The Impact of the Responsiveness of Monetary Policy on the Housing Market

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Abstract of the Dissertation

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Taylor (2007) claims that a not too responsive monetary policy was responsible for the housing boom between 2001 and 2005 and the subsequent financial crisis because it results in the low interest rate during 2002-2004. Using a reduced form model, he shows that the economic situation would have been improved if a more responsive monetary policy had been implemented. In this paper, we set up a two-sector New Keynesian DSGE model, estimated using Bayesian techniques, to evaluate Taylor's hypotheses. First, we did identify a less responsive monetary policy after 2000. Our results partially support Taylors hypothesis, that a more responsive monetary policy can stabilize the housing price during the transition period of 2003-2008, the period from the boom to the bust. But it is not the reason of low interest rate during 2002-2004. Second, a more responsive monetary policy would have generated smaller responses of the real variables to shocks, except for the technology shock in the housing sector. One of the differences with the literature is that we introduce housing market segmentation through different discount factors, leading to a housing market that is occupied only by constrained (impatient) households, which actually makes the impulses responses distinguishable under different responsiveness of the monetary policy. The impulse responses to monetary policy shocks and cost push shocks under this assumption deliver unconventional results. In particular, a contractionary monetary policy shock and a positive cost push shock will bring down the interest rate and inflation respectively. Moreover, the real housing price is reduced with these two shocks, leading to more binding constraints for the impatient households. To clear the housing market, the interest rate is reduced, automatically taxing patient lenders and subsidizing impatient borrowers. The theoretical variance decomposition indicates that the monetary policy shock explains about 25% of the variances in housing prices and the cost push shock explains about 58%, while the variance in the housing output is mainly explained by the technology shock in housing sector. Our conclusion is that monetary policy shocks, rather than the responsiveness of monetary policy, contributed to the housing boom, but with limited effect.

We experiment on monetary policies with different response rates to the housing price. By comparing the welfare changes, we found that the patient households always gain from the new policies. But the impatient households only gain under moderate responses to the housing price. When the monetary policy overreacts to the housing price, they will be worse off because of the reduced utility level even though the utility volatility is smoothed over time.

Contents

List of Figures

List of Tables

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Chapter 1

The Impact of the Responsiveness of Monetary Policy on the Housing Market

1.1 Introduction

The housing market boom in the 2000s has attracted massive attention because it initiated the big economic downturn which was named the 'Great Recession', implying its significance and vast influence. An intensive debate has centered on the question: what factors induced this housing boom and bust? There are mainly two opinions: lowered lending standards of mortgages and the low interest rate. The lowered lending standards could be attributed to both governmental policy which aimed at increasing homeownership and to financial market innovation such as the new types of mortgage loans and the securitization that changed the mortgage loan market structure and hid the real risk exposures. Regarding the low interest rate level, a loose monetary policy and the 'global saving glut' proposed in Bernanke (2005) can be the major contributors.

Let's review the findings in previous literature on this debate. DellAriccia et al. (2012) find evidence of the relationship between the housing market boom and the lowered lending standards. In particular, they find that the the mortgage loan market structures changed a lot during the housing boom period. Bernanke (2010) states that the links between the monetary policy and the housing bubble were very weak and 'more exotic types of mortgages and the associated decline of underwriting standards' were key explanations for the housing boom. Greenspan also once said that it was the lower interest rate for 30 year mortgage, rather than the monetary policy, that induced this housing bubble.

In the literature focusing on the monetary policy, Bordo and Landon-Lane (2013) find that loose monetary policy may have played some role regarding the low inflation in the 2000s housing boom, but the financial innovations had more important effects. Del Negro and Otrok (2007) conclude that the impact of monetary policy on housing prices was small compared to the size of housing price increases. Iacoviello and Neri (2010) attribute less than 20% of the housing price change to the monetary policy. Jaroci?ski and Smets (2008) indicate that the monetary policy in $2002-2004$ may have been too loose and therefore it did contributed to the housing boom. Taylor (2010) claims that a not too responsive monetary policy was partly responsible for the housing boom between 2001 and 2005 and the subsequent financial crisis. Using a reduced form model, he shows that the economic situation would have been improved if a more responsive monetary policy had been implemented. Alessandro Calza and Stracca (2013) emphasize both reasons and conclude that the monetary shock had more significant effects on housing investment and housing price in countries with more developed mortgage markets in a cross-country study. Among all these studies mentioning the loose monetary policy, only Taylor (2007) explicitly attributed it to changes in the responsiveness of monetary policy while others only attributed it to a general monetary policy shock. In this paper, we set up a two-sector New Keynesian DSGE model to test Taylor's hypotheses by identifying the responsiveness of the Taylor rule.

Before we discuss Taylor's hypothesis, we need to clarify three issues. The first one is the definition of the responsiveness of monetary policy. Assuming that the central bank's policy follows a Taylor rule, according to Taylor (2007), the responsiveness of monetary policy refers to the value of the coefficients in the Taylor rule. Taylor (2007) declares the monetary policy during the Great Moderation was more responsive and the policy after 2001 was less responsive. So we conclude that the higher the coefficients in the Taylor rule, the more responsive the monetary policy is. In this paper, we will estimate the model for the two periods of interest: the Great Moderation period (1987-1999) and the period including the housing boom and the Great Recession (2000-2009). Our estimation delivers different coefficients of the Taylor rule for these two sub periods corresponding to a more and a less responsive monetary policy.

The second issue is how to differentiate the responsiveness of monetary policy and monetary policy shock. What are monetary policy shocks? Do the changes in the Taylor rule coefficients belong to the monetary policy shocks? Christiano et al. (1999) provide some thoughts about the interpretation of monetary policy shocks. They propose three interpretations: the first one attributes the shocks to the variation in the preferences of monetary authorities, which changes the Taylor rule coefficients. In our model, the identified smaller Taylor rule coefficients capture the effects of this kind of monetary policy shock so that we do not include such shocks in the monetary policy shocks. In other words, we theoretically take out the effect of smaller Taylor rule coefficients from the total effects of monetary policy shocks. In such a way we can distinguish the effect of responsiveness of monetary policy from the effect of the monetary policy shocks caused by other reasons. Christiano et al. (1999) second and third interpretations are that monetary policy shocks stem from the 'Fed's desire to avoid the social cost of disappointing private agents expectations' and from the measurement error in data the monetary policy maker used respectively. The monetary policy shock in this paper refers to the last two interpretations.

Mishkin (2007) and Bernanke and Gertler (1995) review housing and monetary policy transmissions with an emphasis on the credit channel[1], which brings us to the third issue that needs to be clarified. Bernanke and Gertler (1995) define the credit channel as an enhanced mechanism to 'amplify and propagate the conventional interest rate effects' instead of an independent channel. Also, the financial frictions are indispensable for the amplification effect. The amplification effect not only works for monetary policy shocks, but it may be also applicable for other shocks. The investigation of the propagation mechanism for a negative shock from the credit channel originated with Bernanke and Gertler (1989) whereby the debt deflation decreases the firms' net worth. Kiyotaki and Moore (1997) specify the financial friction to be the collateral constraint. In this paper,the asset value changes the agents' net worth, then amplifies the effects of the shocks. Bernanke et al. (1999) analyzes the frictions in financial market within a DSGE model framework, in which the frictions are the collateral constraints. The collateral constraint is widely applied in more recent research on the housing market (or durable goods) and the monetary policy, such as Cordoba and Ripoll (2004), Iacoviello (2005), Monacelli (2009), Iacoviello and Neri (2010),Eggertsson and Krugman (2012) and Alessandro Calza and Stracca (2013), in which the housing price (or durable good price) plays a central role in the propagation mechanism through the credit channel when facing good or bad shocks. With these model specifications, the economy will be better or even turn into a boom when there is a preferred shock and the economic situation would deteriorate even further when there is a negative shock. In line with those previous prominent studies, we also include the collateral constraint in our model to create the credit channel for the monetary policy transmission to realize the amplification effects.

In order to have a collateral constraint, we must first introduce heterogeneous agents in our model such that there are actual borrowing and lending. In a model with homogenous households, the debt must be 0 to clear the market and the collateral constraint has no effect in the system. We adopt the method to generate heterogeneity in households proposed by Iacoviello (2005)- the households are different in the time preferences. We let the lenders

[1]For a review for all the monetary policy transmission channels, refer to Mishkin (1996) and Bernanke and Blinder (1992).

be patient with a higher discount rate while the impatient households are borrowers with a lower discount rate. The impatient households can borrow against the housing and they are subject to a collateral constraint. The impatient households are constrained in every period because of their smaller discount rate[2]. To maximize the effect of monetary policy through the credit channel, we segment the housing market so that only constrained households can buy homes and consume the services delivered by housing. The patient households in our model behave like bankers who have several income sources but are not interested in housing at all so that they would like to supply funds to impatient households to finance their home buying. This is an extreme case that the constrained households play a central role in the housing market. In Iacoviello and Neri (2010), the patient households can also participate in the housing market leading to the fact that the constrained households play a minor role in the housing market. In such a way, we would not observe obviously distinct effects of different monetary policies. With our model, we could see enlarged effects of the responsiveness of monetary policy on the housing market[3].

Since we model an extreme case in which the constrained households play a central role in the housing market, our model possesses special features that are different from those of the models in the existing literature. The impulse responses to monetary policy shock and cost push shock deliver unconventional results under the housing market segmentation assumption and the no default on household debt assumption. In particular, a contractionary monetary policy shock and a positive cost push shock will bring down the interest rate and inflation respectively. The reason is that the real housing price is reduced with these two shocks, leading to more binding constraints for the impatient households. At the same time, they are the only ones who can clear the housing market. To clear the housing market, the real interest rate is reduced, automatically taxing patient lenders and subsidizing impatient borrowers. This fact may shed some light on the possible fiscal policies when facing a negative shock that may lead to deleveraging. Also, we found that the relationship between quantity of housing demanded and housing price may not be monotonic.

By identifying a set of smaller Taylor rule coefficients for 2000-2009, we distinguish the effect of responsiveness of monetary policy from the effects of other monetary shocks. The counterfactual simulation indicates that a more responsive monetary policy had no effect on

[2]Refer the proof in Iacoviello (2005).

[3]If both types of households participate in the housing market, the patient households will buy most of the housing since they possess more resources. Assume there is change in the monetary policy and the housing price increases (decreases), the increase (decrease) in benefit of obtaining one extra unit of housing for patient households will be smaller than that for impatient households because of the collateral constraint. We can conjecture that the change in housing demand and housing price would be more dramatic if we only allow the impatient households participate in the housing market. Chapter 2 explains the importance of housing market segmentation in details.

lifting up the interest rate. However, it controlled the housing price for the major housing boom and bust period, but with limited effects considering the counterfactual line is close to the data. So we conclude that a less responsive monetary policy was not the major reason of low interest rate and a more responsible monetary policy can reduce the housing price could have helped stabilize the monetary policy during the transition period of 2003-2008. The theoretical variance decomposition indicates that the monetary policy shock explains about 25% of the variances in housing prices and the cost push shock explains about 58%, while the variation in the housing output is mainly explained by the technology shock in the housing sector. Our conclusion is that monetary policy shocks, rather than the responsiveness of monetary policy, mainly contributed to the housing boom, but with limited effect.

The paper is arranged in the following way: section II presents the model, section III sum up the equilibrium conditions, section IV explains the data and estimation method, and section V illustrates the results and some interesting findings. Section VI presents our conclusions.

1.2 Model

The model comprises of two types of households who are different in discount factors, two production sectors and a borrowing constraint tied to the housing value. The two types of households are patient and impatient households. Patient households supply labor, consume consumption good, collect profit and dividends and lend money to impatient households. They do not accumulate housing or consume housing services. Impatient households work, consume and accumulate housing. Since they are impatient, they only accumulate the required down payment for their homes and borrow up against their borrowing limit in each period. In our model, we only concentrate on the housing market for the constrained households and the patient households are not interested in this market at all.

The wholesale good sector uses labor and capital to produce intermediate good. The housing sector produces new homes by combining the labor, land and capital. In addition, we have a retail sector and a central bank. The retail sector generates price stickiness by the monopolistic retailers, who transfer homogenous intermediate good into differentiated final consumption good. The central bank adjusts the nominal interest rate though a Taylor rule. We assume that the money supply will meet the money demand to support the interest rate level set by the central bank.

There is a continuum of households that measured 1 for each type of household and they live infinitely.

1.2.1 Patient Households (Savers)

The patient households maximize their lifetime utility:

$$\max\{\sum_{t=0}^{\infty} E_0\beta^t a_t(\ln(C_t - \gamma C_{t-1}) - \frac{1}{1+\eta}(N_{ct}^{1+\zeta} + N_{ht}^{1+\zeta})^{\frac{1+\eta}{1+\zeta}})\} \tag{1.1}$$

β is the discount factor. C_t, N_{ct}, N_{ht} are patient households' consumption, labor supply in the wholesale good sector and labor supply in the housing sector. The upper case letter means the variables with growth trends. γ is the habit persistent parameter. Iacoviello (2005) stated that 'the habit persistent could help replicate the delayed response of macroeconomic variables to various shocks'. Many authors, including Ireland (2011) and Alessandro Calza and Stracca (2013), apply habit persistent in their models. η is labor supply elasticity of patient households. ζ allows the labor specification between two production sectors. If $\zeta = 0$, the labor in two production sectors is perfect substitute and the patient households tend to supply all the labor in the sector with higher wage. A positive ζ could help avoid such a situation and the labor supply responses less to production shock in either of the production sectors. Also, we do not need to enforce the same wage rate across sectors with a positive

ζ. a_t is a preference shock that affects all variables in the utility function and it follows an AR(1) process:

$$\ln(a_t) = \rho_a \ln(a_{t-1}) + \epsilon_{at} \tag{1.2}$$

with $\epsilon_{at} \sim N(0, \sigma_a)$

The patient households are subject to a budget constraint:

$$C_t - B_t = W_{ct}N_{ct} + W_{ht}N_{ht} - \frac{B_{t-1}R_{t-1}}{\pi_t} + F_t + D_t \tag{1.3}$$

They have several income sources: labor income, dividend from both production firms, profit from retailers and debt payment from borrowers. Their only expenditure is their own consumption. They also supply funds for impatient households. B_t is the real debt representing the amount of money the patient households lend out at time t in units of consumption good. It is negative, inferring a fund outflow. $\frac{B_{t-1}R_{t-1}}{\pi_t}$ is the debt payment from borrowers adjusted by the inflation. Note that $\frac{R_{t-1}}{\pi_t}$ is the real interest rate at time $t-1$, so it is the real interest rate that affects the debt burden instead of the nominal one. W_{ct}, W_{ht} are the patient households' wages from the two production sectors. D_t is the dividend from the competitive firms. F_t is the profit from the retailers.

1.2.2 Impatient Households (Borrowers)

The households maximize their lifetime utility function:

$$\max\{\sum_{t=0}^{\infty} E_0(\beta')^t a_t(\ln(C'_t - \gamma' C_t) + j\ln(H'_t) - \frac{1}{1+\eta'}((N'_{ct})^{1+\zeta'} + (N'_{ht})^{1+\zeta'})^{\frac{1+\eta'}{1+\zeta'}})]\} \tag{1.4}$$

β' is the discount factor for impatient households and $\beta > \beta'$ implies that the patient households are more patient. $C'_t, H'_t, N'_{ct}, N'_{ht}$ are impatient households' final good consumptions, housing stock (or housing service consumption), and labor supply for the two production sectors. γ', η', ζ' are the habit persistent parameter, labor supply elasticity and sectoral labor specification parameter. We allow the two types of households to pertain different values for these parameters.

j is the weight on the utility of housing services consumption. Iacoviello and Neri (2010) set j as the housing preference shock and it could vary trough different periods. We set j as a constant because we only consider it as a preference indicator for housing services compared to consumption good. The housing and housing services are not distinguishable in this model, so H_t represents both. In the utility function, it is housing services; in the budget constraint and collateral constraint, it represents housing units itself. In our model, the housing is homogenous and so is housing services. Because the same housing will deliver the same services no matter how its price will change, impatient households have no reason to become more prefer or less prefer the same housing services with all other parameters keep unchanged. Moreover, people, especially those take housing as collateral, expect to get more from housing price appreciation besides the services it delivers. Modeling j as a shock and allowing it to vary across time means accounting for all the expectations for housing into housing services. It certainly will absorb the effects of other shocks. In such a way, we will overestimate the housing preference shock and underestimate other shocks. So we set j as a constant in this paper. As we are going to estimate the model for the two sub periods, we would expect j to be different for the two sub periods because some parameters that relate to j have different calibrated values. We think it is more proper to investigate the housing preference shock in a model that we can distinguish housing and the services it delivers.

The impatient households consume the final consumption goods, accumulate housing, supply labor and demand fund to finance their housing purchases. The budget constraint is:

$$C'_t + Q_t(H'_t - (1-\delta_h)H'_{t-1}) - B'_t = W'_{ct}N'_{ct} + W'_{ht}N'_{ht} - \frac{B'_{t-1}R_{t-1}}{\pi_t} \tag{1.5}$$

Q_t is the real housing price which is defined as the ratio of nominal housing price and the final consumption good price $\frac{Q^n_t}{P_t}$. The impatient households borrow from the patient households and take housing as collateral. So the impatient households are subject to a collateral constraint which shows their borrowing limit.

$$B_t \leq E_t\{\frac{m(1-\delta_h)H'_t\pi_{t+1}Q_{t+1}}{R_t}\} \tag{1.6}$$

This collateral constraint implies that the total value of debt payment in next period should be less or equal to a certain fraction of the inflation and depreciation adjusted housing value in next period. The collateral constraint will always be binding at steady state because of $\beta > \beta'$.

In Iacoviello and Neri (2010), the housing for both types of households are homogenous. The housing stock of impatient households only accounts for about 15% of all the housing in the market. I test my model without housing market segmentation (the housing are homogenous) and also get similar results for the housing market fraction of impatient households. If impatient households only take a small fraction of housing, a change in responsiveness of monetary policy would not influence the housing market very much because the effects of monetary policy through the credit channel and the collateral constraint is limited[4]. Based on these considerations, we need an environment where the impatient households account for larger fraction in the housing market so that the effect of responsiveness of monetary policy would be amplified to a level we may observe. During the past financial crisis, the constrained consumers must take more than 15% of the total housing stock. So a model in which patient households could consume and save on housing just as the impatient households could not help illustrate our problem. In order to make the impatient households take a relatively large fraction in the housing market, we adopt an extreme case that they are the only participants in the housing market. In such a way, we could see the maximized effect of the responsiveness of monetary policy through the credit channel.

The housing market segmentation also corresponds to the real economy. Consider that the housing market for patient households and impatient households are totally different and segmented. For example, the residents of very expensive houses in Manhattan don't care about the housing in Brooklyn. We are only interested in the Brooklyn housing market, which has more constrained homeowners. So we are interested in a market which mainly occupied by the constrained households. Also, we consider the housing for patient and impatient households as heterogeneous goods. The two different types of housing are in equilibrium in two separate markets. We can ignore the housing market of patient households and focus on the other one. In such a circumstance, the patient households act like bankers: they are more patient and they supply funds for the impatient households to finance their home buying.

There are three production firms: wholesale good firms, housing firms and retailers. The first two are competitive and the last one is monopolistic. The wholesale good firms produce homogeneous intermediate goods. The retailers buy the intermediate goods and transfer them into differentiated goods with linear technology. The households consume final goods which are the indexed differentiated goods.

1.2.3 Competitive Production Sectors

The technology of the two competitive firms are both in Cobb-Douglas style:

$$Y_{ct} = K_{ct-1}^{\alpha_c}(Z_{ct}(N_{ct}^{\nu}(N'_{ct})^{1-\nu}))^{1-\alpha_c} \tag{1.7}$$

$$Y_{ht} = K_{ht-1}^{\alpha_h}L_{t-1}^{\alpha_l}(Z_{ht}(N_{ht}^{\nu}(N'_{ht})^{1-\nu}))^{1-\alpha_h-\alpha l} \tag{1.8}$$

We index the variables according to when it is determined. $K_{ct-1}, K_{ht-1}, L_{t-1}$ enters into

[4] we will discuss this problem in chapter 2.

the production functions implying that the business capital and land used in production at time t is determined at time $t-1$. We assume a constant land supply and normalize $L_t = 1$

The labor demand for the two types of households enters into the production function in Cobb-Douglas fashion and ν represents the income share of patient households. There are two interpretations of ν. The first one is taking ν as the population share of patient households by assuming that the two types of households supply homogenous labor. The second one is considering ν as an indicator of the relative importance of labor. The higher the ν, the higher the wage. In our model, we adopt the second explanation of ν. Z_{ct}, Z_{ht} are the productivity of two production sectors and each of them is composed of a constant growth rate and an $AR(1)$ process.

The dividend of the firms:

$$D_{ct} = \frac{Y_{ct}}{X_t} - W_{ct}N_{ct} - W'_{ct}N'_{ct} - \frac{I_{ct}}{Z_{\mu t}} - \Psi_{ct} \tag{1.9}$$

$$D_{ht} = Q_t Y_{ht} - W_{ht}N_{ht} - W'_{ht}N'_{ht} - I_{ht} - \Psi_{ht} - P_{lt}(L_t - L_{t-1}) \tag{1.10}$$

X_t is the markup of final goods over the wholesale goods and is defined as $X_t = \frac{P_t}{P_t^w}$. $Z_{\mu t}$ is the investment specific shock which influences the efficiency of investment. We assume that the investment specific shock only affects wholesale good sector because the improvement or disturbance in technology always affects the capital efficiency for the non-housing sector. For example, the introduction of a new operational system may improve the efficiency of a production line and a small bug in software could make the computer work less efficiently. These factors exert little influence in housing production sector. The three shocks are:

$$\ln(Z_{ct}) = t\ln(\lambda_c) + \ln(z_{ct}) \tag{1.11}$$

$$\ln(Z_{ht}) = t\ln(\lambda_h) + \ln(z_{ht}) \tag{1.12}$$

$$\ln(Z_{\mu t}) = t\ln(\lambda_\mu) + \ln(z_{\mu t}) \tag{1.13}$$

$$\ln(z_{ct}) = \rho_c \ln(z_{ct-1}) + \epsilon_{ct} \tag{1.14}$$

$$\ln(z_{ht}) = \rho_c \ln(z_{ht-1}) + \epsilon_{ht} \tag{1.15}$$

$$\ln(z_{\mu t}) = \rho_c \ln(z_{\mu t-1}) + \epsilon_{\mu t} \tag{1.16}$$

$\lambda_c, \lambda_h, \lambda_\mu$ are the growth rate of the technologies. $\epsilon_{ct}, \epsilon_{ht}, \epsilon_{\mu t}$ are iid and $\epsilon_{ct} \sim N(0, \sigma_c), \epsilon_{ht} \sim N(0, \sigma_h), \epsilon_{\mu t} \sim N(0, \sigma_\mu)$.

The business investment of both firms are:

$$I_{ct} = K_{ct} - (1 - \delta_{kc})K_{ct-1} \tag{1.17}$$

$$I_{ht} = K_{ht} - (1 - \delta_{kh})K_{ht-1} \tag{1.18}$$

For both firms, they are also facing a capital adjustment cost Ψ_{ct}, Ψ_{ht}:

$$\Psi_{ct} = \frac{\psi}{2\delta_{kc}}(\frac{K_{ct}}{K_{ct-1}} - g_k)^2 \frac{K_{ct-1}}{\lambda_k^t} \tag{1.19}$$

$$\Psi_{ht} = \frac{\psi}{2\delta_{kh}}(\frac{K_{ht}}{K_{ht-1}} - g_c)^2 K_{ht-1} \tag{1.20}$$

I took this form of capital adjustment cost from Iacoviello and Neri (2010) and made some adjustments according to our model specification. This form of capital adjustment cost grantees zero adjustment cost at steady state.

1.2.4 Retailer

We adopt the sticky price set up of Bernanke et al. (1999), which was also applied in Iacoviello (2005): The monopolistic competition is at the retailer level. There is a continuum of retailers with mass of 1 and they are indexed with ι. The retailers buy wholesale goods Y_{ct} in the competitive market at price P_t^w and transfer them into differentiated final good $Y_{ct}(\iota)$ at no cost. we assume the retailer ι will incur an implicit cost when resetting the nominal prices of the differentiated final consumption good he produced at time t. Each retailer sets his own retail price $P_t(\iota)$. The final consumption good is aggregated through the function

$$Y_{ct}^f = (\int_0^1 (Y_t(\iota))^{\frac{\varepsilon-1}{\varepsilon}} d\iota)^{\frac{\varepsilon}{\varepsilon-1}} \tag{1.21}$$

where $\varepsilon > 1$.

