Hence

$$L^\star = \begin{cases} \frac{1-2\delta-\delta^2}{2(2+\delta)}\alpha B, & \text{if } \delta > \frac{\alpha B}{S}-2 \\ \frac{(1-\delta)\alpha B-(1+\delta)S}{2}, & \text{if } \delta \leq \frac{\alpha B}{S}-2 \end{cases}$$

<u>Subcase 2.1</u> $\delta > \frac{\alpha B}{S} - 2$

This is the interior solution case where

$$L^\star = \frac{1-2\delta-\delta^2}{2(2+\delta)}\alpha B \text{ and } y(L^\star,\delta) = \frac{\alpha B}{2+\delta} < S$$

The expected revenue of the website is

$$\begin{aligned} E\pi &= Ly + \tfrac{\delta}{2}\alpha By + \tfrac{\delta}{4}y^2 \\ &= \tfrac{1-2\delta-\delta^2}{2(2+\delta)}\alpha B\tfrac{\alpha B}{2+\delta} + \tfrac{\delta}{2}\alpha B\tfrac{\alpha B}{2+\delta} + \tfrac{\delta}{4}(\tfrac{\alpha B}{2+\delta})^2 \\ &= \tfrac{\alpha^2B^2}{4(2+\delta)} \end{aligned}$$

Thus $E\pi$ is decreasing with δ and hence

$$\delta^\star = 0 \text{ and } L^\star = \frac{\alpha B}{4}$$

The set of entrant seller is $[0, \frac{\alpha B}{2}]$ and the expected revenue of the website is $\frac{\alpha^2B^2}{8}$. The condition $\delta > \frac{\alpha B}{S} - 2$ is equivalent to $\alpha < \frac{2S}{B}$. When $\frac{2S}{B} > 1$, i.e. $B < 2S$, the condition is always satisfied. When $\frac{2S}{B} \leq 1$, i.e. $B \geq 2S$, there exists $\overline{\alpha}$ such that $\overline{\alpha} = \frac{2S}{B}$. If $\alpha < \overline{\alpha}$, the condition holds. If $\alpha \geq \overline{\alpha}$, $L^\star = \frac{(1-\delta)\alpha B-(1+\delta)S}{2}$.

Recall in Case 1, the website obtains an expected revenue $\frac{S(\alpha B-S)}{2}$. Since,

$$\frac{\alpha^2B^2}{8} - \frac{S(\alpha B-S)}{2} = \frac{1}{2}(\frac{\alpha B}{2} - S)^2 \geq 0$$

When $B < 2S$ or $B \geq 2S$ and $\alpha < \overline{\alpha}$, where $\overline{\alpha} = \frac{2S}{B}$, the optimal transaction fee $\delta^\star = 0$ and the optimal listing fee $L^\star = \frac{\alpha B}{4}$. The set of entrant seller is $[0, \frac{\alpha B}{2}]$ and the expected revenue of the website is $\frac{\alpha^2 B^2}{8}$.

<u>**Subcase 2.2**</u> $\delta \leq \frac{\alpha B}{S} - 2$

The optimal listing fee is

$$L^\star = \frac{(1-\delta)\alpha B - (1+\delta)S}{2}$$

The equilibrium choice is the same as in Case 1, i.e.

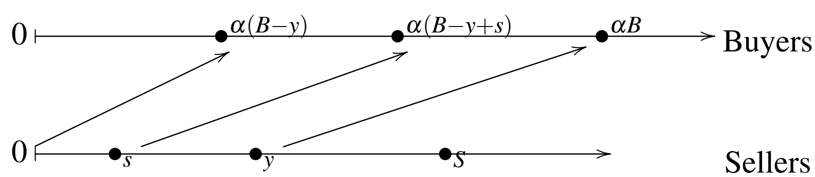
$$\delta^\star = 0 \text{ and } L^\star = \frac{\alpha B - S}{2}$$

The condition $\delta \leq \frac{\alpha B}{S} - 2$ is equivalent to $0 \leq \frac{\alpha B}{S} - 2$.

When $B \geq 2S$ and $\alpha \geq \overline{\alpha}$, where $\overline{\alpha} = \frac{2S}{B}$, the optimal transaction fee $\delta^\star = 0$ and the optimal listing fee $L^\star = \frac{\alpha B}{4}$. The set of entrant seller is $[0, \frac{\alpha B}{2}]$ and the expected revenue of the website is $\frac{(\alpha B - S)S}{2}$.