The market price index for final good is:

$$P_t = (\int_0^1 (P_t(\iota))^{\frac{\varepsilon-1}{\varepsilon}} d\iota)^{\frac{\varepsilon}{\varepsilon-1}} \tag{1.22}$$

Each retailer ι faces a demand for his differentiated good $Y_{ct}(\iota)$

$$Y_{ct}(\iota) = \frac{P_t(\iota)}{P_t} Y_{ct}^f \tag{1.23}$$

The retailers market is organized by the Calvo pricing strategy. In each period, there is a constant fraction $1-\theta$ of retailers who can adjust their price. The rest of the retailers would take the average price from the last period. $\frac{1}{1-\theta}$ is the average length of the price contract. In such a way, we do not need to keep a track of the pricing strategies of all the retailers. Also, all the retailers who have a chance to adjust their price will choose the same pricing strategy. Each retailer takes the demand curve as given and chooses a retail price $P_t(\iota)$. The retailers problem is:

$$\max\{\Sigma_{k=0}^{\infty}\theta^k E_t \Lambda_{t,k} \frac{P_t(\iota) - P_{t+k}^w}{P_{t+k}} Y_{t+k}(\iota)\} \tag{1.24}$$

We define $X_t = \frac{P_t}{P_t^w}$ as the mark up and we have $X = \frac{\varepsilon}{\varepsilon-1}$ at steady state. The profit of retailers $(1-\frac{1}{X_t})Y_{ct}$ is returned to patient households. As we have mentioned above, all the retailers that do adjust their price at time t choose the same pricing strategy P_t^* and the rest of the retailers will charge the average price from last period P_{t-1}. So the price index for time t is:

$$P_t^{1-\varepsilon} = \int_0^1 (P_t(\iota))^{1-\varepsilon} d\iota = \theta P_{t-1}^{1-\varepsilon} + (1-\theta)(P_t^*)^{1-\varepsilon} \tag{1.25}$$

Linearization of the optimal condition for P_t^* and price index of P_t will yield the New Kaynesian Philips curve. Refer the Appendix C for details of derivation of the Philips curve.

1.2.5 Central Bank

We assume the central bank adjusts the nominal interest through a Taylor rule:

$$R_t = \pi_t^{1+r_\pi} (\frac{GDP_t}{g_c GDP_{t-1}})^{r_Y} e_{et}. \tag{1.26}$$

R_t is the gross interest rate and the interest rate target is related to the current inflation and GDP growth rate. e_t is an iid monetary policy shock.

1.3 Equilibrium

1.3.1 Market Clearing Condition

In equilibrium, good market, housing market, debt market, and labor market all clear.

$$C_t + \frac{I_{ct}}{Z_{\mu t}} + I_{ht} + \Psi_{ct} + \Psi_{ht} = Y_{ct} \tag{1.27}$$

$$H_t' + (1-\delta_h) H_{t-1}' = Y_{ht} \tag{1.28}$$

$$B_t + B'_t = 0 \tag{1.29}$$

The total labor demand for wholesale good firms and housing firms are:

$$NN_{ct} = N_{ct}^{\nu}(N'_{ct})^{1-\nu} \tag{1.30}$$

$$NN_{ht} = N_{ht}^{\nu}(N'_{ht})^{1-\nu} \tag{1.31}$$

Please refer the appendix for all the optimal conditions of the model.

1.3.2 Balanced Growth Path

The technologies of the wholesale good sector, the housing sector and investment contain different growth trends. $\lambda_c, \lambda_h, \lambda_\mu$ denote the gross growth rates of the technologies respectively. The growth rate $Y_{ct}, Y_{ct}^f, C_t, C'_t, I_{ht}, K_{ht}, \Psi_{ct}, \Psi_{ht}, F_t D_t$ is g_c. Growth rate of I_{ct}, K_{ct} is g_k Growth rate of H'_t and Y_{ht} is g_h.

$$g_c = \lambda_c \lambda_\mu^{\frac{\alpha_c}{1-\alpha_c}} \tag{1.32}$$

$$g_k = \lambda_c \lambda_\mu^{\frac{1}{1-\alpha_c}} \tag{1.33}$$

$$g_h = \lambda_c^{\alpha_h} \lambda_\mu^{\frac{\alpha_h \alpha_c}{1-\alpha_c}} \lambda_h^{1-\alpha_h-\alpha l} \tag{1.34}$$

$$g_q = \lambda_c^{1-\alpha_h-\alpha l} \lambda_\mu^{\frac{(1-\alpha_h-\alpha l)\alpha_c}{1-\alpha_c}} \lambda_h^{-(1-\alpha_h-\alpha l)} \tag{1.35}$$

We detrend the model by using these growth rate and get a stationary system. We use the lower case letter to represent the variables without the trend. We assume there is no growth in labor at steady state so we transfer upper case N's to lower case n's directly. For the three multipliers we have $\xi_t = \Xi_t g_c^t, \lambda_t = \Lambda_t g_c^t, \lambda'_t = \Lambda'_t g_c^t$. The prices except for housing price in our model do not contain any growth trends since we assume a zero inflation at steady state. After substitute out b_t by the market clearing conditions, we have 32 unknowns:k_{ct},k_{ht},h'_t,b'_t,q_t,y_{ct},y_{ht},n_{ct},n'_{ct},n'_{ht},n_{ht},nn_{ct},nn_{ht},i_{ct},i_{ht},c_t,c'_t,R_t,π_t,x_t, gdp_t,ξ_t,λ_t,λ'_t,ψ_{ct},ψ_{ht},w_{ct},w_{ht},w'_{ct},w'_{ht},p_{lt},rr_t.

The stationary system is composed of 32 equations:

$$\lambda_t = \frac{a_t}{c_t - \gamma \frac{c_{t-1}}{g_c}} - \frac{\beta \gamma a_{t+1}}{g_c c_{t+1} - \gamma c_t} \tag{1.36}$$

$$\lambda'_t = \frac{a_t}{c'_t - \gamma' \frac{c'_{t-1}}{g_c}} - \frac{\beta' \gamma' a_{t+1}}{g_c c'_{t+1} - \gamma c'_t} \tag{1.37}$$

$$a_t (n_{ct}^{1+\zeta} + n_{ht}^{1+\zeta})^{\frac{\eta-\zeta}{1+\zeta}} n_{ct}^{\zeta} = w_{ct} \lambda_t \tag{1.38}$$

$$a_t (n_{ct}^{1+\zeta} + n_{ht}^{1+\zeta})^{\frac{\eta-\zeta}{1+\zeta}} n_{ht}^{\zeta} = w_{ht} \lambda_t \tag{1.39}$$

$$a_t ((n'_{ct})^{1+\zeta'} + (n'_{ht})^{1+\zeta'})^{\frac{\eta'-\zeta'}{1+\zeta'}} (n'_{ct})^{1+\zeta'} = w'_{ct} \lambda'_t \tag{1.40}$$

$$a_t ((n'_{ct})^{1+\zeta'} + (n'_{ht})^{1+\zeta'})^{\frac{\eta'-\zeta'}{1+\zeta'}} (n'_{ht})^{1+\zeta'} = w'_{ht} \lambda'_t \tag{1.41}$$

$$q_t \lambda'_t = j \frac{a_t}{h'_t} + E_t \{ \beta' (1-\delta_h) \frac{q_{t+1} \lambda'_{t+1}}{g_h} + \frac{m(1-\delta_h) \xi_t q_{t+1} \pi_{t+1}}{R_t} g_q \} \tag{1.42}$$

$$1 = E_t \beta \frac{\lambda_{t+1}}{\lambda_t g_c} \frac{r_t}{\pi_{t+1}} \tag{1.43}$$

$$\lambda'_t = E_t \beta' \frac{\lambda_{t+1}}{g_c} \frac{r_t}{\pi_{t+1}} + \xi_t \tag{1.44}$$

$$c'_t + q_t (h'_t - \frac{1-\delta_h}{g_h} h'_{t-1}) - b'_t = w'_{ct} n'_{ct} + w'_{ht} n'_{ht} - \frac{b'_{t-1} R_{t-1}}{\pi_t g_c} \tag{1.45}$$

$$b'_t = m(1-\delta_h) \frac{h'_t q_{t+1} \pi_{t+1}}{R_t} g_q \tag{1.46}$$

$$\frac{1}{z_{\mu_t}} + \frac{\psi}{\delta_{kc}} g_k (\frac{k_{ct}}{k_{ct-1}} - 1) = E_t \beta \frac{\lambda_{t+1}}{\lambda_t g_c} \{ \alpha_c \frac{y_{ct+1}}{x_{t+1} k_{ct}} g_c + \frac{(1-\delta_{kc})}{\lambda_\mu z_{\mu t+1}} + \frac{\psi}{2\delta_{kc}} g_c g_k (\frac{k_{ct+1}^2}{k_{ct}^2} - 1) \} \tag{1.47}$$

$$1 + \frac{\psi}{\delta_{kh}} g_c (\frac{k_{ht}}{k_{ht-1}} - 1) = E_t \beta \frac{\lambda_{t+1}}{\lambda_t g_c} \{ \alpha_h \frac{q_{t+1} y_{ht+1}}{k_{ht}} g_c + (1-\delta_{kh}) + \frac{\psi}{2\delta_{kh}} g_c^2 (\frac{k_{ht+1}^2}{k_{ht}^2} - 1) \} \tag{1.48}$$

$$i_{ct} g_k = k_{ct} g_k - (1-\delta_{kc}) k_{ct-1} \tag{1.49}$$

$$i_{ht} g_c = k_{ht} g_c - (1-\delta_{kh}) k_{ht-1} \tag{1.50}$$

$$y_{ct} = k_{ct-1}^{\alpha_c}(n_{ct}^{\nu}(n'_{ct})^{1-\nu})^{1-\alpha_c}(\frac{1}{g_k})^{\alpha_c} \tag{1.51}$$

$$y_{ht} = k_{ht-1}^{\alpha_h}(n_{ht}^{\nu}(n'_{ht})^{1-\nu})^{1-\alpha_h-\alpha l}(\frac{1}{g_c})^{\alpha_h} \tag{1.52}$$

$$y_{ht}g_h = h'_t g_h - (1-\delta_h)h'_{t-1} \tag{1.53}$$

$$y_{ct} = c_t + c'_t + \frac{i_{ct}}{z_{\mu t}} + i_{ht} + \psi_{ct} + \psi_{ht} \tag{1.54}$$

$$\psi_{ct} = \frac{\psi}{2\delta_{kc}} g_k(\frac{k_{ct}}{k_{ct-1}} - 1)^2 k_{ct-1} \tag{1.55}$$

$$\psi_{ht} = \frac{\psi}{2\delta_{kh}} g_c(\frac{k_{ht}}{k_{ht-1}} - 1)^2 k_{ht-1} \tag{1.56}$$

$$nn_{ct} = n_{ct}^{\nu}(n'_{ct})^{1-\nu} \tag{1.57}$$

$$nn_{ht} = n_{ht}^{\nu}(n'_{ht})^{1-\nu} \tag{1.58}$$

$$w_{ct} = (1-\alpha_c)\nu\frac{y_{ct}}{x_t n_{ct}} \tag{1.59}$$

$$w'_{ct} = (1-\alpha_c)(1-\nu)\frac{y_{ct}}{x_t n'_{ct}} \tag{1.60}$$

$$w_{ht} = (1-\alpha_h-\alpha l)\nu\frac{q_t y_{ht}}{n_{ht}} \tag{1.61}$$

$$w'_{ht} = (1-\alpha_h-\alpha l)(1-\nu)\frac{q_t y_{ht}}{n'_{ht}} \tag{1.62}$$

$$p_{lt} = \beta\frac{\lambda_{t+1}}{\lambda_t g_c}(\frac{\alpha_l q_{t+1} y_{ht+1}}{L_t} + p_{lt+1}) \tag{1.63}$$

$$gdp_t = y_{ct} + q_t y_{ht} \tag{1.64}$$

$$R_t = \pi_t^{1+r_\pi}(\frac{gdp_t}{gdp_{t-1}g_c})^{r_Y} e_t \tag{1.65}$$

$$\hat{\pi_t} = \beta\hat{\pi_{t+1}} - \kappa\hat{X}_t + u_t \tag{1.66}$$

$$rr_t = \hat{R}_t - \hat{\pi_{t+1}} \tag{1.67}$$

1.4 Data and Estimation Strategies

The model is linearized around the balanced growth path and estimated with Bayesian techniques. I chose the prior distribution for the estimated parameters according to the Smets and Wouters (2007) and Iacoviello and Neri (2010). I estimate the posterior distribution by using Metropolis-Hastings algorithm and please refer to An and Schorfheide (2007) for details. The periods of interest are 1987Q1:1999Q4 and 2000Q1:2009Q4 which characterized as the 'Moderation Period' and the period including both the housing boom and the 'Great Recession'.

1.4.1 Data

The data series available for the estimation are business investment, consumption, housing output, housing price, inflation and interest rate. The business investment is defined as the total investment subtracting the residential fixed investment. For the housing price, I choose the Census Bureau constant quality index for the price of new houses sold, the same as Iacoviello and Neri (2010). In the model presented in the previous section, the final good consumption and housing services consumption are separated. So the data for the good consumption is calculated as the total private consumption expenditure minus the housing services expenditure. Because there are no available consumption price index matching the consumption good defined in our model, I constructed the consumer price index by following Liu et al. (2013) and using Torquest formula. I deflate the business investment,consumption and housing price by the constructed consumer price index to get the real values. To get the per capita values for real variables, we divide real business investment, real consumption and housing output by the total number of population, which is civilian noninstitutional population of 16 years old or above. I follow Iacoviello and Neri (2010) and use the residential investment in chained 2009 dollar as a proxy of the housing output. The ratio of housing output and the population is the per capita housing output.

We have the data on hours and wages for the construction sector and good manufacturing sector. In order to get the per capita hours, we divide the total hours in a sector by the total population by following Iacoviello and Neri (2010). From both data we could observe declining trends which conflicts with our assumption that there is no trend in hours and implies that the hour data does not match the model. So we take the working hours as unobserved. As for the wage, we do not assume sticky wages so there is no wage inflation in

Parameters	β'	α_c	α_h	α_l	δ_{kc}	δ_{kh}	X_{ss}	θ	m
Value	0.91	1/3	0.2	0.1	0.025	0.03	1.05	0.75	0.8

Table 1.1: Parameters(from literature)

parameters	β	δ_h	λ_c	λ_h	λ_μ
$value(87-99)$	0.9956	0.031	1.0025	0.9992	1.0054
$vaue(00-09)$	0.9747	0.0435	0.9915	0.9747	1.0060

Table 1.2: Parameters(calibrated)

our model, which also conflicts with the observed data. We treat wage as observed, too.

The four real variables are not stationary so I use the growth rate instead of the level data as observable. The inflation is the quarter on quarter log difference of the constructed consumer price index and the interest rate is the secondary market rate of 3-month treasury bills. In sum, the 6 observables are growth rate of real business investment,growth rate of real consumption,growth rate of housing output, growth rate of real housing price, inflation and interest rate. All variables are demeaned except for the interest rate. The observation equations are:

$$Y_t = \begin{bmatrix} gc_t \\ gi_t \\ gyh_t \\ gq_t \\ r_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} \ln(c_t) - \ln(c_{t-1}) \\ \ln(i_t) - \ln(i_{t-1}) \\ \ln(yh_t) - \ln(yh_{t-1}) \\ \ln(q_t) - \ln(q_{t-1}) \\ r_t \\ \pi_t \end{bmatrix} \tag{1.68}$$

1.4.2 Calibrated Parameters

There are two categories of calibrated parameters. The first one contains the parameters which are unchanged across two subperiods and the second one contains parameters that are different for these two subperiods. The parameters that belong to first group are the discount factor of impatient household β', depreciate rate of capital δ_{kc}, δ_{kh}, loan to value ratio m, the parameter of Calvo-style pricing θ, the steady state mark up x_{ss}, and the capital share in consumption production and in housing production $\alpha_c, \alpha_h, \alpha l$. The parameters that go to the second group are the discount factor for patient household β, the depreciation rate of housing δ_h and the growth rates of technology $\lambda_c, \lambda_h, \lambda_\mu$. The parameters in the first group are taken from literature and the parameters in the second group are calibrated by targeting certain steady state ratios.

Table 1.1 and Table 1.2 lists the calibrated parameter values. The discount factor of impatient households should be lower than that of patient households. According to the empirical work of Carroll and Samwick (1997), the consumer discount factor falls in the range of $[0.91, 0.99]$. Considering the value of β for the two subperiods, I set β' equal to the lower bound 0.91. Also, this value for β' guarantees that the collateral constraints are binding for all our simulations and impulse responses. The capital share α_c in wholesale good sector is 1/3. Iacoviello and Neri (2010) set the share of capital in housing production as 0.2.[5] and the share of land as 0.1. So we have $\alpha_h = 0.2$ and $\alpha_l = 0.1$. The depreciation rate of capital in wholesale good sector is 0.025, following Smets and Wouters (2007) and Favilukis et al. (2012). And the depreciation rate of capital in housing sector is 0.03. The steady state markup x_{ss} is 1.05, which is the same as that in Iacoviello (2005). The loan to value ratio m is set to be 0.80, implying a 20% down payment requirement for the impatient households. The data shows that the average loan to value ratio are 0.768 and 0.761 for the two sub periods respectively[6]. The impatient households are more willing to borrow so we use a slightly higher value for m. The value of θ governs the degree of price stickiness. Following the previous literature like Monacelli (2009), we parameterize θ as 0.75 which suggests one year average length of price contract.

We calibrate λ_μ and λ_c by using the average growth rate of investment and consumption. With the value of g_c, we calibrate the discount factor of patient households to be 0.9956 and 0.9747 by targeting the average annual interest of 5.41% and 2.74% for the two sub periods respectively.

1.4.3 Prior Distribution

Following the standard practices, we choose the beta distribution as priors for the parameters that fall between 0 and 1; inverse gamma distribution for innovations standard deviations. For other parameters that are greater than 0 we use normal distribution or gamma distribution.

Our priors selection is similar to that specified in previous literature. By following Smets and Wouters (2007), we assume the standard deviations of innovations to follow an inverse gamma distribution with mean of 0.1 and standard deviation of 2, which indicates a pretty loose prior. We adopt the priors from Iacoviello and Neri (2010) for the following parameters: The persistence of $AR(1)$ processes follows beta distribution with mean 0.8 and standard deviation 0.1. we set the prior mean for habit persistent (γ and γ') to be 0.5. The priors of labor supply elasticity (η and η') are assumed to follow a gamma distribution with mean

[5] Iacoviello and Neri (2010) has two types of capital in the housing production, each accounts for 0.1 in the production. Since we only have one type of capital, the capital share in the housing production is 0.2.

[6] The data is from the Finance Board's Monthly Survey of Rates and Terms on Conventional Single-Family Non-farm Mortgage Loan.

and standard deviation of 0.5 and 0.1 respectively. The prior of the labor income share ν of patient households follows a beta distribution with mean 0.5 and standard deviation 0.05.

For the housing services consumption weight j in the utility function, I assume it follows a normal distribution. Iacoviello and Neri (2010) calibrate j to 0.12 and Iacoviello (2005) calibrates j to be 0.1. So I assume the prior mean of j to be 0.1 and standard deviation to be 0.075. The prior mean of capital cost parameter ψ is 0.1. Liu et al. (2013) finds that the estimation of capital adjustment cost parameter ψ varies across different research. This parameter is much smaller in the DSGE model with no financial frictions. The estimation of ψ is 0.18 in Liu et al. (2013).

The priors of Taylor rule coefficients r_π, r_y follow a normal distribution. We assume different prior means for the two periods of interests. Various paper have shown that the Taylor rule coefficients of the Great Moderation period and those of the periods after 2000 are different substantially. Classic Taylor rule coefficients by Taylor (1993) assigned the value of 1.5 ($r_\pi = 0.5$)for inflation and 0.5 ($r_y = 0.5$)for output gap. Seyfried (2010) demonstrates that the classical Taylor rule fits the interest rate path much better before 2000. Hofmann and Bogdanova (2012) states that 'The systematic deviation of policy rules from the Taylor rule since the early 2000 has been identified by previous studies' and they showed that the interest rate path implied by Taylor rule lied above the actual one. Labonte (2012) also presents the substantial deviation of policy rules after 2000 from the path predicted by the classical Taylor rule. These are the evidences that the Taylor rule, which fits the interest rate path before 2000, could not fit the interest rate path after that. Taylor (2007) also believes that the monetary policy after the early 2000s becomes less responsive which implies smaller Taylor rule coefficients. Based on these evidences, Taylor rule coefficients experienced substantial changes after 2000. So I set the prior mean for r_π, r_y to be 0.15 for the period of 2000-2009 and 0.5 for period of 1987-1999. The standard deviation is 0.05 for both cases.

1.5 Empirical Results

1.5.1 Posterior Distribution

We use Dynare version 4.3 to do the estimation. Table 1.3 reports the posterior means and the 95% confidence intervals for all the parameters, together with the prior means and standard deviations. We will focus on the results for the period of 2000-2009 and we are only interested in the Taylor rule coefficients for the period of 1987-1999. The weight for housing utility j is 0.2, which is different from the calibrated j in Iacoviello's two papers, 0.1

	Prior			Posterior		
Parameters	Distri.	Mean	SD	Mean	5%	95%
j	*Normal*	0.1	0.075	0.2000	0.1185	0.2805
ψ	*Gamma*	0.1	0.075	0.0052	0.0000	0.0110
γ	*Beta*	0.5	0.1	0.5357	0.3993	0.6689
γ'	*Beta*	0.5	0.1	0.4818	0.4181	0.5470
η	*Gamma*	0.5	0.1	0.4844	0.3241	0.6355
η'	*Gamma*	0.5	0.1	0.5479	0.3757	0.7186
υ	*Beta*	0.5	0.05	0.6683	0.6074	0.7314
ζ	*Normal*	1	0.1	1.0088	0.8469	1.1703
ζ'	*Normal*	1	0.1	1.0018	0.8337	1.1625
$r_\pi(0009)$	*Normal*	0.15	0.05	0.3322	0.2664	0.3951
$r_y(0009)$	*Normal*	0.15	0.05	0.1790	0.1185	0.2336
$r_\pi(8799)$	*Normal*	0.5	0.05	0.6272	0.5500	0.6988
$r_y(8799)$	*Normal*	0.5	0.05	0.5672	0.4696	0.6394
ρ_c	*Beta*	0.8	0.1	0.8501	0.7538	0.9450
ρ_h	*Beta*	0.8	0.1	0.9095	0.8690	0.9515
ρ_a	*Beta*	0.8	0.1	0.9400	0.8919	0.9885
ρ_u	*Beta*	0.8	0.1	0.3089	0.1951	0.4232
ρ_μ	*Beta*	0.8	0.1	0.9947	0.9901	0.9991
σ_c	*Inv.gamma*	0.1	2	0.0248	0.0193	0.0302
σ_h	*Inv.gamma*	0.1	2	0.0403	0.0321	0.0483
σ_a	*Inv.gamma*	0.1	2	0.0354	0.0235	0.0475
σ_u	*Inv.gamma*	0.1	2	0.0166	0.0133	0.0198
σ_e	*Inv.gamma*	0.1	2	0.0149	0.0123	0.0175
σ_μ	*Inv.gamma*	0.1	2	0.0193	0.0151	0.0234

Table 1.3: Results (structural parameters): 2000Q1-2009Q4

and 0.12. Iacoviello (2005) did not include a housing production sector and Iacoviello and Neri (2010) did not exclude the patient households from the housing market. Because of these differences in the model set up and the definition, it is not a surprise that we obtain a different value of j.[7]. The two habit persistent parameters γ, γ' are both around 0.5: both agents demonstrate moderate degree of habit persistency with a slightly higher value for the unconstrained households. The two types of households do not demonstrate that different habit persistent as that in Iacoviello and Neri (2010) because of the segmented housing market. In our model, the patient households save only through the bond market. Even though the impatient households could not save, they have the access to the housing market. The housing itself is an asset and the constrained households save passively by buying homes.

[7]If we calibrate j by targeting the ratio of residential investment and GDP as Iacoviello and Neri (2010) does, we will also get different j's for the two sub periods since the δ_h are different. Refer Appendix B for the ratio of residential investment and GDP for details.

Both households can smooth consumption through their own assets, so they demonstrate moderate habit persistence.

The income share of patient households is about 0.67 which implies that the income fraction of constrained households only accounts for about one third of all the labor income. Despite the fact that the impatient households only take a relatively small fraction of all labor income and they also purchase housing with no more income sources, they still consume about 16% of all the consumption good at steady state because of their high marginal propensity to consume and ability to borrow.

The labor supply elasticity η, η' and the parameters of the labor specification in the two production sectors ζ, ζ' are all around the prior mean, suggesting that there is limited information about these parameters in the data. It is reasonable because we do not have observed data on labor or wages. We have discussed the reason in the previous section.

The Taylor rule coefficients of the period of 2000-2009 are 0.3322 and 0.1790, lower than those of the period of 1987-1999, which are 0.6272 and 0.5672. These two sets of Taylor rule coefficients confirm the fact that the monetary policy was less responsive after 2000. The productivity shock in the housing sector, the preference shock and the investment specific shock are pretty persistent with autocorrelation coefficients all above 0.9. The productivity shock in the wholesale good sector is not as persistent as that in the housing sector. The persistency of cost push shock is low, but it still accounts for important fractions in the variance decompositions of the key economic variables.

1.5.2 Properties of the Model

Prices and The Intertemporal Decisions of Housing

The prices play important roles in our economy. There are three different prices in our model: The nominal housing price Q_t^n, the wholesale good price P_t^w and the price index of final consumption good P_t. From these prices we can derive two relative prices: the real housing price $Qt = \frac{Q_t^n}{P_t}$ and the real price of wholesale good $\frac{P_t^w}{P_t}$ which is also the inverse of the mark up. The real housing price Q_t enters in our model since we represent every variable in real term. The nominal housing price and wholesale good price are flexible while the price of final consumption good is sticky. When a shock hits the economy, the fact that different prices adjust at different rates would cause changes in relative prices. For example, if a shock presses down all the prices, we will observe a drop in the real housing price and an increase in the mark up because the nominal housing price and wholesale good price adjust faster.

The impatient households' choice on housing is governed by the first order conditions of housing, which is closely related to the housing price. There two effects on housing choices regarding the housing price change: the substitution effect and the collateral effect. We use the marginal utility to rewrite the first order condition ($u'_{ct} = \lambda'_t$, $u'_{ht} = j\frac{a_t}{h'_t}$):

$$u'_{ht} = q_t u'_{ct} - \frac{\beta'(1-\delta_h)}{g_h} q_{t+1} u'_{ct+1} - m(1-\delta_h) g_q \xi_t \frac{q_{t+1}}{rr_t} \tag{1.69}$$

The first order condition equation tells us that the marginal cost equals the benefit of obtaining one unit of housing at time t. As we have shown, the housing both delivers housing services and serves as collateral and asset. By getting one more unit of housing, impatient households gain the marginal utility by u'_{ht}. At the same time, they could borrow more against this one unit of housing by the amount of $m(1-\delta_h)(g_q \frac{q_{t+1}}{rr_t})$ at time t as well as get resale value in consumption good units in next period by $\frac{\beta'(1-\delta_h)}{g_h} q_{t+1}$. ξ_t represents the marginal benefit of relaxing the borrowing constraint. Specifically, it is the marginal benefit of housing as collateral. So their product is the collateral effect and is the total marginal benefit of holding this one unit of housing and making it as collateral. Also, one unit of housing can be traded for final consumption good either in current period or in next period. If the impatient households obtain one unit of housing at time t, they must give up q_t unit of final consumption good c'_t by which they lose benefit $q_t u'_{ct}$. Or, they can hold this unit of housing until t+1 when they can resale the housing and trade for for $\frac{(1-\delta_h)q_{t+1}}{g_h}$ units of final consumption good c'_{t+1}. To sum it up, the left hand side of the first order condition is the benefit of obtaining one unit of housing while the right hand side is the cost. To be specific, the sum of the first two items on the right hand side is the net substitution effect of housing while the last item is the collateral effect. So, the net substitution effect together with the collateral effect determine the net cost of obtaining one unit of housing.

Equation (1.63) is the first order condition of housing for impatient households. We can see that the cost of one unit of housing is positively correlated with the current real housing price and negatively correlated with the future real housing price. When the current housing price is moderately high and future housing price keeps increasing, the collateral effect dominates the net substitution effect. we will observe an increase in housing demand because the higher future real housing price brings down the net cost of the housing at time t. When the current real housing price is so high that the net substitution effect outweighes the collateral effect, the cost of obtaining one unit of housing will increase, following a drop in housing demand. Further more, the cost of gaining more housing at current period is positively correlated with the housing depreciation rate, the real interest rate and negatively correlated to the marginal utility of consumption in next period and the shadow price of collateral constraint.

Based on the analysis above, we conjecture that the housing demand curve may not be a simple upward sloping line anymore. If we keep other things unchanged, the relationship

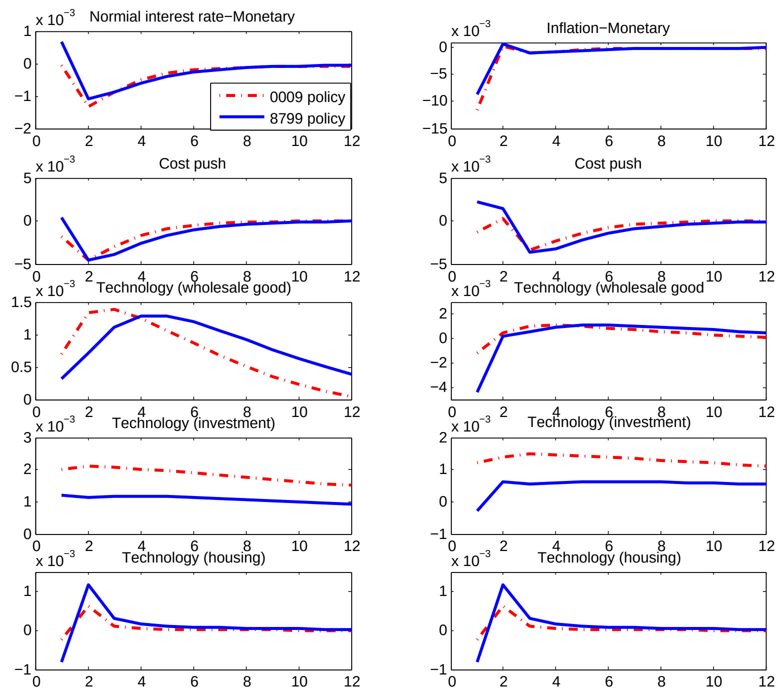

Figure 1.1: Impulse Response-Interest Rate and Inflation

between housing quantity demanded and real housing housing price is not monotonic: the collateral effect dominates when the current real housing price is below certain level and the demand of housing keeps increasing but with a descending rate; the net substitute effect dominates when the current real housing price is above certain level and a drop in housing demand follows.

Impulse Responses

Figure 1.1 to Figure 1.5 show us the impulse responses of variables to different shocks. The red dashed line represents the responses under a less responsive monetary policy, the 0009 policy while the blue solid line represents the responses under a more responsive monetary policy, the 8799 policy. In general, for most real variables, the responses for the same shock become less under a more responsive monetary policy and we could see this from figure 1.2 to figure 1.6, by which we conclude that a more responsive monetary policy makes the real variables respond less to the same shock except for the technology shock in the housing

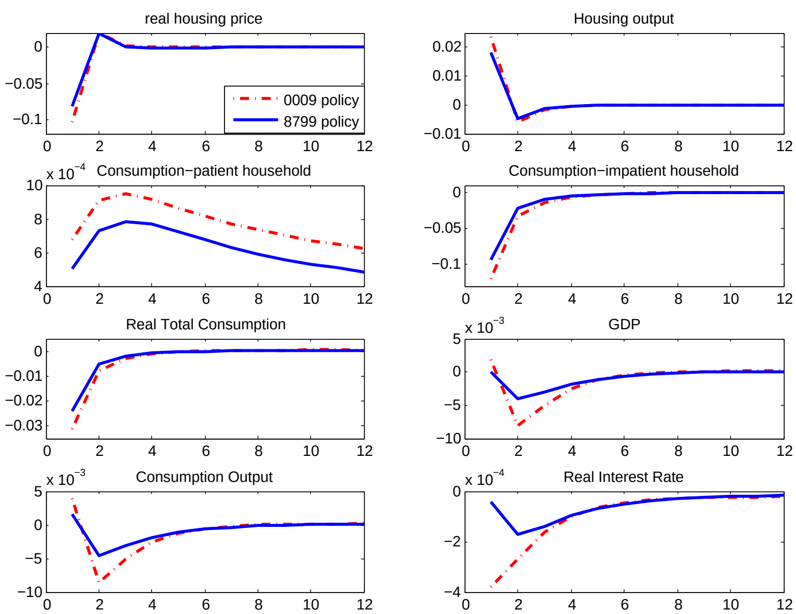

Figure 1.2: Impulse Response-Monetary Policy Shock

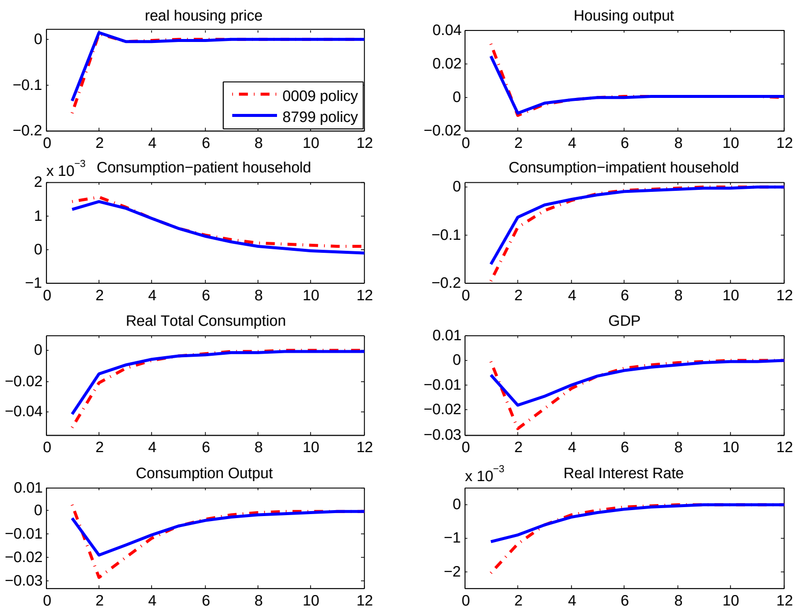

Figure 1.3: Impulse Response-Cost Push Shock

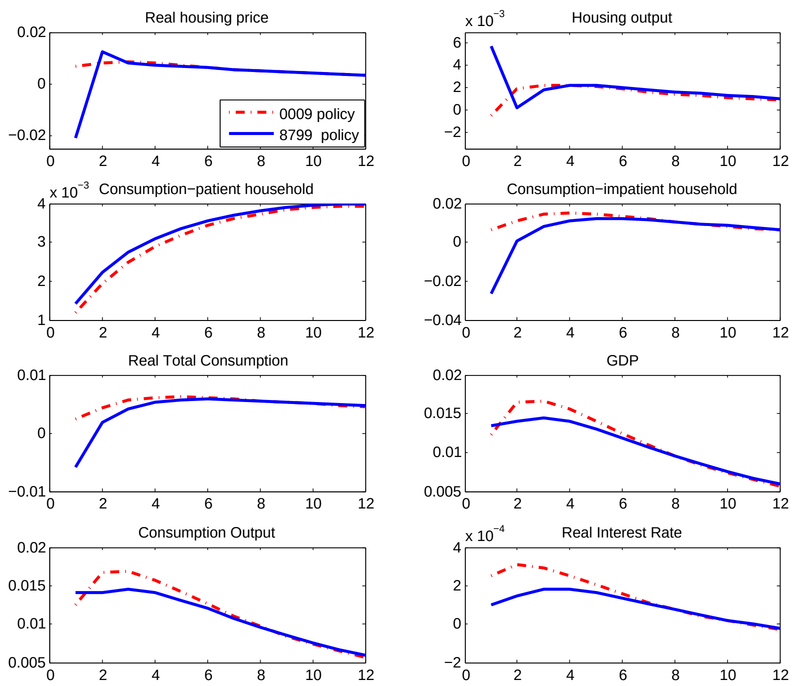

Figure 1.4: Impulse Response-Technology (Wholesale Good)

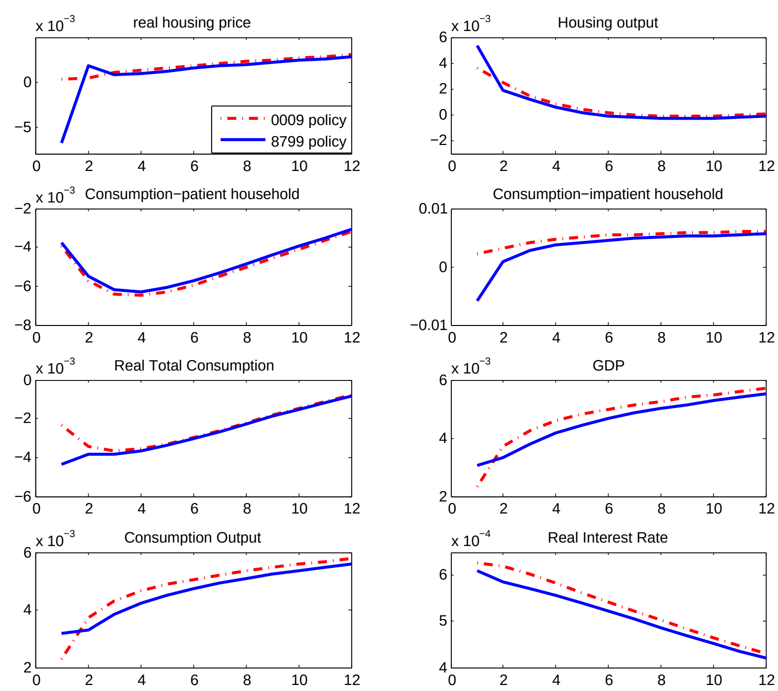

Figure 1.5: Impulse Response-Investment

sector.

Figure 1.2 presents us the impulse responses of variables to a tightened monetary policy. A contraction in monetary policy leads to an increase in nominal interest rate, then there would be a downward pressure on all of the prices in the economy. The higher interest rate reduces business investment by increasing its cost. It also increases the debt burden and the impatient households would cut the demand of both consumption good and housing. The decrease in the total demand for final consumption good brings down the price level P_t. Nominal housing price would also drop but the change in housing demand is not clear because it is determined by both the substitution effect and collateral effect of housing. Since the demand for final good decreases, the demand for intermediate good also decreases which leads to a drop in P_t^w. The nominal housing price Q_t^n and wholesale good price P_t^w adjust faster than the price of final good P_t, we will see a drop in real housing price and a rise in mark up.

Figure 1.2 shows us exactly the changes we mentioned above. The sudden decrease in real housing price tightens the collateral constraint and the increased mark up brings more income for patient households. The impatient households would cut back on the consumption for sure. Since they have no other income sources except for labor income, they would like to supply more labor to increase their total income. So the production for housing and consumption good increase a little bit after the shock. Combining with the housing given up by the constrained households and the increased new production, there are more surplus in the housing market. The impatient households are the only ones who can buy new homes and they are also the only ones to absorb all the surplus in the market. A contraction in monetary policy has brought the impatient household a tightening collateral constraint, they could not buy more new homes even after they cut the consumption and increase the labor supply. At this moment, only a lighter debt burden could make the housing market clear possible. So a drop in real interest rate makes it possible to clear the housing market.

The change in monetary policy has pushed down price level P_t, so the inflation will be lowered. Then, we expect a even lower nominal interest rate level from the first period[8] so that the real interest rate can be reduced. From Figure 1.1 we observe that the interest rate falls below the steady state at the beginning of the period and the real interest rate in figure 1.2 also falls below the steady state level. The patient households were worse off because of the reduced real debt payment. In such a way, the market 'taxes' the patient households and subsidizes the impatient households to clear the housing market and also delivers the unconventional result: a contraction in monetary policy brings down the interest rate instead of lifts it up.

[8]We assume the shock hit the system at time 0 and the we plot the impulse responses from time 1

From the analysis above, we can infer that within our model framework, all the shocks that bring down the real housing price suddenly will work in the same way. They first make the impatient households more constrained but also need them to clear the housing market, so the debt burden must be reduced by a drop in real interest rate. If we remove the segmentation in housing market, the patient households clear the housing market and take all new homes and some used ones from impatient households without causing a big drop in nominal interest rate since the patient households own most of the income in this economy and they take a large fraction of all the housing before the shock happens[9]. However, such models are insensitive to the responsiveness of monetary policy because it is the impatient households who are supposed to respond more differently to different monetary policies. So the market segmentation is an important feature for a model to generate a different result under different monetary policies.

When the system is hitting by a positive cost push shock, we have a similar situation which is demonstrated in Figure 1.3. When there is a positive cost push shock, it directly affects the retail sector and lifts up the final good price, while leaves the housing market unaffected at the moment the shock is hitting the economy. So we will again observe a drop in the real housing price and an increase in the mark up. A decreased real housing price tightened the impatient households collateral constraint so that they need to both cut the consumption and supply more labor. Again, a very low level of real interest rate is needed to induce the impatient households to clear the housing market and the real interest rate is reduced at the same time. Also, the increased labor supply from impatient households pushes up the production a little bit. The patient households benefit from higher profit which is caused by the higher mark up. As a result, we observe an increase in patient households consumption at the beginning. The wholesale good price adjust very quickly and the mark up goes to steady state after one period. Considering the decreased real interest rate and quickly adjusted mark up, we again observe the continuous drop in patient households consumption. Even though the consumption of two types of households change in the opposite way, the decrease in total consumption implies that the drop in impatient households consumption is much more than the increase in that of patient households. Drop in both total consumption and business investment lead to the drop in total demand which results in a lower price level. So we observe a decrease instead of an increase of inflation after a positive cost push shock. The nominal interest rate is pushed down even further so we have a decreased real interest rate.

Even though a positive monetary policy shock and cost push shock bring us unconventional results, they still provide us some insights as we are considering the housing market.

[9]Refer chapter 2 for details about the importance of housing market segmentation.

When the constrained households account for a very large fraction in the housing market and we do not allow default on household debt, the economy behaves very differently. The market automatically 'taxes' the patient households by reducing the real interest rate and subsidizes the impatient households to clear the housing market. In such a way, the economy avoids the debt deflation and liquidity trap and recovers by itself whenever it faces a shock that may cause deleveraging crisis. Though we cannot enforce the non-default assumption on household debt in real life, it still shed lights on how the government could respond to a non-preferred shock with fiscal policy: tax or borrow from the patient borrowers and subsidize the lenders. In crisis, the debt distribution can make a difference. Our findings corresponds to the conclusion of Eggertsson and Krugman (2012): 'more debt may be the rescue of a lot of debt'. How to design and implement the fascial policy is out of the scope of this paper.

When it comes to the technology shock and investment shock, we observe the responses of housing price and housing production under different monetary policies are dramatically different. First, let us focus on the technology shock in the wholesale good sector. Figure 1.4 presents the impulse responses to such a shock. When a positive shock hits the economy under a less responsive monetary policy, the productivity in the wholesale good sector increase which brings down the wholesale good price. The retail price for final good also decreases because of the a lower input cost. Higher wage in wholesale good sector attracts more labor so that the production of housing first experiences a drop at the beginning because of the reduced labor input. The technology shock in wholesale good sector would not affect the nominal housing price at the time when it hits the system, so the real housing price increases as a result of decreased final consumption good price. Considering that the final consumption good becomes cheaper and the higher housing price relaxes the collateral constraint, both types of households increase their consumption, which results in a higher GDP.

When a more responsive monetary policy is implemented, things are different. With a more responsive monetary policy, the central bank is more aggressive on controlling the inflation. Moreover, the increased productivity would lead to an increased GDP growth rate. So we expect a higher interest rate and a much lower price level in period 1, which leads to an increase in the real interest rate. So the marginal cost of obtaining one unit of housing also increases. A drop in housing demand follows. But the lower price of final good also reduced the investment cost, so the production of housing increases in period 1. Increased supply and reduced demand together drive down the housing price in period 1 to clear the housing market. The actually decreased housing price makes the collateral constraint becomes tighter so that the impatient households have to cut the consumption and housing. Low housing price results in the reduction of housing production since the second period. In such a way, a more responsive monetary policy alters the responses of housing price and housing output to the same technology shock.

Now let's look at the impulse responses to an investment shock in Figure 1.5. A positive investment shock increases the efficiency of investment and capital. A positive investment shock could bring more investment. Following the shock, there is an increase in business investment, a decrease in total consumption and an increase in total final good production. Increased business investment lifts up the production in both sectors. The increased business investment pushes up the price level of final good. Also, the higher demand for investment increases the real interest rate, which makes a heavier debt burden. With the less responsive monetary policy, interest rate will not respond to inflation so aggressively. Then interest rate would be higher with a less responsive monetary policy than that under a more responsive monetary policy. We observe that the productions are at lower levels under a less responsive monetary policy. After the investment shock, increased housing production lifts up the housing supply. Under a more responsive monetary policy, the production of housing increases more. Combining a heavier debt burden and a lower housing demand at period 1, the housing price is pushed down. The constrained households benefit from increased labor income from the output increase. The housing production adjust quickly according to the market condition. So the housing price recovers also fast.

The patient households choose to investment more and consume less. Their consumption falls below the steady state. The impatient households cut the consumption under a more responsive monetary policy because of a decreased housing price tightens the collateral constraint. However, the recovered housing price afterwards relaxes the constraint so that their consumption keeps increasing. But it cannot offset a large decrease in patient households consumption so we see a drop in total consumption.

We have to admit that our model could not generate a constant deviation of housing price from the steady state under the monetary shock and cost push shock. The housing price goes back to steady state very quickly. Iacoviello and Neri (2010) states that the housing preference shock helps to generate the constant deviation of housing price. It is not a surprise that the housing price return to steady state from the third period considering that we do not include a housing preference shock in our model.

1.5.3 Shocks and Variables

In this section we are going to talk about the what shocks drive the changes in variables in housing market, which helps us to understand even further the effect of responsiveness of monetary policy on the housing market. Figure 1.6 and figure 1.7 are the historical

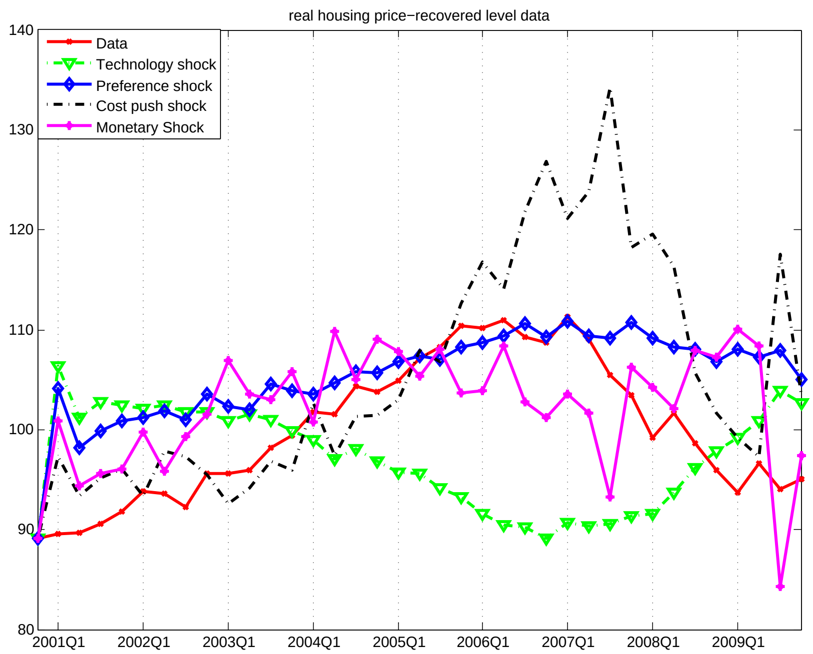

Figure 1.6: Historical Variance Decomposition-Housing Price

Parameters	e_c	e_h	e_a	e_u	e_e	e_μ
y_h	0.25	69.17	16.85	7.79	4.07	1.87
q	1.17	3.64	1.65	58.38	24.71	10.45
b'	4.48	3.44	26.55	1.82	4.28	59.10
r	7.51	0.32	17.73	25.55	2.09	46.81
π	3.67	0.21	6.63	10.08	62.53	16.88
gdp	14.42	0.28	11.97	10.38	0.79	62.17
$c(total)$	4.41	0.08	9.53	24.10	7.96	53.92
c	4.11	0.02	25.78	0.07	0.12	69.9
c'	2.19	0.19	1.89	64.21	20.85	10.67

Table 1.4: Theoretical Variance Decomposition

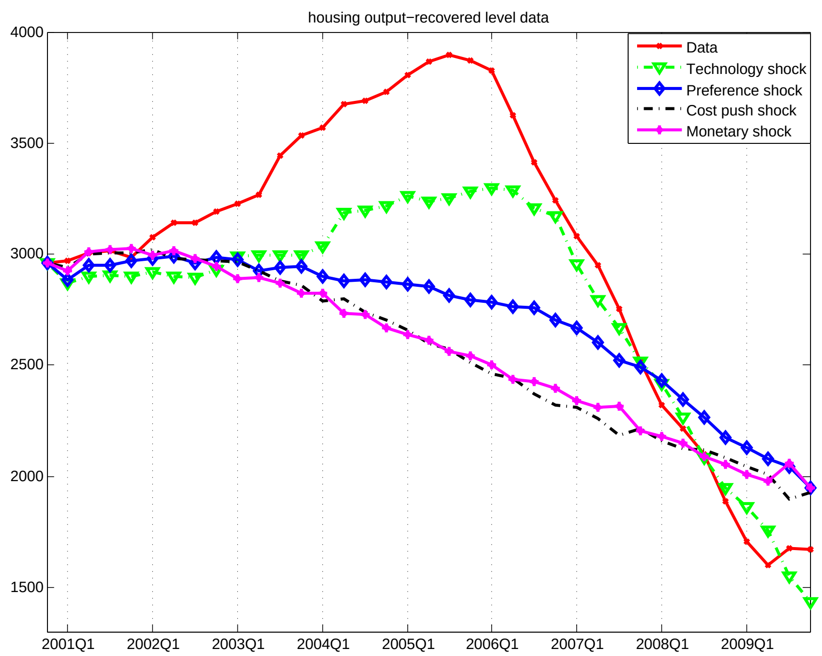

Figure 1.7: Historical Variance Decomposition-Housing Output

decomposition of housing prices and housing output. We can see that the monetary shock drove the housing price before 2003Q1. From 2003 to the end of 2005, the cost push shock accompanying the influence of the preference shock took place as the driving force. From 2006 to the middle of 2007, the monetary shock dominates. After that until 2009 the cost push shock took the lead and during the last year in our sample, the monetary shock drove the housing price again. In sum, the cost push shock and monetary shock took turns to drive the hosing price in our sample period. The preference shock has some effects before 2004, but not as obvious as that of monetary policy shock. Regarding the housing output, the technology shocks in housing sector dominated for the whole period from 2000-2009.