D.2 Efficient Rationing Rule (ERR)

Under the efficient rationing rule (ERR),



Hence the revenue of a seller s is

$$R(s) = -L + (1-\delta)\alpha(B - y + s)$$

A seller s is willing to sell his product iff $R(s) \geq s$ and the last entrant seller, y, satisfies $R(y) = y$. Namely

$$y = \max\{\min[-L + (1-\delta)\alpha B, S], 0\}$$

Since for $L \geq (1-\delta)\alpha B$, there is no sellers are willing to enter the website. Thus without loss of generality, we assume $L < (1-\delta)\alpha B$. Thus,

$$y(L,\delta) = \begin{cases} S, & \text{if } L \leq (1-\delta)\alpha B - S \\ -L + (1-\delta)\alpha B, & \text{if } (1-\delta)\alpha B - S \leq L < (1-\delta)\alpha B \end{cases}$$

Case 1 **Suppose** $L \le (1-\delta)\alpha B - S$

In this case, $y = S$, all sellers enter the website. The website obtains from an entrant seller s

$$\pi(s) = L + \delta\alpha(B - y + s) = L + \delta\alpha(B - S + s)$$

Thus the expected revenue of the website is

$$E\pi = [L + \delta\alpha(B-S)]S + \int_0^S sds = LS + \alpha\delta(B-S)S + \frac{\alpha\delta}{2}S^2$$

Since $E\pi$ is increasing with L, $L = (1-\delta)\alpha B - S$. Thus

$$E\pi = [(1-\delta)\alpha B - S]S + \alpha\delta(B-S)S + \frac{\alpha\delta}{2}S^2 = \alpha BS - S^2 - \frac{\alpha\delta}{2}S^2$$

Therefore, $\delta^\star = 0$, $L^\star = \alpha B - S$, $y^\star = S$, and $E\pi = (\alpha B - S)S$.

Case 2 **Suppose** $(1-\delta)\alpha B - S \le L < (1-\delta)\alpha B$

In this case, $y = -L + (1-\delta)\alpha B \in (0, S]$. The website obtains from an entrant seller s

$$\begin{aligned}
\pi(s) &= L + \delta\alpha(B - y + s) \\
&= L + \delta\alpha[B + L - (1-\delta)\alpha B + s] \\
&= (1+\delta\alpha)L + \delta\alpha B - \delta(1-\delta)\alpha^2 B + \delta\alpha s
\end{aligned}$$

Hence, the expected revenue of the website is

$$\begin{aligned}
E\pi &= [(1+\delta\alpha)L + \delta\alpha B - \delta(1-\delta)\alpha^2 B]y + \delta\alpha \textstyle\int_0^y sds \\
&= [(1+\delta\alpha)L + \delta\alpha B - \delta(1-\delta)\alpha^2 B]y + \tfrac{\delta\alpha}{2}y^2
\end{aligned}$$

The first order condition of $E\pi$ with respect to L

$$\frac{\partial E\pi}{\partial L} = (1+\delta\alpha)y + [(1+\delta\alpha)L + \delta\alpha B - \delta(1-\delta)\alpha^2 B]\frac{\partial y}{\partial L} + \delta\alpha y\frac{\partial y}{\partial L}$$

Since $\frac{\partial y}{\partial L} = -1$, we have

$$\begin{aligned}
\tfrac{\partial E\pi}{\partial L} &= (1+\delta\alpha)y - [(1+\delta\alpha)L + \delta\alpha B - \delta(1-\delta)\alpha^2 B] - \delta\alpha y \\
&= y - (1+\delta\alpha)L - \delta\alpha B + \delta(1-\delta)\alpha^2 B \\
&= \alpha B[1 - 2\delta + \delta(1-\delta)\alpha] - (2+\delta\alpha)L
\end{aligned}$$

Therefore $\frac{\partial E\pi}{\partial L} \ge 0$, iff $L \le \frac{1-(2-\alpha)\delta - \alpha\delta^2}{2+\delta\alpha}\alpha B$.