Table 1.4 lists the theoretical variance decomposition of the key variables in our model. It shows us that the cost push shock explains 58% of variances in real housing price while the monetary shock explains about 25%. The fact that the cost push shock explains a large fraction of the variance in housing price again illustrates the importance of the price stickiness. The nominal housing price is flexible while the final consumption good price is sticky. So the different adjustment rate of these two prices becomes the major force of the housing price change. The iid monetary shock explains more of the housing price variation in our model compare to Iacoviello and Neri (2010), in which the iid monetary shock only explains 11.5%.[10]Moreover, Iacoviello and Neri (2010) did not separate the effect of the responsiveness of monetary policy from the effect of a general monetary shock. If they did, the iid monetary shock would have explained even less variations in housing price.

The fact that the monetary policy shock drove the housing price before 2003Q1 implies that it is the monetary policy shock instead of the change in responsiveness of monetary policy that should be responsible for the housing price increase before 2003Q1. According to the interpretation of monetary shocks from Christiano et al. (1999), the Fed 'desire to avoid the social cost of disappointing private agents expectations' and the measurement error in data that FDMOC used mainly led to the housing boom during 2001-2003. The first interpretation is more proper in explaining the possible scenario: In 2001, there was an end of dot-com boom and a sharp decline in stock price. The 911 attack in the same year aggravated the recession. The private agents formed some strong expectations toward the

[10]Iacoviello and Neri (2010) claims that the monetary shock explains less than 20% of the variance of housing price. The monetary shock in their paper includes the highly persistent inflation objective shock in the monetary policy. They combined these two shocks as one. The iid monetary shock explains 11.5% and the inflation objective shock explains 3.6% of the housing price variation. So the monetary shock in total explains about 15% of the variation of the housing price. After they include the highly persistent inflation objective shock into the monetary shock, it still explains less than it could in our model. Moreover, the price stickiness in our model is less that in theirs. We set $\theta = 0.75$, which implies that the average length of price contract is one year while they estimated $\theta = 0.83$ which implies a six-quarter average length of price contract. They have a higher degree of price stickiness but the monetary policy explains less variations of real housing price.

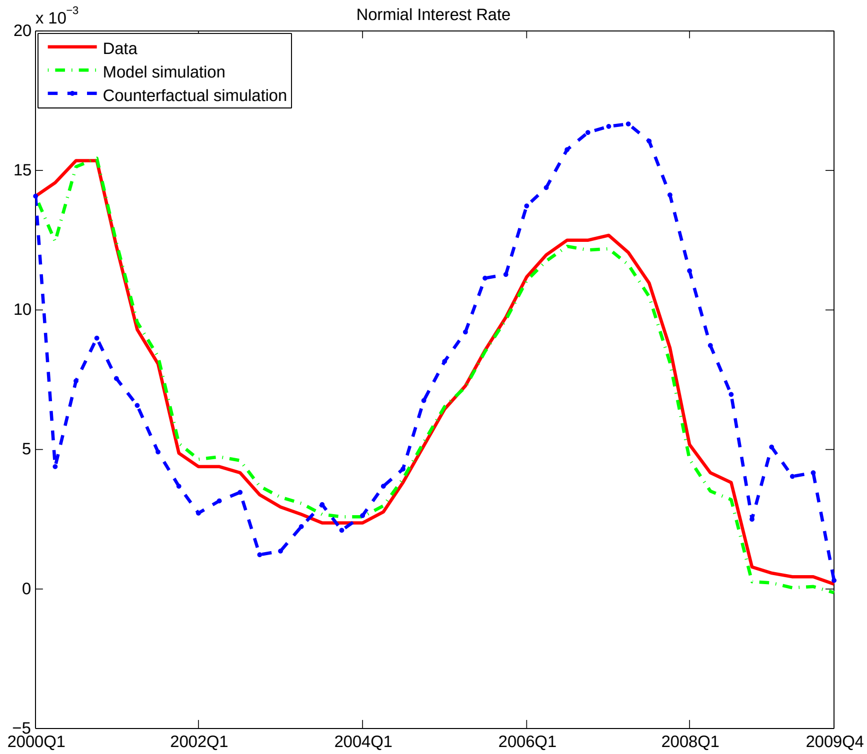

Figure 1.8: Interest Rate

interest rate and the Fed took them into account when making decisions for the monetary policies. So we allege that it is the monetary policy shock instead of the less responsive monetary policy that should be responsible for the low interest rate and increasing housing price before 2003Q1.

1.5.4 Effects of the Responsiveness of Monetary Policy-Counterfactual Analysis

Responsiveness of Monetary Policy-the Interest Rate and Inflation

Figure 1.8 and Figure 1.9 shows the dynamic simulation and counterfactual simulation of interest rate and inflation. The red solid line represents the data. Green dashed line is the dynamic simulation with the parameters estimated for period of 2000-2009. Blue dotted dashed line is the counterfactual simulation with the 8799 monetary policy. From the figures we could see that the green line matches with the red line very well, implying that our model and estimates fit the data. With a more responsive monetary policy, the interest rate is below the data value before 2004Q1 and above the data value after 2004Q1. We

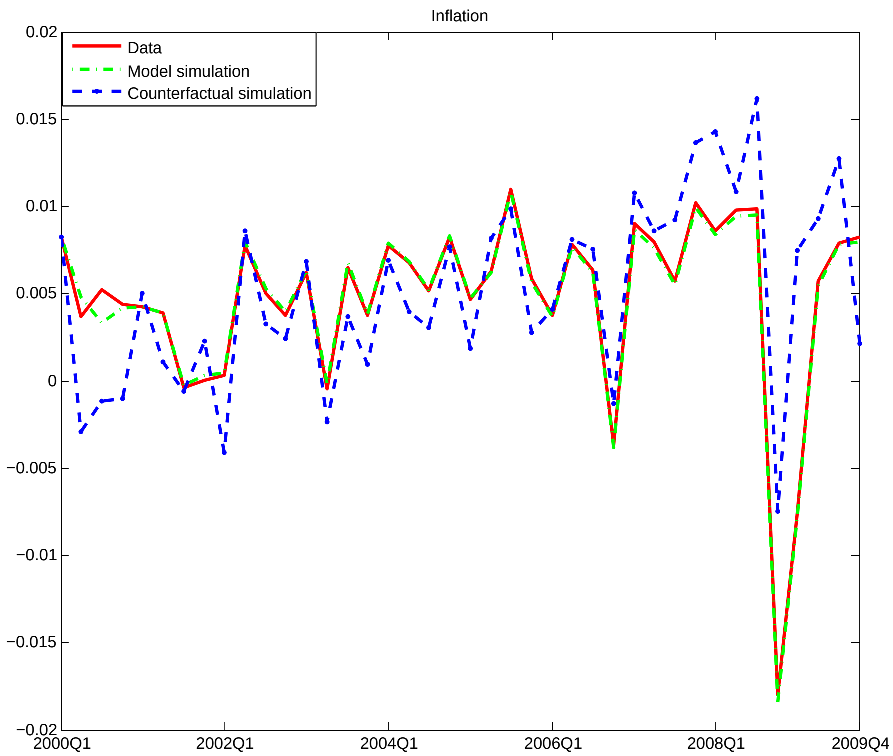

Figure 1.9: Inflation

need to pay attention to the period between 2002-2004 when the interest rate was claimed as a result of a less responsive monetary policy. As we can see from Figure 1.8, a more responsive monetary policy does not raise the interest rate during the period of 2002-2004. On the contrary, the interest rate went to a even lower level with a more responsive monetary policy. However, the interest rate does raise to a higher level after 2004Q1 under the more responsive monetary policy. Taylor (2007) states that the low interest rate level from 2002 to 2006 are due the less responsive monetary policy. We have a different conclusion: the low interest rate during 2002Q1-2004Q1 was not caused by the less responsive monetary policy while that after 2004Q1 was.

Our findings regarding the interest rate path predicted by the counterfactual Taylor rule are different from those in the literature. The reason lies in the fact that we have a dynamic model so that the GDP growth rate and inflation would have changed to different paths with a more responsive monetary policy rule. The paths of these two variables keep unchanged in the literatures which predicts interest rate by only using the simple Taylor rule. We can verify this from Figure 1.9, the counterfactual simulation of inflation is very different from the data. In our dynamic model, not only the Taylor rule coefficients changed, but the paths of the variables that the interest rate depends on also changed. In such a situation, we may not necessarily get a higher interest rate level with a more responsive monetary policy.

If we predict the interest rate by only the Taylor rule, we apply a more responsive monetary policy on the same GDP growth rate and same inflation. There is no surprise that it gives us a higher level of interest rate for the same periods. Our economy is operated dynamically and we could not expect the GDP growth rate and inflation would keep unchanged with a different monetary policy rule. Also, the fact that our estimated model fits the interest rate data better than the single equation Taylor rule estimation gives us more confidence about our results.

Responsiveness of Monetary Policy and the Housing Market

According to the data on real housing price and housing output, we define the housing boom period from 2001Q1 to 2006Q1 during which both variables keeps increasing. Figure 1.10 and figure 1.11 show us that the different paths of housing price and housing output (recovered level data by using simulated growth rate) with different monetary policies. The green line matches with the red line on both graphs again tells us that our model simulations match the data. The blue line represents the counterfactual paths under a more responsive monetary policy. From Figure 1.10 we could clearly see that the housing price would be controlled at a lower level from 2001 even though it is not far away from data. However, a more responsive monetary policy reinforced the boom during the first part of housing boom

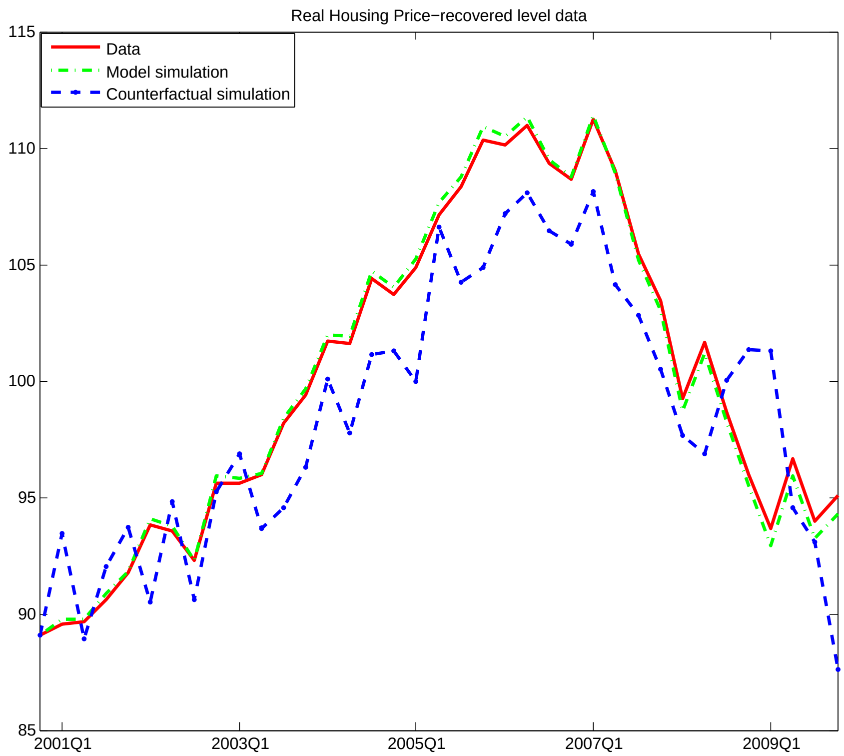

Figure 1.10: Housing Price

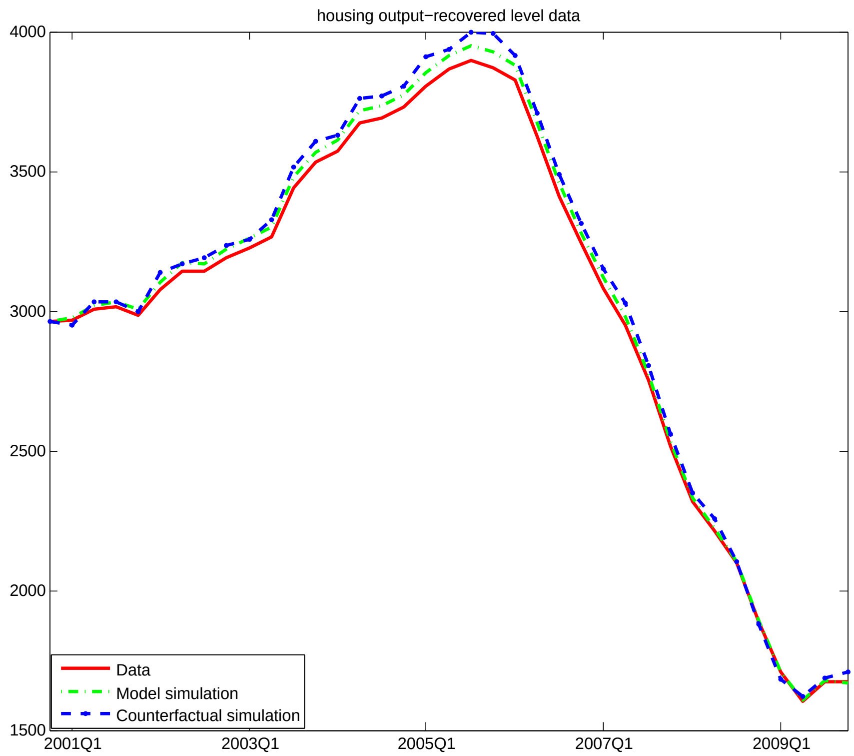

Figure 1.11: Housing Output

in 2001. From 2002 to 2006, the housing price can be controlled to a lower level by a more responsive monetary policy. So a more responsive monetary policy is helpful to control the housing boom. And it also stabilize the housing price during the transition period (transit from a boom to a bust)from 2001-2008. The housing price is pushed up in the end of 2008 then following a even bigger dip in 2009, which implies a worse recovery after the housing boom. Regarding the housing output, we have to admit that the effect of a more responsive monetary policy is almost unobservable on it because the counterfactual simulation is almost the same as the model simulation.

In general, the less responsive monetary policy after 2000 is not the principle reason of the housing boom but a more responsive monetary policy can control the housing market boom with limited effects within our model framework. However, since the effect of monetary policy on housing market works mainly through the credit channel and it has been enlarged in our model by the housing market segmentation, we conjecture that a more responsive monetary policy would not play an important role in controlling the housing boom in other models.

1.6 Conclusion

We set up a two-sector DSGE model by using Bayesian techniques to test Taylor's hypotheses: The less responsive monetary policy was responsible for the housing boom and the subsequent crisis and a more responsive monetary policy could improve the economic situation. We identify a less responsive Taylor rule for the period of study, which allows us to separate the effects of the responsiveness of monetary policy and the effects of monetary shocks caused by other reasons. The impulse responses show us that most of the real variables respond less to the same shocks except for the technology shock in the housing sector under a more responsive monetary policy. One of the differences with the literature is that we introduce housing market segmentation through different discount factors, leading to a housing market that is occupied only by constrained (impatient) households. The impulse responses to monetary policy shocks and cost push shocks under this assumption deliver unconventional results. In particular, a contractionary monetary policy shock and a positive cost push shock will bring down the interest rate and inflation respectively. Moreover, the real housing price is reduced with these two shocks, leading to more binding constraints for the impatient households. To clear the housing market, the real interest rate is reduced, automatically taxing patient lenders and subsidizing impatient borrowers. These findings shed some light on the possible policies when we facing shocks that may lead to deleveraging crisis. Also, we find out that the relationship between the quantity of housing demanded and housing price may not be monotonic.

By comparing the the model simulation and data we conclude that our model prediction matches the data well. Our results partially support Taylors hypothesis. The less responsive monetary policy is not the reason for the low interest during 2002-2004. We do find, however, that a more responsive monetary policy may have stabilized housing prices during the transition period of 2002-2008. The theoretical variance decomposition indicates that the monetary shock explains about 25% of the variances in housing prices and the cost push shock explains about 58%, while the variance in the housing output is mainly explained by the technology shock in housing sector. Our conclusion is that monetary policy shocks, rather than the responsiveness of monetary policy, contributed to the housing boom. A more responsive monetary policy can help stabilize the housing price during 2002-2008, but it will make a worse recovery in 2009.

Our model enforces that the collateral constraint is binding all the time. It is a very strong assumption and we achieve this by choosing a low value for the discount rate of impatient households. We could let the collateral constraint occasionally binding and see effects of the responsiveness of monetary policy in future research. Also, as we have mentioned in previous section, we do not distinguish housing units and the services it delivers which impedes us from the investigation of housing preferences shock. Lacking of housing preference shock prevents our model from generating a persistent housing price change under the monetary shock and cost push shock.

Chapter 2

The Importance of the Housing Market Segmentation

2.1 Model Without the Housing Market Segmentation

By following Iacoviello and Neri (2010), the model without the housing market segmentation allows the patient households to participate in the housing market and both types of households trade homogenous housing in the same market. So only the patient households problem and the housing market clearing condition are different from the model in chapter 1.

Patient households problem: The patient households maximize their lifetime utility:

$$\max\{\sum_{t=0}^{\infty} E_0\beta^t a_t(\ln(C_t - \gamma C_{t-1}) + j\ln H_t - \frac{1}{1+\eta}(N_{ct}^{1+\zeta} + N_{ht}^{1+\zeta})^{\frac{1+\eta}{1+\zeta}})\} \tag{2.1}$$

The patient households are subject to a budget constraint:

$$C_t - B_t + Q_t(H_t - (1-\delta_h)H_{t-1}) = W_{ct}N_{ct} + W_{ht}N_{ht} - \frac{B_{t-1}R_{t-1}}{\pi_t} + F_t + D_t \tag{2.2}$$

Housing market clearing condition:

$$H_t + H'_t + (1-\delta_h)(H_{t-1} + H'_{t-1}) = Y_{ht} \tag{2.3}$$

The rest of the model is the same as that in chapter 1. We use the same data, same calibrated parameters and estimation strategies to estimate the model for the two sub periods: 1987-1999 and 2000-2009.

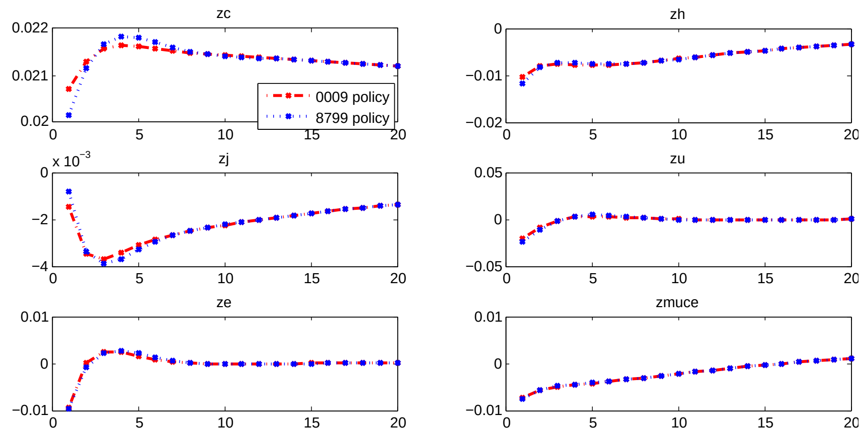

Figure 2.1: Impulse Response (No Housing Market Segmentation)-Housing Price

2.2 Impulse Responses

One of the important reasons thatwe introduce the housing market segmentation is that the variables, especially the housing market variables, do not respond to the shocks very differently under different monetary policies. The change in the responsiveness of monetary policy will not affect the housing market much if the patient households participate in the same housing market.

Figure 2.1 and 2.2 show the impulse responses of the housing price and the housing output to different shocks under different responsiveness of monetary policies. The impulse responses of the two monetary policies are indistinguishable because the blue line and the red line almost coincide with each other. For the housing price, the impulse responses to the technology shock in wholesale good sector are somewhat different. However, the scale is very small: the difference is less than 0.1%. The difference of impulse responses with different monetary policy rules for these two variables in the model with housing market segmentation varies between 0.3% and 2% (please refer to the graphs in chapter 1 for details).

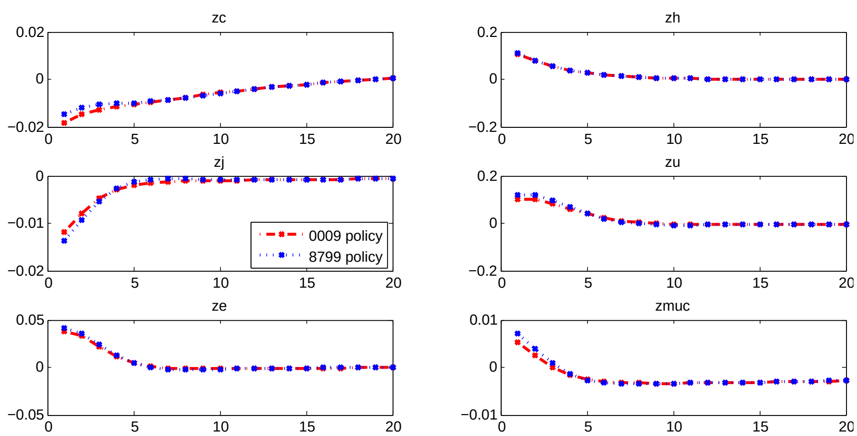

Figure 2.2: Impulse Response (No Housing Market Segmentation)-Housing Output

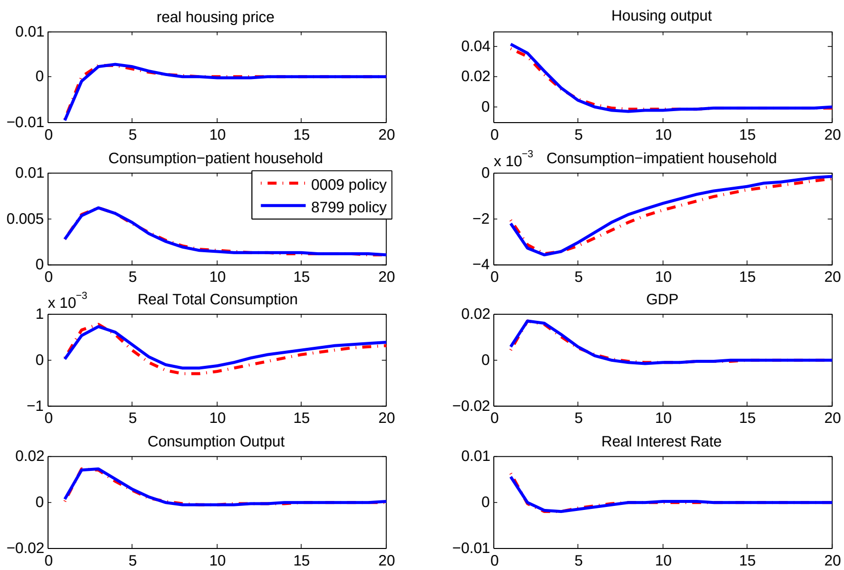

Figure 2.3: Impulse Response (No Housing Market Segmentation)-Monetary Shock

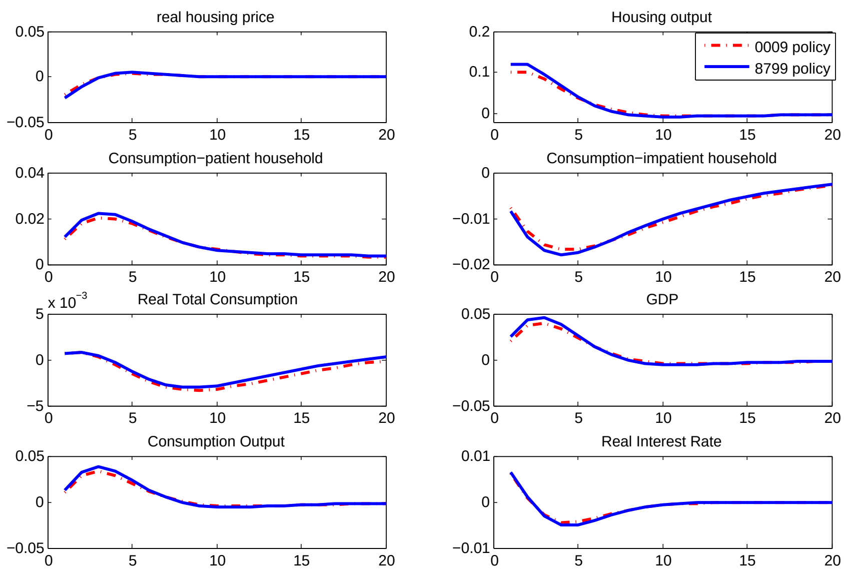

Figure 2.4: Impulse Response (No Housing Market Segmentation)-Cost Push Shock

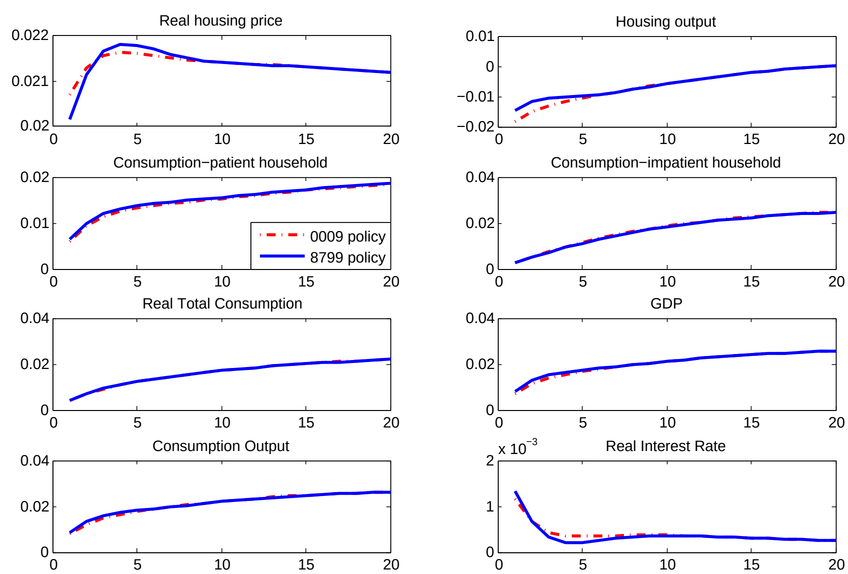

Figure 2.5: Impulse Response (No Housing Market Segmentation)-Technology Shock (Wholesale Good)

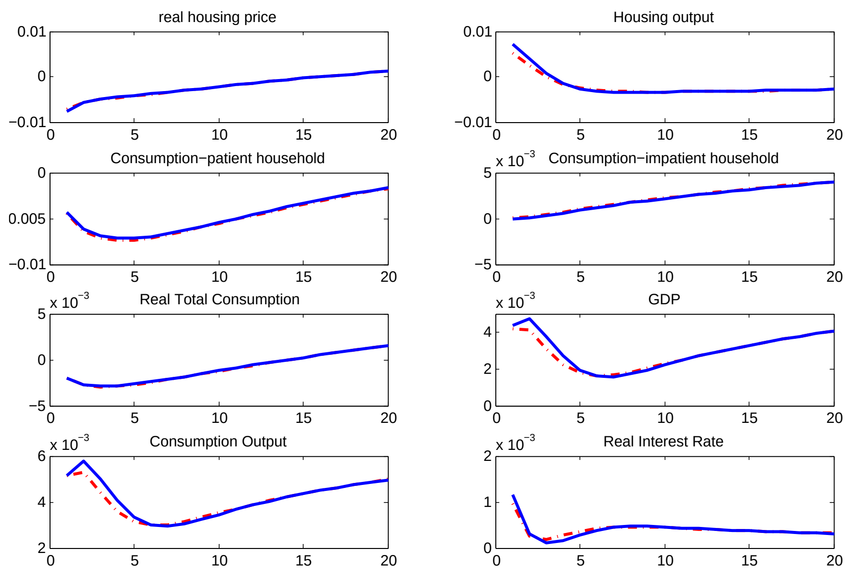

Figure 2.6: Impulse Response (No Housing Market Segmentation)-Investment Shock

Figure 2.3 to Figure 2.6 presents the impulse responses of some key variables under the two monetary policy rules. From the graphs, we again cannot see very big differences between the responses. If different responsiveness of monetary policy rules does not make very different impulse responses, they will not make different pathes of the key variables over time. We can conclude that the housing market segmentation is very important for us to observe the effects of the responsiveness of monetary policy.

There is another thing worthy noting: the impulse response of the real interest rate does not fall below 0 after a positive monetary shock or cost push shock. In Chapter 1, we have discussed that the real interest would be going down to clear the housing market when the market is occupied by only constrained households. With a deleveraging shock, the housing price decreases so the collateral constrains are tightened. The market must bring down the debt burden for impatient households to make them absorb all the surplus in the housing market. The real interest rate must be pushed down a lot to make this happen. If the the patient households participate in the housing market, we see a different picture. Patient households have more income sources and are at a better position in the case of a deleveraging shock. A smaller decrease in interest rate could induce them to absorb all the surplus in the housing market. In such a situation, we will not see a reduced real interest rate after the deleveraging shock.

2.3 Why Is the Housing Market Segmentation Important?

The responsiveness of monetary policy does not have much effects on the economy without the housing market segmentation. There are three reasons. First of all, the housing surplus is less in a model without the housing market segmentation after a deleveraging shock. On one hand, patient and impatient households pertain different tolerance for the expected housing price changes. We can see this from their first order conditions, equation (2.4) and (2.5). When the price is expected to increase (decrease), the impatient households are more likely to buy (sell) because of they have more benefit or (loss) from collateral constraint, leading to more shortage (surplus) of housing in the market. In a model with a housing market occupied by only impatient households, there are more housing surplus when there is a deleveraing shock. On the other hand, the housing owned by the constrained households only accounts for a very small portion of housing stock when the patient households also participate in the housing market, which is not true during the housing boom and bust period. In Iacoviello and Neri (2010), they own about 15% of all the housing stock. Combing these two aspects, the surplus of housing is much less when the patient households also participate in the housing market under deleveraging shocks. Second, the patient households have bigger capacity to absorb the surplus housing in the market without causing dramatic changes in other variables. Based on the first two arguments, the patient households act like a buffer in the housing market when there are shocks which exert influences on the housing market, making it response less to changes in monetary policies. Third, according to Mishkin (2007), the effect of monetary policy is mainly through the credit channel. The patient households are not constrained, so the monetary policy, in general, would not affect them very much. On the contrary, the impatient households finance their homes by borrowing, they are the ones who will be mainly influenced by the change of the responsiveness of monetary policy.

$$u'_{ht} = q_t u'_{ct} - \frac{\beta'(1-\delta_h)}{g_h} q_{t+1} u'_{ct+1} - m(1-\delta_h) g_q \xi_t \frac{q_{t+1}}{rr_t} \tag{2.4}$$

$$u_{ht} = q_t u_{ct} - \frac{\beta(1-\delta_h)}{g_h} q_{t+1} u_{ct+1} \tag{2.5}$$

We can imagine that in a model with patient households also participating in the housing market, the housing surplus will not change much under two different policies since they are not sensitive to the interest rate change, so does the housing price as well as other real variables. When we include the housing market segmentation, we see a total different picture. The impatient households are very sensitive to the interest rate change, which makes the housing price fluctuate more. The housing surplus or shortage would be very different under different responsiveness of monetary policies during the housing boom and bust period, so does the housing price. The housing price relaxes or tightens the collateral constraint so that other variables in the economy would behave differently. Then we observe different responses for different responsiveness of monetary policy in a model with housing market segmentation.

2.4 Conclusion

By comparing the impulses responses to different shocks under the different responsiveness of monetary policy rules, the variables do not respond to shocks very differently under without the housing market segmentation. The patient households would have served as buffers in the housing market and made the economy response less to the shocks. We need the housing market segmentation to generate distinguishable responses to shocks when the responsiveness of monetary policy changes.

Chapter 3

Monetary Policy Responses to the Housing Price-A Welfare Analysis

3.1 Introduction

There has been considerable debate on the role of the monetary policy in stabilizing the asset prices so as to benefit the whole economy. The prevention of asset bubbles is one of them. Should monetary policy directly respond to a general asset price? The importance of asset price is broadly associated with the economic activities and is summarized in Gilchrist and Leahy (2002). First of all, the asset price change has the wealth effect by relaxing or tightening the budget constraint because of the fluctuation in its resale value. Second, the asset price affects the agents' net worth so as to enlarge the effects of shocks and policies. Among the previous studies which focus on the stock market and investigate whether the monetary policy should respond to the movements in stock market, Bernanke and Gertler (2001) and Gilchrist and Leahy (2002) both conclude that the monetary policy should not respond to the stock price. However, Gali (2013) alleges that it is optimal to respond to a general asset price bubbles. He also noticed that central bank needs to balance the stabilization of current aggregate demand and future aggregate demand: higher interest rate before and during the asset bubble can help control the current demand. But the high interest rate will continue depress the demand after the asset price bubble and lead to a slower recovery.

Comparing to other asset prices, housing price is related to households more closely. Households can borrow against housing, so the housing price not only affect the budget constraint, but also the borrowing capacity. The housing satisfies the households in two ways. On one hand, it delivers the housing services to households. On the other hand, the housing price depreciation and appreciation tightens or relaxes the borrowing constraint, leading to a more tightened or relaxed budget constraint. A small change in monetary policy could be amplified through the housing price change. From this point of view, the monetary policy can be an effective tool and used to adjust the housing market development. Literatures propose different attitudes towards this problem. Iacoviello (2005) found that responding to the housing price is unimportant. Allen and Rogoff (2011) stated that 'monetary policy as well as macro-prudential policies need to be used to guard against real estate bubbles'. Finocchiaro and Heideken (2013) proved that it is optimal to react to the housing price even though the gains are negligible.

Based on the arguments above, we are facing two problems when the monetary policy responds directly to the housing market changes. First, according to Gali (2013), it is a dilemma to balance the demand before and after the housing boom. Second, how to determine a proper response to the housing price when the effects of a small change in monetary policy can be amplified through the housing market? The magnitude of the response of the monetary policy to the housing market is very important. The overreaction to the housing price can foster a boom or depress the future economy. Also, frequent change in monetary policy can result in high volatility in the housing market, which is not helpful for the stabilization of the economy.

In this paper, we will discuss the problem that how the welfare of the two types of households change if the central bank responds directly to the housing price with the housing market segmentation. In Chapter 1, we have seen that a more responsive monetary policy could stabilize the housing price during the boom and bust cycle. Instead of changing the responsiveness of the monetary policy, we change the Taylor rule to the following equation:

$$r_t = \pi_t^{1+r_\pi} (\frac{GDP_t}{g_c GDP_{t-1}})^{r_y} (\frac{Q_t}{g_q Q_{t-1}})^{r_q} \tag{3.1}$$

We assume that the monetary policy directly responds to the housing price by adding the housing price growth rate in the Taylor rule. r_q is the coefficient for the housing price, which means that when the housing price growth rate changes by 1%, the interest rate should change by r_q%. We apply the estimation from chapter 1 for the period of 2000-2009. When $r_q = 0$, we will have exactly the same model as it is in chapter 1.

3.2 Welfare Evaluation

3.2.1 Welfare Measurement

Before we evaluate the welfare, we first need to declare the definition of the welfare in this paper. The individual welfare of each type of households is the summation of utility over time:

Welfare of patient households W_P:

$$W_P = E_t \sum_{t=0}^{T} \beta^t [\ln(C_t - \gamma C_{t-1})] - \frac{(N_{ct}^{1+\zeta} + N_{ht}^{1+\zeta})^{\frac{1+\eta}{1+\zeta}}}{1+\eta}] \tag{3.2}$$

Welfare of impatient households W_I

$$W_I = E_t \sum_{t=0}^{T} \beta^t [\ln(C_t - \gamma C_{t-1})] + j\ln(H_t) - \frac{(N_{ct}^{1+\zeta} + N_{ht}^{1+\zeta})^{\frac{1+\eta}{1+\zeta}}}{1+\eta}] \tag{3.3}$$

By following Carrasco-Gallego and Rubio (2013), we also use consumption equivalents to represent the welfare change. Consumption equivalents tell us that what the fraction of the consumption the agents should give up or obtain to archive the benefits of a new policy. A positive value of consumption equivalent means a welfare gain while a negative value means a welfare loss. Our benchmark model is the model in Chapter 1 where $r_q = 0$. The consumption equivalence of the two types of agents are:

$$CE_P = exp((1-\beta)(W_P^{r_q \neq 0} - W_p^{r_q = 0})) - 1 \tag{3.4}$$

$$CE_I = exp((1-\beta')(W_I^{r_q \neq 0} - W_I^{r_q = 0})) - 1 \tag{3.5}$$

3.2.2 Welfare Evaluation

Carrasco-Gallego and Rubio (2013) states that "the literature typically finds that the macro prudential reactions to exogenous shocks can make some people better off (typically borrowers), but not every type of households". Based on this argument, we think it is important to evaluate the disaggregate welfare of each type of households separately so that we can see the how the new policies affect each group.

Figure 3.1 and 3.2 shows the counterfactual simulations of housing price and housing output with different values of r_q respectively. Not surprisingly, Figure 3.1 tells us that the higher the response rate of monetary policy to the housing price, the more stable of the housing price, especially during 2003-2008 when the housing price experienced a shooting up and a sharp decline. However, the new monetary policy has very limited effects on the housing output. The results here aligned with those we found in chapter 1, the monetary policy change affects the housing price but has limited effects on housing output.

Figure 3.3 plots the changes of consumption equivalents with different values of r_q. The vertical axis represents the welfare change in percentage of consumption change and the horizonal axis shows the different values of r_q. The blue and green lines represent the welfare

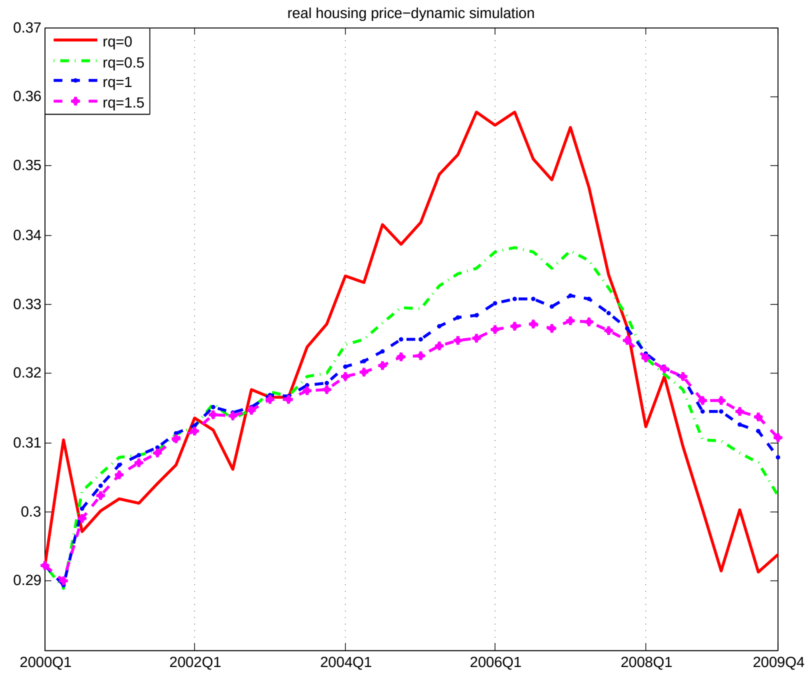

Figure 3.1: Housing Price and r_q

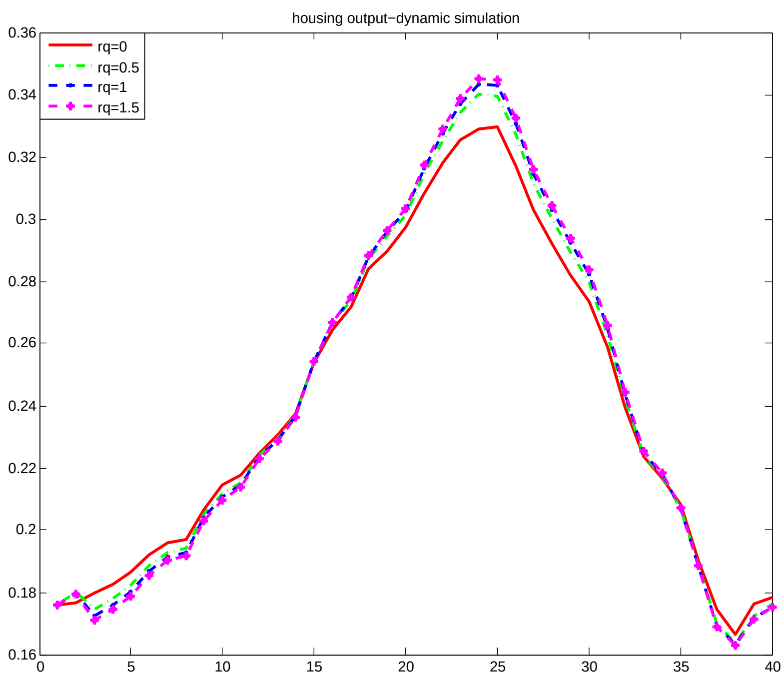

Figure 3.2: Housing Output and r_q

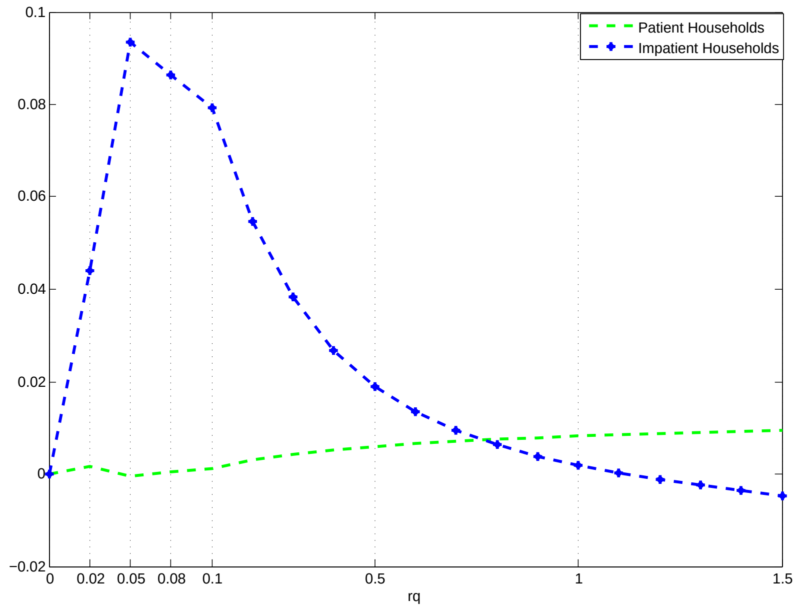

Figure 3.3: Welfare Change and r_q

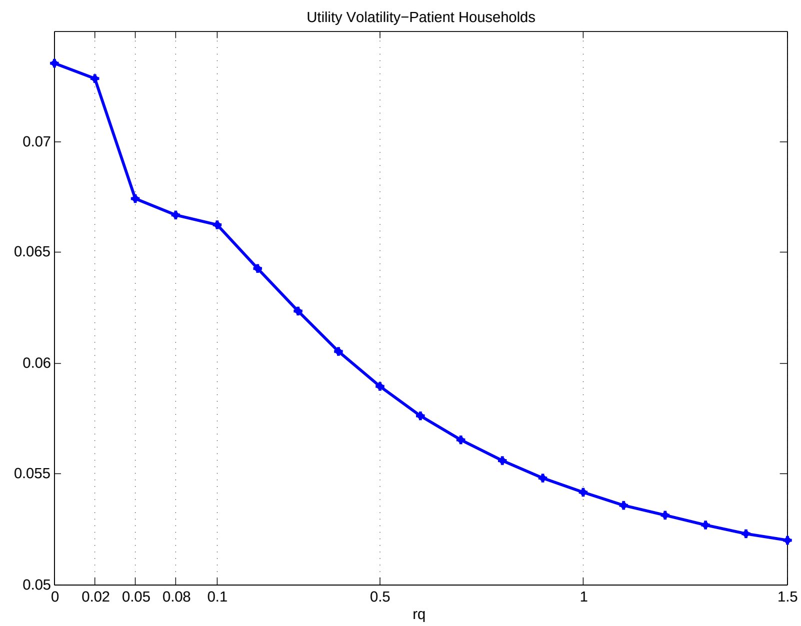

Figure 3.4: Volatility of Utility-Patient Households

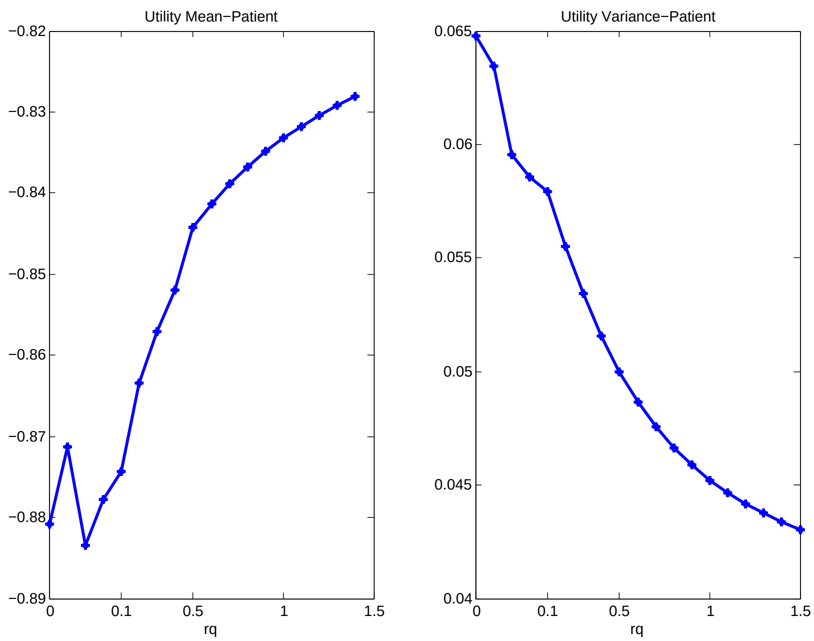

Figure 3.5: Variance and Mean of Utility-Patient Households

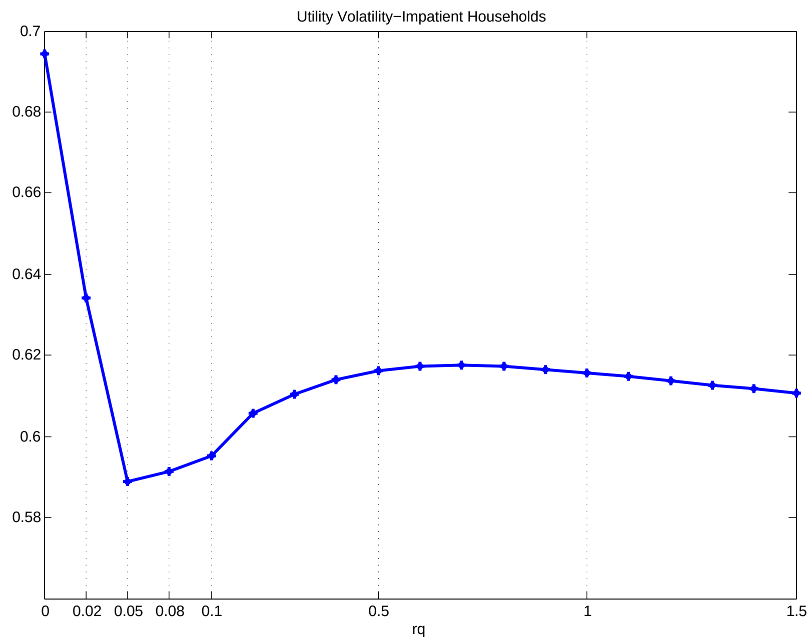

Figure 3.6: Volatility of Utility-Impatient Households

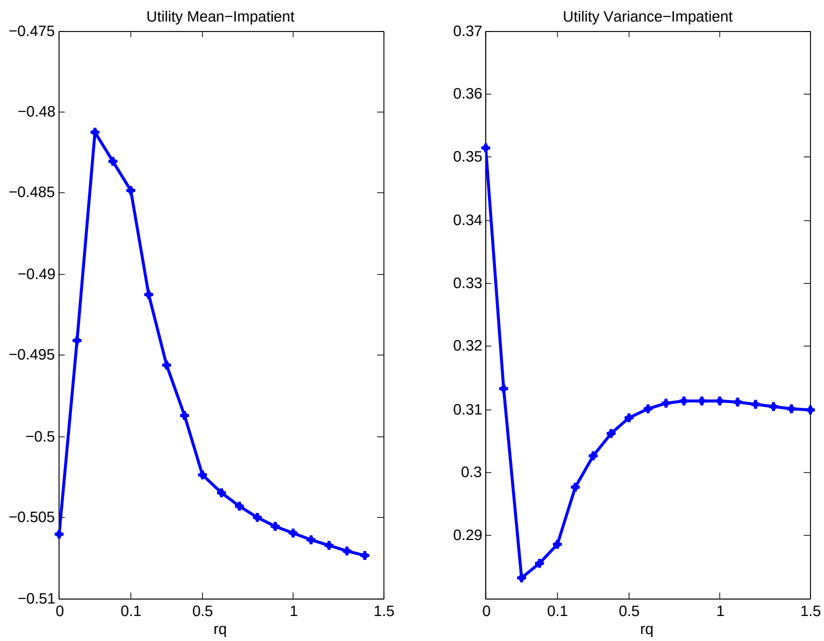

Figure 3.7: Variance and Mean of Utility-Impatient Households

gain or loss of impatient and patient households respectively. We can see that the welfare of patient households keeps increasing with the increase of response to the housing price. But the magnitude of welfare gain is very small. The highest welfare gain is less than 1%. The welfare change of impatient households is not monotonic with the increase of r_q. The hump shape of the blue line implies that there exists an optimal response of monetary policy to the housing price. The consumption equivalents peak at $r_q = 0.05$ and the welfare gain is about 9%. The welfare gain experiences a sharp decrease after $r_q = 0.1$ and it disappears when the $r_q = 1$. It becomes a welfare loss after that. We can conclude from the graph that a moderate response of monetary policy to the housing price promotes the impatient households welfare gains. However, the overreaction to the housing price will result in a welfare loss for them.