Since $\frac{1-(2-\alpha)\delta-\alpha\delta^2}{2+\delta\alpha}\alpha B < (1-\delta)\alpha B$, the optimal listing fee is

$$L^\star = \max\{\frac{1-(2-\alpha)\delta-\alpha\delta^2}{2+\delta\alpha}\alpha B, (1-\delta)\alpha B - S\}$$

or more precisely,

$$L^\star = \begin{cases} \frac{1-(2-\alpha)\delta-\alpha\delta^2}{2+\delta\alpha}\alpha B, & \text{if } \delta \geq \frac{B}{S} - \frac{2}{\alpha} \\ (1-\delta)\alpha B - S, & \text{if } \delta < \frac{B}{S} - \frac{2}{\alpha} \end{cases}$$

Subcase 2.1 Suppose $\delta \geq \frac{B}{S} - \frac{2}{\alpha}$

In the interior solution case,

$$\begin{aligned} y(L^\star, \delta) &= -\frac{1-(2-\alpha)\delta-\alpha\delta^2}{2+\delta\alpha}\alpha B + (1-\delta)\alpha B \\ &= \frac{\alpha B}{2+\delta\alpha}[-1+(2-\alpha)\delta+\alpha\delta^2+(1-\delta)(2+\delta\alpha)] \\ &= \frac{\alpha B}{2+\delta\alpha} \end{aligned}$$

Thus the expected revenue of website at $L^\star$ is

$$\begin{aligned} E\pi &= [(1+\delta\alpha)L+\delta\alpha B-\delta(1-\delta)\alpha^2 B]y+\frac{\delta\alpha}{2}y^2 \\ &= [(1+\delta\alpha)\frac{1-(2-\alpha)\delta-\alpha\delta^2}{2+\delta\alpha}\alpha B+\delta\alpha B-\delta(1-\delta)\alpha^2 B]\frac{\alpha B}{2+\delta\alpha}+\frac{\delta\alpha}{2}(\frac{\alpha B}{2+\delta\alpha})^2 \\ &= (\frac{\alpha B}{2+\delta\alpha})^2\{(1+\delta\alpha)[1-(2-\alpha)\delta-\alpha\delta^2]+\delta(2+\delta\alpha)-\delta(1-\delta)\alpha(2+\delta\alpha)+\frac{\delta\alpha}{2}\} \\ &= (\frac{\alpha B}{2+\delta\alpha})^2(1+\frac{1}{2}\alpha\delta) \\ &= \frac{\alpha^2 B^2}{2(2+\delta\alpha)} \end{aligned}$$

Since $E\pi$ is decreasing with δ, thus $\delta^\star = 0$ and $L^\star = \frac{\alpha}{2}B$. In this case the set of entrant seller is $[0, \frac{\alpha}{2}B]$. The expected revenue of the website is $\frac{\alpha^2 B^2}{4}$.

The expected revenue is higher than that of Case 1, since

$$\frac{\alpha^2 B^2}{4} - (\alpha B - S)S = (\frac{\alpha B}{2} - S)^2 \geq 0$$

The condition $\delta \geq \frac{B}{S} - \frac{2}{\alpha}$ is equivalent to $\alpha \leq \frac{2S}{B}$.

Thus when $B < 2S$ or $B \geq 2S$ and $\alpha < \overline{\alpha}$, where $\overline{\alpha} = \frac{2S}{B}$, the equilibrium is stated as above.

Subcase 2.2 Suppose $\delta < \frac{B}{S} - \frac{2}{\alpha}$

In this corner solution case,

$$L^\star = (1-\delta)\alpha B - S \Rightarrow y^\star = S$$

The equilibrium is the same as Case 1, i.e., $\delta^\star = 0$, $L^\star = \alpha B - S$, $y^\star = S$, and $E\pi = (\alpha B - S)S$.■

Appendix E

Proof of Lemma 5

If we allow $L^\star = \frac{1-\delta-\delta^2}{2(2+\delta)}(B_1+B_2) = \frac{1-\delta-\delta^2}{2(2+\delta)}(1+t)B_1 < 0$[1], we should also consider the case where $\delta \geq \frac{\sqrt{5}-1}{2}$. In this case,

$$\frac{\partial E\pi}{\partial \delta} \geq 0, \text{ iff } \frac{(5+9\delta+3\delta^2)(1-\delta-\delta^2)}{(2+\delta)^2(1-3\delta^2)} \geq \frac{2(1+t^2)}{(1+t)^2}$$

and $\frac{\partial E\pi}{\partial \delta} \geq 0$, iff

$$f(\delta) \equiv \delta(4-11\delta-12\delta^2-3\delta^3) \leq \frac{(3-t)(3t-1)}{(t-1)^2} \equiv g(t)$$