Why the welfare gain of impatient households is hump shaped while that of patient households keeps increasing with the increase of r_q? The volatility and mean level of utility of each type of households can explain this question. These are the two aspects that affect the welfare change. If the new policy can bring higher levels of utility, the households are generally better off through the whole period. Also, the households prefer smoother utilities so they are better off when the utility volatility is reduced. From Figure 3.4 and 3.5 we can see that the utility volatility of patient households does decrease with the increase of r_q while the average utility level keeps increasing except for a small decrease when $r_q = 0.05$. Combining higher levels of utility and decreased volatility, the patient households definitely gain under the new policies.

As for the impatient households, we are facing a more complicated situation. From Figure 3.6, we can see that the utility volatility first decreases and reaches the lowest point at $r_q = 0.05$, which corresponds to the peak of the consumption equivalents. The consumption equivalents of impatient households decreases after that but it is still positive until $r_q = 1$. The utility volatility is also lifted up after $r_q = 0.05$ but it becomes stable at a level that is much lower than its starting point after r_q reaches 0.5. However, the consumption equivalents of impatient households keeps decreasing after r_q reaches 0.5. Why? We can explain this by looking at the graph of mean utility in Figure 3.7. The mean utility level of impatient households peaks at $r_q = 0.05$, the it falls down. So the decreasing consumption equivalents after r_q reaches 0.05 is mainly caused by the falling mean utility level. After r_q reaches 1, the mean utility level falls below the level of the benchmark model when $r_q = 0$, then we observe a negative consumption equivalents. Based on these facts, we find out that when the monetary policy overreacts to the housing price ($r_q \geq 1$), it stabilizes the housing price but also depresses it to a low level. The loss from a depressed housing prices outweighes the benefits from stabilized housing prices for the impatient households.

In sum, the response of monetary policy to the housing prices can stabilize the housing price by keeping it at a lower level during the boom and bust period. Higher housing price with $r_q = 0$ can expand the borrowing capacity so that the impatient households are better off. However, the high volatility of utility offsets some benefit of higher housing price when the monetary policy does not respond to it. On the contrary, if the monetary policy responds too much to the housing price, the impatient households are worse off because of a lower level of utility caused by depressed housing prices even though the utility volatility is also low. There is a welfare trade-off under the new policy rules with $r_q \neq 0$. Moderate response of monetary policy to the housing price is optimal for the impatient households by keeping the housing price as well as utility volatility at a proper level.

3.3 Conclusion

From the experiments on monetary policies with different responses to the housing price, we conclude that the higher the response, the more stable of the housing price. The welfare change of two types of households are different under new monetary policies. The higher the response to the housing price, the more benefits the patient households gain because of a smoothed utility as well as a higher utility level. The hump shape of consumption equivalents of impatient households implies that there is an optimal response of monetary policy to the housing price. According to our experiments, the consumption equivalents of impatient households peaks at $r_q = 0.05$. However, it is still positive after that even if it has a negative slope until r_q reaches 1, then it becomes negative. The hump shaped consumption equivalents can be explained by the U shaped utility volatility and the humped shaped average utility levels.

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Appendix A

Data

- Consumption: Personal Consumption Expenditure (seasonally adjusted) minus the housing service expenditures.
- Personal Consumption Price Index: Constructed by using Torquest formula.
- Total Investment: Gross Private Domestic Investment (seasonally adjusted annual rate).
- Residential Fixed investment: Private Residential Fixed Investment (seasonally adjusted).
- Business Investment: Total investment-Residential Fixed Investment.
- Housing Output: Residential fixed investment in chained 2009 dollars.
- Housing Price: Price Indexes of New Single-Family Houses Sold Including Lot Value.
- Inflation: Quarter on quarter log differences on the constructed consumption price index.
- Interest Rate: Secondary market rate of 3-month treasury bills (quarterly average).

Appendix B

Complete Model with the Housing Market Segmentation

B Model

B.1 Patient Household

$$\max\{\sum_{t=0}^{\infty} \beta^t a_t(\ln(C_t - \gamma C_{t-1}) - \frac{1}{1+\eta}(N_{ct}^{1+\zeta} + N_{ht}^{1+\zeta})^{\frac{1+\eta}{1+\zeta}}]\} \tag{B.1}$$

subject to

$$C_t - B_t = W_{ct}N_{ct} + W_{ht}N_{ht} + D_t - \frac{B_{t-1}R_{t-1}}{\pi_t} + F_t \tag{B.2}$$

B.2 Impatient Household

$$\max\{\sum_{t=0}^{\infty} (\beta')^t a_t[\ln(C_t' - \gamma' C_t') + j\ln(H_t') - \frac{1}{1+\eta'}((N_{ct}')^{1+\zeta'} + (N_{ht}')^{1+\zeta'})^{\frac{1+\eta'}{1+\zeta'}}]\} \tag{B.3}$$

subject to

$$C_t' + Q_t(H_t' - (1-\delta_h)H_{t-1}') - B_t' = W_{ct}'N_{ct}' + W_{ht}'N_{ht}' - \frac{B_{t-1}'R_{t-1}}{\pi_t} \tag{B.4}$$

$$B_t \leq E_t\{\frac{m(1-\delta_h)H_t'\pi_{t+1}Q_{t+1}}{R_t}\} \tag{B.5}$$

- C_t, C_t': Consumption.

- H'_t: Housing stock
- γ and γ' are habit persistent parameters.
- Q_t: Real housing price.
- B_t, B'_t: Real debt.
- π_t: Inflation.
- F_t: Profit from retailer.
- a_t: Preference shock.
- D_t:Dividend from competitive firms.
- η and η': Labor supply elasticities.
- $N_{ct}, N_{ht}, N'_{ct}, N'_{ht}$:Labor supply in consumption good sector and labor supply in housing sector.

The preference shock follows an $AR(1)$

$$\ln a_t = \rho_a \ln a_{t-1} + \epsilon_{at} \tag{B.6}$$

B.3 Competitive firms

the production function of wholesale good sector is

$$Y_{ct} = K_{ct-1}^{\alpha_c}(Z_{ct}(N_{ct}^{\nu}(N'_{ct})^{1-\nu}))^{1-\alpha_c} \tag{B.7}$$

The production function of the housing sector

$$Y_{ht} = K_{ht-1}^{\alpha_h} L_{t-1}^{\alpha l}(Z_{ht}(N_{ht}^{\nu}(N'_{ht})^{1-\nu}))^{1-\alpha_h-\alpha l} \tag{B.8}$$

The dividend of the consumption sector

$$D_{ct} = \frac{Y_{ct}}{X_t} - W_{ct}N_{ct} - W'_{ct}N'_{ct} - \frac{I_{ct}}{Z_{\mu t}} - \Psi_{ct} \tag{B.9}$$

$$D_{ht} = Q_t Y_{ht} - W_{ht}N_{ht} - W'_{ht}N'_{ht} - I_{ht} - \Psi_{ht} - P_{lt}(L_t - L_{t-1}) \tag{B.10}$$

$Z_{\mu t}, Z_{ct}, Z_{ht}$ are investment specific shock and two production shocks.

$$I_{ct} = K_{ct} - (1 - \delta_{kc})K_{ct-1} \tag{B.11}$$

$$I_{ht} = K_{ht} - (1 - \delta_{kh})K_{ht-1} \tag{B.12}$$

X_t is the mark up and is defined as $X_t = \frac{P_t}{P_t^w}$, δ_{kc}, δ_{kh} are depreciation rate of capital in the wholesale good sector and the housing sector.

Ψ_{ct} and Ψ_{ht} are the capital adjustment cost and defined as:

$$\Psi_{ct} = \frac{\psi}{2\delta_{kc}}(\frac{K_{ct}}{K_{ct-1}} - g_k)^2 \frac{K_{ct-1}}{\lambda_\mu^t} \tag{B.13}$$

$$\Psi_{ht} = \frac{\psi}{2\delta_{kh}}(\frac{K_{ht}}{K_{ht-1}} - g_c)^2 K_{ht-1} \tag{B.14}$$

The value function of the wholesale good sector

$$V_{ct} = \sum_{j=0}^{\infty} E_t \Lambda_{t,t+j} D_{ct+j} \tag{B.15}$$

The value function of the housing sector

$$V_{ht} = \sum_{j=0}^{\infty} E_t \Lambda_{t,t+j} D_{ht+j} \tag{B.16}$$

the $\Lambda_{t,t+j}$ is the stochastic discount factor. Because the patient households own the firms, so the stochastic discount factor depends on the depreciation rate and marginal utility of patient households $\Lambda_{t,t+j} = \beta^j \frac{\Lambda_t}{\Lambda_{t+j}}$

B.4 Retailers

My retailer part is the same as Iacoviello (2005) and Bernanke et al. (1999). The retailers buy the wholesale good in a competitive market and transform the wholesale good into final consumption good with a linear production technology and with no labor cost. The only cost for retailers is the cost to buy the wholesale good.

The final good market is monopolistic. There is a continuum of retailers with mass of 1. Each retailer is indexed with ι and produces differentiated good $Y_t(\iota)$. At each time t, only a constant fraction of retailers $1 - \theta$ could adjust their prices $P_t(\iota)$ and the rest of the retailers will charge the price in last period. The indexed final good Y_{ct}^f is:

$$Y_{ct}^f = (\int_0^1 (Y_t(\iota))^{\frac{\epsilon-1}{\varepsilon}} d\iota)^{\frac{\epsilon}{\epsilon-1}} \tag{B.17}$$

The price index is:

$$P_t^{1-\epsilon} = \int_0^1 (P_t(\iota))^{1-\epsilon} d\iota = \theta P_{t-1}^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon} \tag{B.18}$$

discussion of the Y_{ct}^f and Y_{ct}: we have $Y_{ct} = \int_0^1 Y_t(z)dz$ and $Y_{ct}^f = (\int_0^1 (Y_t(z))^{\frac{\epsilon-1}{\epsilon}} d\iota)^{\frac{\epsilon}{\epsilon-1}}$. Both are CES functions and the first order linear approximation are the same. Since our analysis is the first order approximation around the steady state, we could represent both as Y_{ct}.

The real profit of all the retailers:

$$F_t = \frac{P_t - P_t^w}{P_t} Y_{ct} = (1 - \frac{P_t^w}{P_t}) Y_{ct} = (1 - \frac{1}{X_t}) Y_{ct} \tag{B.19}$$

We could get the Philips curve:

$$\hat{\pi_t} = \beta \hat{\pi_{t+1}} - \frac{(1-\theta)(1-\beta\theta)}{\theta} \hat{X}_t \tag{B.20}$$

For detailed information, refer to Appendix C "Philips curve with retailer"

B.5 Central Banks

The central bank implement the policy rule:

$$R_t = \pi_t^{1+r_\pi} (\frac{GDP_t}{g_c GDP_{t-1}})^{r_Y} e_t \tag{B.21}$$

B Optimal Conditions

$\Lambda_t, \Lambda_t', \Xi_t$ are the multipliers of the three constraints.

B.1 Patient Household

$$\Lambda_t = \frac{a_t}{C_t - \gamma C_{t-1}} - \frac{\beta \gamma a_{t+1}}{C_{t+1} - \gamma C_t} \tag{B.22}$$

$$a_t (N_{ct}^{1+\zeta} + N_{ht}^{1+\zeta})^{\frac{\eta-\zeta}{1+\zeta}} N_{ct}^{\zeta} = W_{ct} \Lambda_t \tag{B.23}$$

$$a_t (N_{ct}^{1+\zeta} + N_{ht}^{1+\zeta})^{\frac{\eta-\zeta}{1+\zeta}} N_{ht}^{\zeta} = W_{ht} \Lambda_t \tag{B.24}$$

$$1 = E_t \beta \frac{\Lambda_{t+1}}{\Lambda_t} \frac{R_t}{\pi_{t+1}} \tag{B.25}$$

B.2 Impatient Household

$$\Lambda'_t = \frac{a_t}{C'_t - \gamma' C'_{t-1}} - \frac{\beta' \gamma' a_{t+1}}{C'_{t+1} - \gamma C'_t} \tag{B.26}$$

$$a_t((N'_{ct})^{1+\zeta'} + (N'_{ht})^{1+\zeta'})^{\frac{\eta'-\zeta'}{1+\zeta'}} (N'_{ct})^{\zeta'} = W'_{ct}\Lambda'_t \tag{B.27}$$

$$a_t((N'_{ct})^{1+\zeta'} + (N'_{ht})^{1+\zeta'})^{\frac{\eta'-\zeta'}{1+\zeta'}} (N'_{ht})^{\zeta'} = W'_{ht}\Lambda'_t \tag{B.28}$$

$$\Lambda'_t Q_t = E_t\{\beta' Q_{t+1}\Lambda'_{t+1}(1-\delta_h) + \Xi_t \frac{m(1-\delta_h)Q_{t+1}\pi_{t+1}}{R_t}\} + \frac{j a_t}{H'_t} \tag{B.29}$$

$$\Lambda'_t = E_t \beta' \Lambda_{t+1} \frac{R_t}{\pi_{t+1}} + \Xi_t \tag{B.30}$$

B.3 Competitive Firms

$$\Psi_{ct} = \frac{\psi}{2\delta_{kc}}(\frac{K_{ct}}{K_{ct-1}} - g_k)^2 \frac{K_{ct-1}}{\lambda_\mu^t} \tag{B.31}$$

$$\Psi_{ht} = \frac{\psi}{2\delta_{kh}}(\frac{K_{ht}}{K_{ht-1}} - g_c)^2 K_{ht-1} \tag{B.32}$$

$$W_{ct} = (1-\alpha_c)\nu \frac{Y_{ct}}{X_t N_{ct}} \tag{B.33}$$

$$W'_{ct} = (1-\alpha_c)(1-\nu) \frac{Y_{ct}}{X_t N'_{ct}} \tag{B.34}$$

$$W_{ht} = (1-\alpha_h - \alpha l)\nu \frac{Q_t Y_{ht}}{N_{ht}} \tag{B.35}$$

$$W'_{ht} = (1-\alpha_h - \alpha l)(1-\nu) \frac{Q_t Y_{ht}}{N'_{ht}} \tag{B.36}$$

$$P_{lt} = \beta \frac{\Lambda_{t+1}}{\Lambda_t}\{\frac{\alpha_l Q_{t+1} Y_{ht+1}}{L_t} + P_{lt+1}\} \tag{B.37}$$

$$\frac{1}{Z_{\mu t}} + \frac{\psi}{\delta_{kc}}(\frac{K_{ct}}{K_{ct-1}} - g_k) = E_t \beta \frac{\Lambda_{t+1}}{\Lambda_t}\{\alpha_c \frac{Y_{ct+1}}{X_{t+1}K_{ct}} + \frac{1-\delta_{kc}}{Z_{\mu t+1}} + \frac{\psi}{2\delta_{kc}\lambda_\mu^{t+1}}(\frac{K^2_{ct+1}}{K^2_{ct}} - g_k^2)\} \tag{B.38}$$

$$1+\frac{\psi}{\delta_{kh}}(\frac{K_{ht}}{K_{ht-1}}-g_c)=E_t\beta\frac{\Lambda_{t+1}}{\Lambda_t}\{\alpha_h\frac{Q_{t+1}Y_{ht+1}}{K_{ht}}+(1-\delta_{kh})+\frac{\psi}{2\delta_{kh}}(\frac{K^2_{ht+1}}{K^2_{ht}}-g^2_c)\} \tag{B.39}$$

B.4 Market Clearing Condition

Total labor supply in consumption sector and housing sector are NN_{ct} and NN_{ht}

$$NN_{ct}=N^{\nu}_{ct}(N'_{ct})^{1-\nu} \tag{B.40}$$

$$NN_{ht}=N^{\nu}_{ht}(N'_{ht})^{1-\nu} \tag{B.41}$$

$$C_t+C'_t+\frac{I_{ct}}{Z_{\mu t}}+I_{ht}+\Psi_{ct}+\Psi_{ht}=Y_{ct} \tag{B.42}$$

resource constraints come from patient household and impatient household's budget constraint and $\rho_t=1$ at equilibrium.

$$H'_t+(1-\delta_h)H'_{t-1}=Y_{ht} \tag{B.43}$$

$$B_t+B'_t=0 \tag{B.44}$$

In equilibrium, the share of the stock holding is always equal to 1.

B Shocks

$$\ln(Z_{ct})=t\ln(\lambda_c)+\ln(z_{ct}) \tag{B.45}$$

$$\ln(z_{ct})=\rho_c\ln(z_{ct-1})+\epsilon_{ct} \tag{B.46}$$

$$\ln(Z_{ht})=t\ln(\lambda_h)+\ln(z_{ht}) \tag{B.47}$$

$$\ln(z_{ct})=\rho_h\ln(z_{ht-1})+\epsilon_{ht} \tag{B.48}$$

$$\ln(Z_{\mu t})=t\ln(\lambda_\mu)+\ln(z_{\mu t}) \tag{B.49}$$

$$\ln(z_{\mu t}) = \rho_\mu \ln(z_{\mu t-1}) + \epsilon_{\mu t} \tag{B.50}$$

$$\ln(a_t) = \rho_a \ln(a_{t-1}) + \epsilon_{at} \tag{B.51}$$

$$\ln(u_t) = \rho_u \ln(u_{t-1}) + \epsilon_{ut} \tag{B.52}$$

$$\ln(e_t) = \epsilon_{et} \tag{B.53}$$

B Stationary System

There are 32 unknowns after subsisting out some variables.

The variables are: $K_{ct}, K_{ht}, H'_t, B'_t, Q_t, Y_{ct}, Y_{ht}, N_{ct}, N'_{ct}, N'_{ht}, N_{ht}, NN_{ct}, NN_{ht}, I_{ct}, I_{ht}, C_t, C'_t, R_t, \pi_t, X_t, GDP_t, \Lambda_t, \Lambda'_t, \Xi_t, \Psi_{ct}, \Psi_{ht}, W_{ct}, W_{ht}, W'_{ct}, W'_{ht}, P_l, rr$.

The variables are transformed to be stationary: $y_{ct} = \frac{Y_{ct}}{g_c^t}, y_{ct}^f = \frac{Y_{ct}^f}{g_c^t}, y_{ht} = \frac{Y_{ht}}{g_h^t}, k_{ct} = \frac{K_{ct}}{g_k^t}, k_{ht} = \frac{K_{ht}}{g_c^t}, i_{ct} = \frac{I_{ct}}{g_k^t}, i_{ht} = \frac{I_{ht}}{g_c^t}, c_t = \frac{C_t}{g_c^t}, q_t = \frac{Q_t}{g_q^t}, z_{ct} = \frac{Z_{ct}}{\lambda_c^t}, z_{ht} = \frac{Z_{ht}}{\lambda_h^t}, z_{\mu t} = \frac{Z_{\mu t}}{\lambda_\mu^t}, g_c = \frac{g_k}{\lambda_\mu}, \xi_t = \Xi_t g_c, \lambda_t = \Lambda_t g_c, \lambda'_t = \Lambda'_t g_c$

$N_{ct}, N_{ht}, N'_{ct}, N'_{ht}, NN_{ct}, NN_{ht}, R_t, \pi_t, X$ are stationary.

The stationary system is:

$$\lambda_t = \frac{a_t}{c_t - \gamma \frac{c_{t-1}}{g_c}} - \frac{\beta \gamma a_{t+1}}{g_c c_{t+1} - \gamma c_t} \tag{B.54}$$

$$\lambda'_t = \frac{a_t}{c'_t - \gamma' \frac{c'_{t-1}}{g_c}} - \frac{\beta' \gamma' a_{t+1}}{g_c c'_{t+1} - \gamma c'_t} \tag{B.55}$$

$$a_t (n_{ct}^{1+\zeta} + n_{ht}^{1+\zeta})^{\frac{\eta-\zeta}{1+\zeta}} n_{ct}^{\zeta} = w_{ct} \lambda_t \tag{B.56}$$

$$a_t (n_{ct}^{1+\zeta} + n_{ht}^{1+\zeta})^{\frac{\eta-\zeta}{1+\zeta}} n_{ht}^{\zeta} = w_{ht} \lambda_t \tag{B.57}$$

$$a_t ((n'_{ct})^{1+\zeta'} + (n'_{ht})^{1+\zeta'})^{\frac{\eta'-\zeta'}{1+\zeta'}} (n'_{ct})^{1+\zeta'} = w'_{ct} \lambda'_t \tag{B.58}$$

$$a_t ((n'_{ct})^{1+\zeta'} + (n'_{ht})^{1+\zeta'})^{\frac{\eta'-\zeta'}{1+\zeta'}} (n'_{ht})^{1+\zeta'} = w'_{ht} \lambda'_t \tag{B.59}$$

$$q_t \lambda'_t = j \frac{a_t}{h'_t} + E_t\{\beta'(1-\delta_h) \frac{q_{t+1}\lambda'_{t+1}}{g_h} + \frac{m(1-\delta_h)\xi_t q_{t+1}\pi_{t+1}}{R_t} g_q\} \tag{B.60}$$

$$1 = E_t \beta \frac{\lambda_{t+1}}{\lambda_t g_c} \frac{r_t}{\pi_{t+1}} \tag{B.61}$$

$$\lambda'_t = E_t \beta' \frac{\lambda_{t+1}}{g_c} \frac{R_t}{\pi_{t+1}} + \xi_t \tag{B.62}$$

$$c'_t + q_t(h'_t - \frac{1-\delta_h}{g_h} h'_{t-1}) - b'_t = w'_{ct} n'_{ct} + w'_{ht} n'_{ht} - \frac{b'_{t-1} R_{t-1}}{\pi_t g_c} \tag{B.63}$$

$$b'_t = m(1-\delta_h) \frac{h'_t q_{t+1} \pi_{t+1}}{R_t} g_q \tag{B.64}$$

$$\frac{1}{z_{\mu t}} + \frac{\psi}{\delta_{kc}} g_k (\frac{k_{ct}}{k_{ct-1}} - 1) = E_t \beta \frac{\lambda_{t+1}}{\lambda_t g_c} \{\alpha_c \frac{y_{ct+1}}{x_{t+1} k_{ct}} g_c + \frac{1-\delta_{kc}}{\lambda_\mu z_{\mu t+1}} + \frac{\psi}{2\delta_{kc}} g_c g_k (\frac{k^2_{ct+1}}{k^2_{ct}} - 1)\} \tag{B.65}$$

$$1 + \frac{\psi}{\delta_{kh}} g_c (\frac{k_{ht}}{k_{ht-1}} - 1) = E_t \beta \frac{\lambda_{t+1}}{\lambda_t g_c} \{\alpha_h \frac{q_{t+1} y_{ht+1}}{k_{ht}} g_c + (1-\delta_{kh}) + \frac{\psi}{2\delta_{kh}} g_c^2 (\frac{k^2_{ht+1}}{k^2_{ht}} - 1)\} \tag{B.66}$$

$$i_{ct} g_k = k_{ct} g_k - (1-\delta_{kc}) k_{ct-1} \tag{B.67}$$

$$i_{ht} g_c = k_{ht} g_c - (1-\delta_{kh}) k_{ht-1} \tag{B.68}$$

$$y_{ct} = k^{\alpha_c}_{ct-1} (n^\nu_{ct} (n'_{ct})^{1-\nu})^{1-\alpha_c} (\frac{1}{g_k})^{\alpha_c} \tag{B.69}$$

$$y_{ht} = k^{\alpha_h}_{ht-1} (n^\nu_{ht} (n'_{ht})^{1-\nu})^{1-\alpha_h-\alpha l} (\frac{1}{g_c})^{\alpha_h} \tag{B.70}$$

$$y_{ht} g_h = h'_t g_h - (1-\delta_h) h'_{t-1} \tag{B.71}$$

$$y_{ct} = c_t + c'_t + \frac{i_{ct}}{z_{\mu t}} + i_{ht} + \psi_{ct} + \psi_{ht} \tag{B.72}$$

$$\psi_{ct} = \frac{\psi}{2\delta_{kc}} g_k (\frac{k_{ct}}{k_{ct-1}} - 1)^2 k_{ct-1} \tag{B.73}$$

$$\psi_{ht} = \frac{\psi}{2\delta_{kh}} g_c (\frac{k_{ht}}{k_{ht-1}} - 1)^2 k_{ht-1} \tag{B.74}$$

$$nn_{ct} = n^\nu_{ct} (n'_{ct})^{1-\nu} \tag{B.75}$$

$$nn_{ht} = n_{ht}^{\nu}(n'_{ht})^{1-\nu} \tag{B.76}$$

$$w_{ct} = (1-\alpha_c)\nu\frac{y_{ct}}{x_t n_{ct}} \tag{B.77}$$

$$w'_{ct} = (1-\alpha_c)(1-\nu)\frac{y_{ct}}{x_t n'_{ct}} \tag{B.78}$$

$$w_{ht} = (1-\alpha_h-\alpha l)\nu\frac{q_t y_{ht}}{n_{ht}} \tag{B.79}$$

$$w'_{ht} = (1-\alpha_h-\alpha l)(1-\nu)\frac{q_t y_{ht}}{n'_{ht}} \tag{B.80}$$

$$p_{lt} = \beta\frac{\lambda_{t+1}}{\lambda_t g_c}(\frac{\alpha_l q_{t+1} y_{ht+1}}{L_t} + p_{lt+1}) \tag{B.81}$$

$$gdp_t = y_{ct} + q_t y_{ht} \tag{B.82}$$

$$R_t = \pi_t^{1+r_\pi}(\frac{gdp_t}{gdp_{t-1}})^{r_Y} e_t \tag{B.83}$$

$$rr = \hat{R}_t - \hat{\pi_{t+1}} \tag{B.84}$$

$$\hat{\pi}_t = \beta\hat{\pi_{t+1}} - \kappa\hat{X}_t + u_t \tag{B.85}$$

B Steady State

At steady state, we have $\pi_t = 1, \beta = \frac{g_c}{r}, x = \frac{\varepsilon}{\varepsilon-1}, \psi_{ct} = \psi_c = 0, \psi_{ht} = \psi_h = 0, \xi = \lambda'(1-\frac{\beta'}{\beta})$. The growth rates are

$$g_c = \lambda_c \lambda_\mu^{\frac{\alpha_c}{1-\alpha_c}} \tag{B.86}$$

$$g_k = \lambda_c \lambda_\mu^{\frac{1}{1-\alpha_c}} \tag{B.87}$$

$$g_h = \lambda_c^{\alpha_h} \lambda_\mu^{\frac{\alpha_h \alpha_c}{1-\alpha_c}} \lambda_h^{1-\alpha_h-\alpha l} \tag{B.88}$$

$$g_q = \lambda_c^{1-\alpha_h} \lambda_\mu^{\frac{(1-\alpha_h-\alpha l)\alpha_c}{1-\alpha_c}} \lambda_h^{-(1-\alpha_h-\alpha l)} \tag{B.89}$$

$$d_5 = \frac{g_c - \gamma}{g_c - \beta\gamma} \tag{B.90}$$

$$d_6 = \frac{g_c - \gamma'}{g_c - \beta'\gamma'} \tag{B.91}$$

$$\lambda = \frac{1}{cd_5} \tag{B.92}$$

$$\lambda' = \frac{1}{c'd_6} \tag{B.93}$$

$$A_1 = \frac{\alpha_c \beta}{1 - \frac{\beta}{g_k}(1 - \delta_{kc})} \frac{1}{x} \tag{B.94}$$

$$A_3 = 1 - \frac{\beta'(1 - \delta_h)}{g_h} - \frac{m(1 - \delta_h)(\beta - \beta')}{g_h} \tag{B.95}$$

$$A_4 = \frac{\alpha_h \beta}{1 - \frac{\beta}{g_c}(1 - \delta_{kh})} \tag{B.96}$$

$$A_5 = \frac{g_h - (1 - \delta_h)}{g_h}, \tag{B.97}$$

$$A_6 = 1 + [A_5 - \frac{m(1 - \delta_h)(\beta - 1)}{g_h} - (1 - \alpha_h - \alpha_l)(1 - \nu)A_5] \frac{j}{A_3} d_6 \tag{B.98}$$

$$A_8 = \frac{(1 - \alpha_c)(1 - v)}{x} \tag{B.99}$$

$$d_1 = \frac{g_k - (1 - \delta_{kc})}{g_k} \tag{B.100}$$

$$d_2 = \frac{g_c - (1 - \delta_{kh})}{g_c} \tag{B.101}$$

$$A_9 = 1 - d_1 A_1 \tag{B.102}$$

$$A_{10} = 1 + d_1 A_4 A_5 d_6 \frac{j}{A_3} \tag{B.103}$$

$$A_{11} = A_9 - A_{10} A_{12} \tag{B.104}$$

$$A_{12} = \frac{A_8}{A_6} \tag{B.105}$$

$$A_{13} = A_4 A_5 j \frac{d_6 A_{12}}{A_3} \tag{B.106}$$

$$\phi = \frac{1-\alpha_c}{1-\alpha_h} \frac{1}{x A_5 j \frac{d_6 A_{12}}{A_3}} \tag{B.107}$$

$$q = d_6 \frac{j}{1 - \frac{\beta'(1-\delta_h)}{g_h} - \frac{m(1-\delta_h)(\beta-\beta')}{g_h}} \frac{c'}{h'} = \frac{j}{A_3} \frac{c'}{h'} d_6 \tag{B.108}$$

$$b' = \frac{m(1-\delta_h) q h'}{r} g_q \tag{B.109}$$

$$k_c = \frac{\alpha_c \beta}{1 - \frac{\beta}{g_c}(1-\delta_{kc})} \frac{1}{x} y_c = A_1 y_c \tag{B.110}$$

$$k_h = \frac{\alpha_h \beta}{1 - \frac{\beta}{g_c}(1-\delta_{kh})} q y_h = A_4 q y_h \tag{B.111}$$

$$y_h = \frac{g_h - (1-\delta_h)}{g_h} h' = A_5 h' \tag{B.112}$$

$$i_c = \frac{g_k - (1-\delta_{kc})}{g_k} k_c = d_1 k_c \tag{B.113}$$

$$i_h = \frac{g_c - (1-\delta_{kh})}{g_c} k_h = d_2 k_h \tag{B.114}$$

With $qh' = \frac{j}{A_3} d_6 c'$

$$k_h = A_4 q y_h = A_4 A_5 q h' = A_4 A_5 j \frac{d_6 c'}{A_3} \tag{B.115}$$

from equation $c + c' + i_c + i_h = y_c$ we have

$$c + d_1 A_1 y_c + d_1 A_4 A_5 j \frac{d_6 c'}{A_3} = y_c \tag{B.116}$$

The budget constraint of impatient households:

$$c' + q(h' - \frac{1-\delta_h}{g_h} h') = b' - \frac{b'R}{\pi z_c} + (1-\alpha_h-\alpha l)(1-\nu) q y_h + (1-\alpha_c)(1-\nu)\frac{y_c}{x} \tag{B.117}$$

$$(1+[A_5-\frac{m(1-\delta_h)(\beta-1)}{g_h}-(1-\alpha_h-\alpha l)(1-\nu)A_5]\frac{jd_6}{A_3})c'=\frac{(1-\alpha_c)(1-v)}{x}y_c \quad \text{(B.118)}$$

We could rewrite the equation above as:

$$A_6c'=A_8y_c \quad \text{(B.119)}$$

then we could solve for c and c'

$$c'=\frac{A_8}{A_6}=A_{12}y_c \quad \text{(B.120)}$$

$$c=(A_9-A_{10}A_{12})y_c=A_{11}y_c \quad \text{(B.121)}$$

$$k_h=A_4A_5j\frac{d_6A_{12}}{A_3}y_c=A_{13}y_c \quad \text{(B.122)}$$

$$i_c=d_1A_1y_c=A_{14}y_c \quad \text{(B.123)}$$

$$i_h=d_1A_{13}y_c=A_{15}y_c \quad \text{(B.124)}$$

$$(n_c^{1+\zeta}+n_h^{1+\zeta})^{\frac{\eta-\zeta}{1+\zeta}}n_c^{1+\zeta}c=(1-\alpha_c)\nu\frac{y_c}{x}\lambda \quad \text{(B.125)}$$

$$(n_c^{1+\zeta}+n_h^{1+\zeta})^{\frac{\eta-\zeta}{1+\zeta}}n_h^{1+\zeta}c=(1-\alpha_h-\alpha l)qy_h\lambda \quad \text{(B.126)}$$

The ratio of the two above equation:

$$(\frac{n_c}{n_h})^{1+\zeta}=\frac{1-\alpha_c}{1-\alpha_h-\alpha l}\frac{1}{xA_5j\frac{d_6A_{12}}{A_3}}=\phi \quad \text{(B.127)}$$

we get $n_c^{1+\zeta}=\phi n_h^{1+\zeta}$ and put it in equation (162)

$$(1+\phi)^{\frac{\eta-\zeta}{1+\zeta}}n_h^{\eta-\zeta}\phi n_h^{1+\zeta}A_{11}d_5y_c=\frac{(1-\alpha_c)\nu}{x} \quad \text{(B.128)}$$

we could get n_h

$$n_h=(\frac{(1-\alpha_c)\nu}{x(1+\phi)^{\frac{\eta-\zeta}{1+\zeta}}\phi d_5A_{11}})^{\frac{1}{1+\eta}} \quad \text{(B.129)}$$

$$((n_c')^{1+\zeta'}+(n_h')^{1+\zeta'})^{\frac{\eta'-\zeta'}{1+\zeta'}}(n_c')^{1+\zeta'}c'=(1-\alpha_c)(1-\nu)\frac{y_c}{x}\lambda' \quad \text{(B.130)}$$

$$((n'_c)^{1+\zeta'} + (n'_h)^{1+\zeta'})^{\frac{\eta'-\zeta'}{1+\zeta'}} (n'_h)^{1+\zeta'} c' = (1-\alpha_h-\alpha_l) q y_h \lambda' \tag{B.131}$$

The ratio of the two above equation:

$$(\frac{n'_c}{n'_h})^{1+\zeta'} = \frac{1-\alpha_c}{1-\alpha_h-\alpha_l} \frac{1}{x A_5 j \frac{d_6 A_{12}}{A_3}} = \phi \tag{B.132}$$

we get $(n'_c)^{1+\zeta} = \phi (n'_h)^{1+\zeta}$ and put it in equation (167)

$$(1+\phi)^{\frac{\eta'-\zeta'}{1+\zeta'}} n_h^{\eta'-\zeta'} \phi n_h^{1+\eta} A_{12} d_6 y_c = \frac{(1-\alpha_c)(1-\nu)}{x} \tag{B.133}$$

we could get n'_h

$$n'_h = (\frac{(1-\alpha_c)(1-\nu)}{x(1+\phi)^{\frac{\eta'-\zeta'}{1+\zeta'}} \phi d_6 A_{12}})^{\frac{1}{1+\eta'}} \tag{B.134}$$

The we get n'_c by $n'_c = \phi^{\frac{1}{1+\zeta'}} n'_h$
Then we could get levels:

$$y_c = (\frac{A_1}{g_k})^{\frac{\alpha_c}{1-\alpha_c}} n_c^{\nu} (n'_c)^{1-\nu} \tag{B.135}$$

$$y_h = (A_{13} y_c)^{\alpha_h} (n_h^{\nu} (n'_h)^{1-\nu})^{1-\alpha_h} g_c^{-\alpha_h} \tag{B.136}$$

$$h' = \frac{y_h}{A_5} \tag{B.137}$$

$$q = \frac{j}{A_3} \frac{c'}{h'} d_6 \tag{B.138}$$

Appendix C

New Keynesian Philips Curve

Bernanke et al. (1999) used a retailer to generate the price stickiness. The entrepreneurs produce intermediate good and face a perfect competitive market.Retailers buy the intermediate good at wholesale price P_t^w and transform them into differentiated final good. There is a continuum of retailers with mass of 1. $Y_t(\iota)$ is the quantity of output sold by the retailer z and it is measured in units of wholesale goods. Y_t^f is usable final good.

$$Y_{ct}^f = (\int_0^1 (Y_{ct}(\iota))^{\frac{\epsilon-1}{\epsilon}} d\iota)^{\frac{\epsilon}{\epsilon-1}} \tag{C.1}$$

$P_t(z)$ is the price charged by the retailer z. Each retailer has $1-\theta$ probability to adjust the price at time t. The price index for each period is:

$$P_t = (\int_0^1 (P_t(\iota))^{1-\epsilon})^{\frac{1}{1-\epsilon}} \tag{C.2}$$

The demand curve for each individual retailer is

$$Y_{ct}(\iota) = (\frac{P_t(\iota)}{P_t})^{-\epsilon} Y_{ct}^f \tag{C.3}$$

The intermediate good Y_{ct} grows at Z_{ct}, so does $Y_t(z)$ and Y_{ct}^f

The real marginal cost for each retailer are the same

$$MC = \frac{P_t^w}{P_t} = \frac{1}{X_t} \tag{C.4}$$

Retailer z reset the price $P_t(\iota)$ at time t to maximize the profit:

$$\max\{\Sigma_{k=0}^{\infty} \theta^k E_t \Lambda_{t,k} \frac{P_t(\iota) - P_{t+k}^w}{P_{t+k}} Y_{t+k}(\iota)\} \tag{C.5}$$

$\Lambda_{t,k}$ is the stochastic discount factor and it is the firm's owner's substitution rate of infratemporal marginal utility.

$$\Lambda_{t,k} = \beta_k \frac{\Lambda_{t+k}}{\Lambda_t} \tag{C.6}$$

$Y_{ct}(\iota)$ inherited the unit root from the shock Z_{ct}. The stationary problem of the railer becomes:

$$\max\{\Sigma_{k=0}^{\infty}(\beta\theta)^k \frac{\lambda_{t+k}}{\lambda_k}\frac{Z_{ct}}{Z_{ct+k}} E_t \frac{P_t(\iota) - P^w_{t+k}}{P_{t+k}} y_{t+k}(\iota) Z_{ct+k}\} \tag{C.7}$$

we could cancel out Z_{ct+k} and Z_t is unrelated to the retailer's optimal choice, we could ignore it. The retailer's problems is:

$$\max\{\Sigma_{k=0}^{\infty}(\beta\theta)^k E_t \frac{\lambda_{t+k}}{\lambda_t}\frac{P_t(\iota) - P^w_{t+k}}{P_{t+k}}(\frac{P_t(\iota)}{P_t})^{-\epsilon} y^f_{ct}\} \tag{C.8}$$

Take first order condition with respect to $P_t(\iota)$. I use P_t^* to represent the optimal price and y^*_{t+k} to represent retailer ι's optimal output:

$$\frac{1-\epsilon}{P_t^*}\Sigma_{k=0}^{\infty}(\beta\theta)^k \frac{\lambda_{t+k}}{\lambda_t} y^*_{t+k}(\frac{P_t^*}{P_{t+k}} - \frac{\epsilon}{\epsilon-1}\frac{P^w_{t+k}}{P_{t+k}}) = 0 \tag{C.9}$$

let $M = \frac{\epsilon}{\epsilon-1} = \frac{1}{X}$ the FOC becomes:

$$\Sigma_{k=0}^{\infty}(\beta\theta)^k \frac{\lambda_{t+k}}{\lambda_t} y^*_{t+k}(\frac{P_t^*}{P_{t+k}} - M\frac{1}{X_{t+k}}) \tag{C.10}$$

The optimal price of retailer's is :

$$P_t^* = \frac{M\sum_{k=0}^{\infty}(\beta\theta)^k \frac{\lambda_{t+k}}{\lambda_t} y^*_{ct+k}\frac{1}{X_{t+k}}}{\sum_{k=0}^{\infty}(\beta\theta)^k \frac{\lambda_{t+k}}{\lambda_t} y^*_{ct+k}\frac{1}{P_{t+k}}} \tag{C.11}$$

divide P_t on both sides of the equation,rearrange it and substitute y^*_{t+k} with $(\frac{P_t^*}{P_{t+k}})^{-\epsilon} y^f_{ct+k}$:

$$\frac{P_t^*}{P_t}\sum_{k=0}^{\infty}(\beta\theta)^k \frac{\lambda_{t+k}}{\lambda_t}(\frac{P_t^*}{P_{t+k}})^{-\epsilon} y^f_{ct+k}\frac{1}{P_{t+k}} = \frac{M}{P_t}\sum_{k=0}^{\infty}(\beta\theta)^k \frac{\lambda_{t+k}}{\lambda_t}(\frac{P_t^*}{P_{t+k}})^{-\epsilon} y^f_{ct+k}\frac{1}{X_{t+k}} \tag{C.12}$$

We could cancel out $(P_t^*)^{-\epsilon}$ on both sides of the above equation:

$$\frac{P_t^*}{P_t}\sum_{k=0}^{\infty}(\beta\theta)^k \frac{\lambda_{t+k}}{\lambda_t} P^{\epsilon-1}_{t+k} y^f_{ct+k} = \frac{M}{P_t}\sum_{k=0}^{\infty}(\beta\theta)^k \frac{\lambda_{t+k}}{\lambda_t} P^{\epsilon}_{t+k} y^f_{ct+k}\frac{1}{X_{t+k}} \tag{C.13}$$

log linearize the equation:

$$LHS = \sum(\beta\theta)^k P^{\epsilon-1} y_c^f (\hat{P}_t^* - \hat{P}_t) + \sum(\beta\theta)^k P^{\epsilon-1} y_c^f (\hat{\lambda_{t+k}} - \hat{\lambda}_t + (\epsilon-1)\hat{P_{t+k}} + \hat{y_{ct+k}^f}) \quad (C.14)$$

At steady state,the mark up $X_{t+k} = X = \frac{1}{M}$.

$$RHS = \sum(\beta\theta)^k P^{\epsilon-1} y_c^f (-\hat{P}_t) + \sum(\beta\theta)^k P^{\epsilon-1} y_c^f (\hat{\lambda_{t+k}} - \hat{\lambda}_t + \epsilon\hat{P_{t+k}} + \hat{y_{ct+k}^f} - \hat{X_{t+k}}) \quad (C.15)$$

we let $LHS = RHS$ and get:

$$\Sigma_{k=0}^{\infty}(\beta\theta)^k(\hat{P}_t^* - \hat{P_{t+k}}) = \Sigma_{k=0}^{\infty}(-\hat{X_{t+k}}) \quad (C.16)$$

If we move $\Sigma_{k=0}^{\infty}\hat{P_{t+k}}$ to the right hand side and subtract $\Sigma_{k=0}^{\infty}\hat{P_{t-1}}$ in both sides of the equation:

$$\Sigma_{k=0}^{\infty}(\beta\theta)^k(\hat{P}_t^* - \hat{P_{t-1}}) = \Sigma_{k=0}^{\infty}(\hat{P_{t+k}} - \hat{P_{t-1}} - \hat{X_{t+k}}) \quad (C.17)$$

The steady state inflation is 1 and we define the inflation rate as

$$\hat{\pi}_t = \log(\frac{P_t/P}{P_{t-1}/P}) = \hat{P}_t - \hat{P_{t-1}} \quad (C.18)$$

the Price index can be written as:

$$P_t^{1-\epsilon} = (1-\theta)(P_t^*)^{1-\epsilon} + \theta(P_{t-1})^{1-\epsilon} \quad (C.19)$$

Log linearize the above equation we get:

$$(1-\epsilon)\hat{P}_t = (1-\theta)(1-\epsilon)\hat{P}_t^* + \theta(1-\epsilon)\hat{P_{t-1}} \quad (C.20)$$

we cancel out $(1-\epsilon)$ and subtract $(\hat{P_{t-1}}$ on both sides of equation we get

$$\hat{P}_t - \hat{P_{t-1}} = (1-\theta)\hat{P}_t^* + (\theta-1)\hat{P_{t-1}} \quad (C.21)$$

rearrange the equation:

$$\hat{\pi}_t = (1-\theta)(\hat{P}_t^* - \hat{P_{t-1}}) \quad (C.22)$$

$$\hat{P_{t+k}} - \hat{P_{t-1}} = \Sigma_{j=0}^{k}\hat{\pi_{t+j}} \quad (C.23)$$

Then the log linearized FOC could be rewritten as:

$$\hat{P}_t^* - \hat{P_{t-1}} = (1-\beta\theta)\Sigma_{k=0}^{\infty}(\beta\theta)^k(-\hat{X_{t+k}}) + (1-\beta\theta)\Sigma_{k=0}^{\infty}(\beta\theta)^k\Sigma_{j=0}^{k}\hat{\pi_{t+j}} \quad (C.24)$$

$$\hat{P^*_{t+1}} - \hat{P}_t = (1-\beta\theta)\Sigma_{k=0}^{\infty}(\beta\theta)^k(-\hat{X_{t+k+1}}) + (1-\beta\theta)\Sigma_{k=0}^{\infty}(\beta\theta)^k\Sigma_{j=0}^{k}\hat{\pi_{t+j+1}} \tag{C.25}$$

$$\beta\theta(\hat{P^*_{t+1}} - \hat{P}_t) = (1-\beta\theta)\Sigma_{k=1}^{\infty}(\beta\theta)^k(-\hat{X_{t+k}}) + (1-\beta\theta)\Sigma_{k=1}^{\infty}(\beta\theta)^k\Sigma_{j=1}^{k}\hat{\pi_{t+j+1}} \tag{C.26}$$

Write the equation (18) recursively:

$$\hat{P}^*_t - \hat{P_{t-1}} = -(1-\beta\theta)\hat{X}_t + (1-\beta\theta)\Sigma_{k=0}^{\infty}\hat{\pi}_t + \beta\theta E_t(\hat{P^*_{t+1}} - \hat{P}_t) \tag{C.27}$$

$$\frac{\hat{\pi}_t}{1-\theta} = -(1-\beta\theta)\hat{X}_t + \pi_t + \beta\theta E_t\frac{\hat{\pi_{t+1}}}{1-\theta} \tag{C.28}$$

$$\hat{\pi}_t = \beta E_t\hat{\pi_{t+1}} - \frac{(1-\theta)(1-\beta\theta)}{\theta}\hat{X}_t \tag{C.29}$$

Appendix D

Complete Model with No Housing Market Segmentation

D Model

D.1 Patient Household

$$\max\{\sum_{t=0}^{\infty}\beta^t a_t(\ln(C_t-\gamma C_{t-1})+j\ln(H_t)-\frac{1}{1+\eta}(N_{ct}^{1+\zeta}+N_{ht}^{1+\zeta})^{\frac{1+\eta}{1+\zeta}})\} \tag{D.1}$$

subject to

$$C_t-B_t+Q_t(H_t-(1-\delta_h)H_{t-1})=W_{ct}N_{ct}+W_{ht}N_{ht}-\frac{B_{t-1}R_{t-1}}{\pi_t}+F_t+D_t \tag{D.2}$$

D.2 Impatient Household

$$\max\{\sum_{t=0}^{\infty}(\beta')^t a_t(\ln(C'_t-\gamma' C_t)+j\ln(H'_t)-\frac{1}{1+\eta'}((N'_{ct})^{1+\zeta'}+(N'_{ht})^{1+\zeta'})^{\frac{1+\eta'}{1+\zeta'}})\} \tag{D.3}$$

subject to

$$C'_t+Q_t(H'_t-(1-\delta_h)H'_{t-1})-B'_t=W'_{ct}N'_{ct}+W'_{ht}N'_{ht}-\frac{B'_{t-1}R_{t-1}}{\pi_t} \tag{D.4}$$

$$B_t\leq E_t\{\frac{m(1-\delta_h)H'_t\pi_{t+1}Q_{t+1}}{R_t}\} \tag{D.5}$$

D.3 Competitive firms, Retailers and Central Bank

The set up of the competitive firms, retailers and central bank are the same as that in the model with the housing market segmentation.

D Optimal conditions

The optimal conditions are the same as those in the model with the housing market segmentation except for that we have one more unknown H_t. We also have one more equations for it:

$$\Lambda_t Q_t = E_t \beta Q_{t+1} \Lambda_{t+1}(1-\delta_h) + \frac{j a_t}{H_t} \tag{D.6}$$

Since the patient households also own and trade housing, the market clearing condition for housing becomes:

$$H_t + H'_t - (1-\delta_h)(H_{t-1} + H'_{t-1}) = Y_{ht} \tag{D.7}$$

D Unknowns and Equations

There are 33 unknowns after subsisting out several variables.