Since $\delta \geq \frac{\sqrt{5}-1}{2}$, $f(\delta) \leq -5$, and $\lim_{t\to\infty} g(t) = -3$, $\frac{\partial E\pi}{\partial \delta} \geq 0$. Thus,

$$\delta^\star = 1 \text{ and } L^\star = -\frac{1}{6}(1+t)B_1$$

In this case,

$$y_1 = y_2 = -L^\star = \frac{1}{6}(1+t)B_1$$

and

$$E\pi = \frac{(B_1+B_2)^2}{12} = \frac{(1+t)^2}{12}B_1^2$$

Let us compare this result with other possible choices when $0 \leq \delta \leq \frac{\sqrt{5}-1}{2}$.

Suppose $1 \leq t \leq 3$, when choosing $\delta = 0$, the expected revenue of the website is

$$E\pi = L(y_1+y_2) = L(1-\delta)(B_1+B_2) - 2L^2$$

[1]Notice: $L^\star = (1-\delta)B_1$ is only optimal when $t \geq 3$ and $0 \leq \delta \leq \overline{\delta} < 1$. Thus, $L^\star$ is always positive in this case.

thus $L = \frac{B_1+B_2}{4}$. In this case,

$$y_1 = B_1 - L = \frac{3B_1 - B_2}{4} \text{ and } y_2 = B_2 - L = \frac{3B_2 - B_1}{4}$$

thus

$$E\pi = L(y_1 + y_2) = \frac{(B_1 + B_2)^2}{8}$$

which exceeds $\frac{(B_1+B_2)^2}{12}$.

When $3 \leq t \leq 9$, the website can obtain $\frac{(t-1)^2}{2}(\frac{3}{2} - \sqrt{\frac{5}{4} - \frac{2}{t-1}})B_1^2$ by choosing

$$\delta^\star = \overline{\delta} = \sqrt{\frac{5}{4} - \frac{2}{t-1}} - \frac{1}{2} \text{ and } L^\star = (\frac{3}{2} - \sqrt{\frac{5}{4} - \frac{2}{t-1}})B_1$$

which exceeds $\frac{(1+t)^2}{12}B_1^2$.