The variables are :

- K_{ct},K_{ht},H_t,H'_t,B'_t,Q_t,Y_{ct},Y_{ht},N_{ct},N'_{ct},N'_{ht},N_{ht},NN_{ct},NN_{ht},I_{ct},I_{ht},C_t,C'_t,R_t,π_t,X_t, GDP_t,Λ_t,Λ'_t,Ξ_t,Ψ_{ct},Ψ_{ht},W_{ct},W_{ht},W'_{ct},W'_{ht}, P_{lt} rr,.

$$\Lambda_t = \frac{a_t}{C_t - \gamma C_{t-1}} - \frac{\beta\gamma a_{t+1}}{C_{t+1} - \gamma C_t} \tag{D.8}$$

$$\Lambda'_t = \frac{a_t}{C'_t - \gamma' C'_{t-1}} - \frac{\beta'\gamma' a_{t+1}}{C'_{t+1} - \gamma C'_t} \tag{D.9}$$

$$a_t (N_{ct}^{1+\zeta} + N_{ht}^{1+\zeta})^{\frac{\eta-\zeta}{1+\zeta}} N_{ct}^{\zeta} = W_{ct}\Lambda_t \tag{D.10}$$

$$a_t (N_{ct}^{1+\zeta} + N_{ht}^{1+\zeta})^{\frac{\eta-\zeta}{1+\zeta}} N_{ht}^{\zeta} = W_{ht}\Lambda_t \tag{D.11}$$

$$a_t ((N'_{ct})^{1+\zeta'} + (N'_{ht})^{1+\zeta'})^{\frac{\eta'-\zeta'}{1+\zeta'}} (N'_{ct})^{\zeta'} = W'_{ct}\Lambda'_t \tag{D.12}$$

$$a_t((N'_{ct})^{1+\zeta'} + (N'_{ht})^{1+\zeta'})^{\frac{\eta'-\zeta'}{1+\zeta'}}(N'_{ht})^{\zeta'} = W'_{ht}\Lambda'_t \tag{D.13}$$

$$\Lambda_t Q_t = E_t \beta Q_{t+1}\Lambda_{t+1}(1-\delta_h) + \frac{j a_t}{H_t} \tag{D.14}$$

$$Q_t \Lambda'_t = E_t\{\beta' Q_{t+1}\Lambda'_{t+1}(1-\delta_h) + \Xi_t \frac{m(1-\delta_h)Q_{t+1}\pi_{t+1}}{R_t}\} + j\frac{a_t}{H'_t} \tag{D.15}$$

$$1 = E_t \beta \frac{\Lambda_{t+1}}{\Lambda_t}\frac{R_t}{\pi_{t+1}} \tag{D.16}$$

$$\Lambda'_t = E_t \beta' \Lambda'_{t+1}\frac{R_t}{\pi_{t+1}} + \Xi_t \tag{D.17}$$

$$C'_t + Q_t(H'_t - (1-\delta_h)H'_{t-1}) - B'_t = W'_{ct}N'_{ct} + W'_{ht}N'_{ht} - \frac{B'_{t-1}R_{t-1}}{\pi_t} \tag{D.18}$$

$$B'_t R_t = m(1-\delta_h)H'_t Q_{t+1}\pi_{t+1} \tag{D.19}$$

$$\frac{1}{Z_{\mu t}} + \frac{\psi}{\delta_{kc}}(\frac{K_{ct}}{K_{ct-1}} - g_k) = E_t \beta \frac{\Lambda_{t+1}}{\Lambda_t}\{\alpha_c \frac{Y_{ct+1}}{X_{t+1}K_{ct}} + \frac{1-\delta_{kc}}{Z_{\mu t+1}} + \frac{\psi}{2\delta_{kc}\lambda_\mu^{t+1}}(\frac{K^2_{ct+1}}{K^2_{ct}} - g_k^2)\} \tag{D.20}$$

$$1 + \frac{\psi}{\delta_{kh}}(\frac{K_{ht}}{K_{ht-1}} - g_c) = E_t \beta \frac{\Lambda_{t+1}}{\Lambda_t}\{\alpha_h \frac{Q_{t+1}Y_{ht+1}}{K_{ht}} + (1-\delta_{kh}) + \frac{\psi}{2\delta_{kh}}(\frac{K^2_{ht+1}}{K^2_{ht}} - g_c^2)\} \tag{D.21}$$

$$I_{ct} = K_{ct} - (1-\delta_{kc})K_{ct-1} \tag{D.22}$$

$$I_{ht} = K_{ht} - (1-\delta_{kh})K_{ht-1} \tag{D.23}$$

$$Y_{ct} = K^{\alpha_c}_{ct-1}(Z_{ct}N^{\nu}_{ct}(N'_{ct})^{1-\nu})^{1-\alpha_c} \tag{D.24}$$

$$Y_{ht} = K^{\alpha_h}_{ht-1}L^{\alpha_l}_{t-1}(Z_{ht}N^{\nu}_{ht}(N'_{ht})^{1-\nu})^{1-\alpha_h-\alpha_l} \tag{D.25}$$

$$Y_{ht} = H_t + H'_t + (1-\delta_h)(H_{t-1} + H'_{t-1}) \tag{D.26}$$

$$Y_{ct} = C_t + C'_t + \frac{I_{ct}}{Z_{\mu t}} + I_{ht} + \Psi_{ct} + \Psi_{ht} \tag{D.27}$$

$$\Psi_{ct} = \frac{\psi}{2\delta_{kc}}(\frac{K_{ct}}{K_{ct-1}} - g_k)^2 \frac{K_{ct-1}}{\lambda_k^t} \tag{D.28}$$

$$\Psi_{ht} = \frac{\psi}{2\delta_{kh}}(\frac{K_{ht}}{K_{ht-1}} - g_c)^2 K_{ht-1} \tag{D.29}$$

$$NN_{ct} = N_{ct}^{\nu}(N'_{ct})^{1-\nu} \tag{D.30}$$

$$NN_{ht} = N_{ht}^{\nu}(N'_{ht})^{1-\nu} \tag{D.31}$$

$$W_{ct} = (1-\alpha_c)\nu \frac{Y_{ct}}{X_t N_{ct}} \tag{D.32}$$

$$W'_{ct} = (1-\alpha_c)(1-\nu)\frac{Y_{ct}}{X_t N'_{ct}} \tag{D.33}$$

$$W_{ht} = (1-\alpha_h - \alpha l)\nu \frac{Q_t Y_{ht}}{N_{ht}} \tag{D.34}$$

$$W'_{ht} = (1-\alpha_h - \alpha l)(1-\nu)\frac{Q_t Y_{ht}}{N'_{ht}} \tag{D.35}$$

$$P_{lt} = \beta \frac{\Lambda_{t+1}}{\Lambda_t}(\frac{\alpha_l Q_{t+1} Y_{ht+1}}{L_t} + P_{lt+1}) \tag{D.36}$$

$$GDP_t = Y_{ct} + Q_t Y_{ht} \tag{D.37}$$

$$R_t = R_{t-1}^{r_R}(\pi_t^{1+r_\pi}(\frac{GDP_t}{g_c GDP_{t-1}})^{r_Y})^{1-r_R} e_t \tag{D.38}$$

$$\hat{\pi}_t = \beta \hat{\pi_{t+1}} - \kappa \hat{X}_t + u_t \tag{D.39}$$

$$rr = \hat{R}_t - \hat{pi}_t \tag{D.40}$$

D Shocks

We have the same shocks as in the model with the housing market segmentation.

D Stationary System

The variables are transformed to be stationary:$y_{ct} = \frac{Y_{ct}}{g_c^t}, y_{ct}^f = \frac{Y_{ct}^f}{g_c^t}, y_{ht} = \frac{Y_{ht}}{g_h^t}, k_{ct} = \frac{K_{ct}}{g_k^t}, k_{ht} = \frac{K_{ht}}{g_c^t}, i_{ct} = \frac{I_{ct}}{g_k^t}, i_{ht} = \frac{I_{ht}}{g_c^t}, c_t = \frac{C_t}{g_c^t}, q_t = \frac{Q_t}{g_q^t}, z_{ct} = \frac{Z_{ct}}{\lambda_c^t}, z_{ht} = \frac{Z_{ht}}{\lambda_h^t}, \xi_t = \Xi_t g_c, \lambda_t = \Lambda_t g_c, \lambda_t' = \Lambda_t' g_c, g_k = g_c \lambda_\mu$

$N_{ct}, N_{ht}, N_{ct}', N_{ht}', NN_{ct}, NN_{ht}, R_t, \pi_t, X$ are stationary

$$\lambda_t = \frac{a_t}{c_t - \gamma \frac{c_{t-1}}{g_c}} - \frac{\beta \gamma a_{t+1}}{g_c c_{t+1} - \gamma c_t} \tag{D.41}$$

$$\lambda_t' = \frac{a_t}{c_t' - \gamma' \frac{c_{t-1}'}{g_c}} - \frac{\beta' \gamma' a_{t+1}}{g_c c_{t+1}' - \gamma c_t'} \tag{D.42}$$

$$a_t (n_{ct}^{1+\zeta} + n_{ht}^{1+\zeta})^{\frac{\eta-\zeta}{1+\zeta}} n_{ct}^{\zeta} = w_{ct} \lambda_t \tag{D.43}$$

$$a_t (n_{ct}^{1+\zeta} + n_{ht}^{1+\zeta})^{\frac{\eta-\zeta}{1+\zeta}} n_{ht}^{\zeta} = w_{ht} \lambda_t \tag{D.44}$$

$$a_t ((n_{ct}')^{1+\zeta'} + (n_{ht}')^{1+\zeta'})^{\frac{\eta'-\zeta'}{1+\zeta'}} (n_{ct}')^{1+\zeta'} = w_{ct}' \lambda_t' \tag{D.45}$$

$$a_t ((n_{ct}')^{1+\zeta'} + (n_{ht}')^{1+\zeta'})^{\frac{\eta'-\zeta'}{1+\zeta'}} (n_{ht}')^{1+\zeta'} = w_{ht}' \lambda_t' \tag{D.46}$$

$$q_t \lambda_t = j \frac{a_t}{h_t} + E_t \beta (1 - \delta_h) \frac{q_{t+1} \lambda_{t+1}}{g_h} \tag{D.47}$$

$$q_t \lambda_t' = j \frac{a_t}{h_t'} + E_t \{ \beta' (1 - \delta_h) \frac{q_{t+1} \lambda_{t+1}'}{g_h} + \frac{m(1 - \delta_h) \xi_t q_{t+1} \pi_{t+1}}{R_t} g_q \} \tag{D.48}$$

$$1 = E_t \beta \frac{\lambda_{t+1}}{\lambda_t g_c} \frac{r_t}{\pi_{t+1}} \tag{D.49}$$

$$\lambda_t' = E_t \beta' \frac{\lambda_{t+1}}{g_c} \frac{r_t}{\pi_{t+1}} + \xi_t \tag{D.50}$$

$$c_t' + q_t (h_t' - \frac{1 - \delta_h}{g_h} h_{t-1}') - b_t' = w_{ct}' n_{ct}' + w_{ht}' n_{ht}' - \frac{b_{t-1}' R_{t-1}}{\pi_t g_c} \tag{D.51}$$

$$b_t' = m(1 - \delta_h) \frac{h_t' q_{t+1} \pi_{t+1}}{R_t} g_q \tag{D.52}$$

$$\frac{1}{z_{\mu t}} + \frac{\psi}{\delta_{kc}} g_k(\frac{k_{ct}}{k_{ct-1}} - 1) = E_t \beta \frac{\lambda_{t+1}}{\lambda_t g_c} \{\alpha_c \frac{y_{ct+1}}{x_{t+1} k_{ct}} g_c + \frac{(1-\delta_{kc})}{\lambda_\mu z_{\mu t+1}} + \frac{\psi}{2\delta_{kc}} g_c g_k (\frac{k_{ct+1}^2}{k_{ct}^2} - 1)\} \quad (D.53)$$

$$1 + \frac{\psi}{\delta_{kh}}(\frac{k_{ht}}{k_{ht-1}} - 1) = E_t \beta \frac{\lambda_{t+1}}{\lambda_t g_c} \{\alpha_h \frac{q_{t+1} y_{ht+1}}{k_{ht}} g_c + (1-\delta_{kh}) + \frac{\psi}{2\delta_{kh}} g_c^2 (\frac{k_{ht+1}^2}{k_{ht}^2} - 1)\} \quad (D.54)$$

$$i_{ct} g_k = k_{ct} g_k - (1-\delta_{kc}) k_{ct-1} \quad (D.55)$$

$$i_{ht} g_c = k_{ht} g_c - (1-\delta_{kh}) k_{ht-1} \quad (D.56)$$

$$y_{ct} = k_{ct-1}^{\alpha_c} (n_{ct}^{\nu} (n'_{ct})^{1-\nu})^{1-\alpha_c} (\frac{1}{g_k})^{\alpha_c} \quad (D.57)$$

$$y_{ht} = k_{ht-1}^{\alpha_h} (n_{ht}^{\nu} (n'_{ht})^{1-\nu})^{1-\alpha_h-\alpha_l} (\frac{1}{g_c})^{\alpha_h} \quad (D.58)$$

$$y_{ht} g_h = h_t g_h + h'_t g_h - (1-\delta_h)(h_{t-1} + h'_{t-1}) \quad (D.59)$$

$$y_{ct} = c_t + c'_t + \frac{i_{ct}}{z_{\mu t}} + i_{ht} + \psi_{ct} + \psi_{ht} \quad (D.60)$$

$$\psi_{ct} = \frac{\psi}{2\delta_{kc}} g_k (\frac{k_{ct}}{k_{ct-1}} - 1)^2 k_{ct-1} \quad (D.61)$$

$$\psi_{ht} = \frac{\psi}{2\delta_{kh}} g_c (\frac{k_{ht}}{k_{ht-1}} - 1)^2 k_{ht-1} \quad (D.62)$$

$$nn_{ct} = n_{ct}^{\nu} (n'_{ct})^{1-\nu} \quad (D.63)$$

$$nn_{ht} = n_{ht}^{\nu} (n'_{ht})^{1-\nu} \quad (D.64)$$

$$w_{ct} = (1-\alpha_c)\nu \frac{y_{ct}}{x_t n_{ct}} \quad (D.65)$$

$$w'_{ct} = (1-\alpha_c)(1-\nu) \frac{y_{ct}}{x_t n'_{ct}} \quad (D.66)$$

$$w_{ht} = (1-\alpha_h - \alpha l)\nu \frac{q_t y_{ht}}{n_{ht}} \quad (D.67)$$

$$w'_{ht} = (1 - \alpha_h - \alpha l)(1 - \nu)\frac{q_t y_{ht}}{n'_{ht}} \tag{D.68}$$

$$p_{lt} = \beta\frac{\lambda_{t+1}}{\lambda_t g_c}(\frac{\alpha_l q_{t+1} y_{ht+1}}{L_t} + p_{lt+1}) \tag{D.69}$$

$$gdp_t = y_{ct} + q_t y_{ht} \tag{D.70}$$

$$R_t = \pi_t^{1+r_\pi}(\frac{gdp_t}{gdp_{t-1}g_c})^{r_Y} e_t \tag{D.71}$$

$$\hat{\pi}_t = \beta\hat{\pi_{t+1}} - \kappa\hat{X}_t + u_t \tag{D.72}$$

$$rr_t = \hat{R}_t - \hat{\pi_{t+1}} \tag{D.73}$$

D Steady State

At steady state, we have $\pi_t = 1, \beta = \frac{g_c}{r}, x = \frac{\varepsilon}{\varepsilon - 1}, \psi_{ct} = \psi_c = 0, \psi_{ht} = \psi_h = 0, \xi = \lambda'(1 - \frac{\beta'}{\beta})$.

$$d_5 = \frac{g_c - \gamma}{g_c - \beta\gamma} \tag{D.74}$$

$$d_6 = \frac{g_c - \gamma'}{g_c - \beta'\gamma'} \tag{D.75}$$

$$\lambda = \frac{1}{cd_5} \tag{D.76}$$

$$\lambda' = \frac{1}{c'd_6} \tag{D.77}$$

$$A_1 = \frac{\alpha_c\beta}{1 - \frac{\beta}{g_k}(1 - \delta_{kc})}\frac{1}{x} \tag{D.78}$$

$$A_2 = 1 - \frac{\beta(1 - \delta_h)}{g_h} \tag{D.79}$$

$$A_3 = 1 - \frac{\beta'(1 - \delta_h)}{g_h} - \frac{m(1 - \delta_h)(\beta - \beta')}{g_h} \tag{D.80}$$

$$A_4 = \frac{\alpha_h \beta}{1 - \frac{\beta}{g_c}(1 - \delta_{kh})} \tag{D.81}$$

$$A_5 = \frac{g_h - (1 - \delta_h)}{g_h}, \tag{D.82}$$

$$A_6 = 1 + [A_5 - \frac{m(1 - \delta_h)(\beta - 1)}{g_h} - (1 - \alpha_h - \alpha_l)(1 - \nu)A_5]\frac{j}{A_3}d_6 \tag{D.83}$$

$$A_7 = (1 - \alpha_h - \alpha_l)(1 - \nu)A_5\frac{j}{A_2}d_5 \tag{D.84}$$

$$A_8 = \frac{(1 - \alpha_c)(1 - v)}{x} \tag{D.85}$$

$$d_1 = \frac{g_k - (1 - \delta_{kc})}{g_k} \tag{D.86}$$

$$d_2 = \frac{g_c - (1 - \delta_{kh})}{g_c} \tag{D.87}$$

$$A_9 = \frac{1 - d_1 A_1}{1 + d_2 A_4 A_5 \frac{j}{A_2} d_5} \tag{D.88}$$

$$A_{10} = \frac{1 + d_2 A_4 A_5 d_6 \frac{j}{A_3}}{1 + d_2 A_4 A_5 \frac{j}{A_2} d_5} \tag{D.89}$$

$$A_{11} = A_9 - A_{10}A_{12} \tag{D.90}$$

$$A_{12} = \frac{A_7 A_9 + A_8}{A_6 + A_7 A_{10}} \tag{D.91}$$

$$A_{13} = A_4 A_5 j(\frac{d_5 A_{11}}{A_2} + \frac{d_6 A_{12}}{A_3}) \tag{D.92}$$

$$A_{14} = \frac{A_3}{A_2}\frac{d_5}{d_6}\frac{A_{11}}{A_{12}} \tag{D.93}$$

$$\phi = \frac{1 - \alpha_c}{1 - \alpha_h - \alpha_l} \frac{1}{x A_5 j(\frac{d_5 A_{11}}{A_2} + \frac{d_6 A_{12}}{A_3})} \tag{D.94}$$

$$q = d_6 \frac{j}{1 - \frac{\beta'(1-\delta_h)}{g_h} - \frac{m(1-\delta_h)(\beta-\beta')}{g_h}} \frac{c'}{h'} = \frac{j}{A_3}\frac{c'}{h'}d_6 \tag{D.95}$$

$$b' = \frac{m(1-\delta_h)qh'}{r} g_q \tag{D.96}$$

$$k_c = \frac{\alpha_c \beta}{1 - \frac{\beta}{g_k}(1-\delta_{kc})} \frac{1}{x} y_c = A_1 y_c \tag{D.97}$$

$$k_h = \frac{\alpha_h \beta}{1 - \frac{\beta}{g_c}(1-\delta_{kh})} q y_h = A_4 q y_h \tag{D.98}$$

$$y_h = \frac{g_h - (1-\delta_h)}{g_h} h' = A_5 h' \tag{D.99}$$

$$i_c = \frac{g_k - (1-\delta_{kc})}{g_k} k_c = d_1 k_c \tag{D.100}$$

$$i_h = \frac{g_c - (1-\delta_{kh})}{g_c} k_h = d_2 k_h \tag{D.101}$$

With $qh' = A_5 j(\frac{d_5 c}{A_2} + \frac{d_6 c'}{A_3})$

$$k_h = A_4 q y_h = A_4 A_5 q h' = A_4 A_5 j(\frac{d_5 c}{A_2} + \frac{d_6 c'}{A_3}) \tag{D.102}$$

from equation $c + c' + i_c + i_h = y_c$ we have

$$c + d_1 A_1 y_c + d_2 A_4 A_5 j(\frac{d_5 c}{A_2} + \frac{d_6 c'}{A_3}) = y_c \tag{D.103}$$

The budget constraint of impatient households:

$$c' + q(h' - \frac{1-\delta_h}{g_h} h') = b' - \frac{b'R}{\pi z_c} + (1-\alpha_h-\alpha_l)(1-\nu) q y_h + (1-\alpha_c)(1-\nu)\frac{y_c}{x} \tag{D.104}$$

$$\begin{aligned}(1 + [A_5 - \frac{m(1-\delta_h)(\beta-1)}{g_h} - (1-\alpha_h-\alpha_l)(1-\nu)A_5]\frac{j d_6}{A_3})c' = \frac{(1-\alpha_c)(1-v)}{x} y_c \\ + (1-\alpha_h-\alpha_l)(1-\nu)A_5 \frac{j}{A_2} d_5 c\end{aligned} \tag{D.105}$$

We could rewrite the equation above as:

$$A_6 c' = A_8 y_c + A_7 c \tag{D.106}$$

then we could solve for c and c’

$$c' = \frac{A_7 A_9 + A_8}{A_7 A_{10} + A_6} = A_{12} y_c \tag{D.107}$$

$$c = (A_9 - A_{10} A_{12}) y_c = A_{11} y_c \tag{D.108}$$

$$k_h = A_4 A_5 j(\frac{d_5 c}{A_2} + \frac{d_6 c'}{A_3}) = A_{13} y_c \tag{D.109}$$

$$i_c = d_1 A_1 y_c = A_{14} y_c \tag{D.110}$$

$$i_h = d_1 A_{13} y_c = A_{15} y_c \tag{D.111}$$

$$(n_c^{1+\zeta} + n_h^{1+\zeta})^{\frac{\eta-\zeta}{1+\zeta}} n_c^{1+\zeta} c = (1-\alpha_c)\nu \frac{y_c}{x} \lambda \tag{D.112}$$

$$(n_c^{1+\zeta} + n_h^{1+\zeta})^{\frac{\eta-\zeta}{1+\zeta}} n_h^{1+\zeta} c = (1-\alpha_h-\alpha_l) q y_h \lambda \tag{D.113}$$

The ratio of the two above equation:

$$(\frac{n_c}{n_h})^{1+\zeta} = \frac{1-\alpha_c}{1-\alpha_h-\alpha_l} \frac{1}{x A_5 j(\frac{d_5 c}{A_2} + \frac{d_6 c'}{A_3})} = \phi \tag{D.114}$$

we get $n_c^{1+\zeta} = \phi n_h^{1+\zeta}$ and put it in equation (162)

$$(1+\phi)^{\frac{\eta-\zeta}{1+\zeta}} n_h^{\eta-\zeta} \phi n_h^{1+\zeta} A_{11} d_5 y_c = \frac{(1-\alpha_c)\nu}{x} \tag{D.115}$$

we could get n_h

$$n_h = (\frac{(1-\alpha_c)\nu}{x(1+\phi)^{\frac{\eta-\zeta}{1+\zeta}} \phi d_5 A_{11}})^{\frac{1}{1+\eta}} \tag{D.116}$$

$$((n'_c)^{1+\zeta'} + (n'_h)^{1+\zeta'})^{\frac{\eta'-\zeta'}{1+\zeta'}} (n'_c)^{1+\zeta'} c' = (1-\alpha_c)(1-\nu) \frac{y_c}{x} \lambda' \tag{D.117}$$

$$((n'_c)^{1+\zeta'} + (n'_h)^{1+\zeta'})^{\frac{\eta'-\zeta'}{1+\zeta'}} (n'_h)^{1+\zeta'} c' = (1-\alpha_h-\alpha_l) q y_h \lambda' \tag{D.118}$$

The ratio of the two above equation:

$$(\frac{n'_c}{n'_h})^{1+\zeta'} = \frac{1-\alpha_c}{1-\alpha_h-\alpha_l} \frac{1}{x A_5 j(\frac{d_5 c}{A_2} + \frac{d_6 c'}{A_3})} = \phi \tag{D.119}$$

we get $(n'_c)^{1+\zeta} = \phi (n'_h)^{1+\zeta}$ and put it in equation (167)

$$(1+\phi)^{\frac{\eta'-\zeta'}{1+\zeta'}} n_h^{\eta'-\zeta'} \phi n_h^{1+\eta} A_{12} d_6 y_c = \frac{(1-\alpha_c)(1-\nu)}{x} \tag{D.120}$$

we could get n_h'

$$n_h' = \left(\frac{(1-\alpha_c)(1-\nu)}{x(1+\phi)^{\frac{\eta'-\zeta'}{1+\zeta'}} \phi d_6 A_{12}}\right)^{\frac{1}{1+\eta'}} \tag{D.121}$$

The we get n_c' by $n_c' = \phi^{\frac{1}{1+\zeta'}} n_h'$
Then we could get levels:

$$y_c = \left(\frac{A_1}{g_k}\right)^{\frac{\alpha_c}{1-\alpha_c}} n_c^{\nu} (n_c')^{1-\nu} \tag{D.122}$$

$$y_h = (A_{13} y_c)^{\alpha_h} (n_h^{\nu} (n_h')^{1-\nu})^{1-\alpha_h} g_c^{-\alpha_h} \tag{D.123}$$

$$\frac{h}{h'} = \frac{A_3}{A_2} \frac{d_5}{d_6} \frac{A_{11}}{A_{12}} = A_{14} \tag{D.124}$$

$$h' = \frac{y_h}{A_5(1+A_{14})} \tag{D.125}$$

$$q = \frac{j}{A_3} \frac{c'}{h'} d_6 \tag{D.126}$$