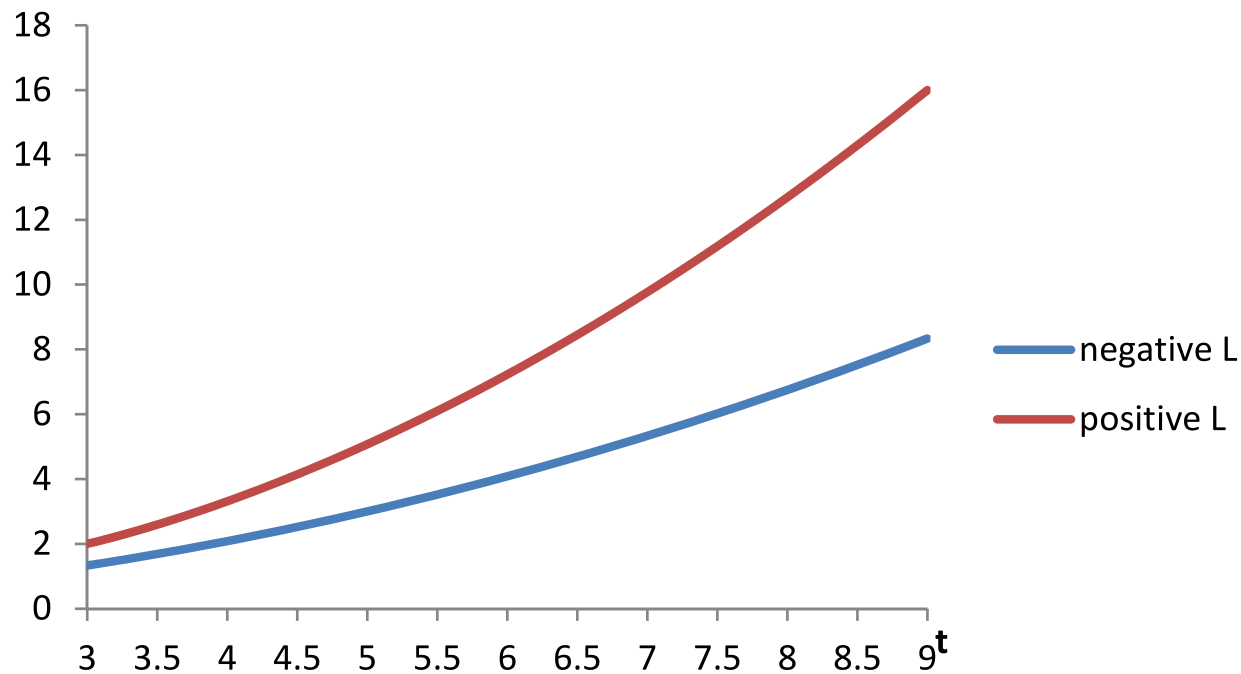


Figure E.1: $3 \leq t \leq 9$

When $t \geq 9$, the website can obtain $(t-1)[1+\frac{t-3}{3}\sqrt{\frac{t-3}{3(t-1)}}]B_1^2$ by choosing

$$\delta^\star = \sqrt{\frac{t-3}{3(t-1)}} \text{ and } L^\star = (1-\sqrt{\frac{t-3}{3(t-1)}})B_1$$

which exceeds $\frac{(1+t)^2}{12}B_1^2$.

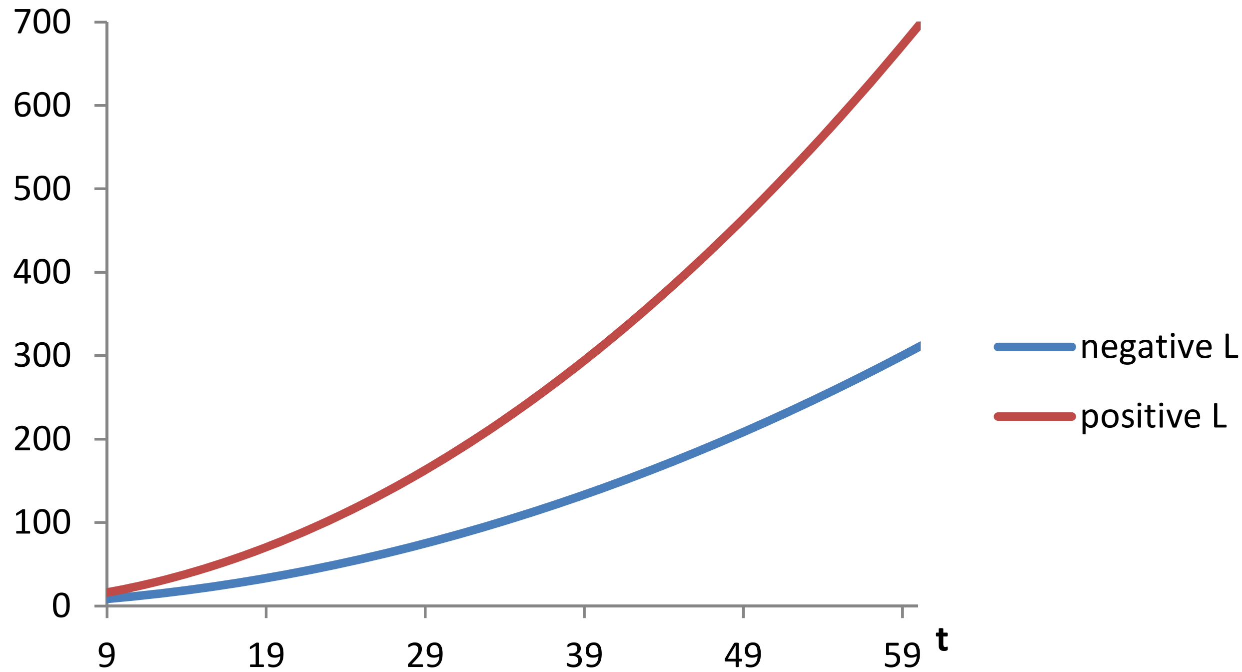


Figure E.2: $t \geq 9$

Therefore even if $L < 0$ is allowed, it is never optimal. ■

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