Risk Measurement and Management in the Global Markets with the Tempered Stable Distributions

A Dissertation Presented

by

Tetsuo Kurosaki

to

The Graduate School

in Partial Fulfillment of the

Requirements

for the Degree of

Doctor of Philosophy

in

Applied Mathematics and Statistics

(Quantitative Finance)

Stony Brook University

December 2013

Stony Brook University

The Graduate School

Tetsuo Kurosaki

We, the dissertation committee for the above candidate for the

Doctor of Philosophy degree, hereby recommend

acceptance of this dissertation.

Svetlozar T. Rachev
Professor, College of Business, Applied Mathematics and Statistics (Advisor)

Xiaolin Li
Professor, Applied Mathematics and Statistics (Chair)

Haipeng Xing
Associate Professor, Applied Mathematics and Statistics

Noah Smith
Assistant Professor, College of Business (External Member)

This dissertation is accepted by the Graduate School

Charles Taber
Dean of the Graduate School

Abstract of the Dissertation

Risk Measurement and Management in the Global Markets with the Tempered Stable Distributions

by

Tetsuo Kurosaki

Doctor of Philosophy

in

Applied Mathematics and Statistics

(Quantitative Finance)

Stony Brook University

2013

Risk measurement and management are now seen in a global context. Accordingly, several new concepts and techniques have been proposed in the field of quantitative finance to address new types of risk such as systemic or systematic risk. In this dissertation, we present three topics related to risk measurement and management in global markets. We particularly focus on applications of tempered stable distributions to asset returns for the purpose of improving risk measurement and management.

In the first part, we measure the systematic risk in global banking stock markets by using stocks of global systemically important financial institutions (G-SIFIs). Because G-SIFIs are identified by the financial regulator, measuring the G-SIFI risks is critical for assessment of the

stability of the global financial system. For time series analysis, we adopt an autoregressive moving average (ARMA) generalized autoregressive conditional heteroscedasticity (GARCH) model with the multivariate normal tempered stable distributed innovations and demonstrate that it is a more realistic model to use with G-SIFI stocks. For measuring the risk, we take different approaches including CoVaR and its extension to average value at risk (AVaR), which we refer to as CoAVaR. We discuss the relationship among different risk measures.

In the second part, we propose mean–CoAVaR portfolio optimization to mitigate the potential loss caused by systematic risk. This is a strategy to minimize the portfolio's CoAVaR with a given expected return. Through empirical studies of portfolios comprising G-SIFI stocks, we confirm that the mean–CoAVaR strategy is effective during a financial crisis.

In the third part, we examine a time series of global currency exchange rates by using currencies circulating in the member countries of the Organization for Economic Co-operation and Development (OECD). We propose a better model to describe the dynamics of exchange rates, by comparing GARCH and Markov‒switching models through both in-sample and out-of-sample tests. Also, the multivariate modeling for OECD currency exchange rates is discussed. We conclude that the tempered stable GARCH model is recommendable, especially for risk management purposes.

Table of Contents

Chapter 1 Introduction 1

1.1 Risk Measurement and Management in the Global Markets 1

1.1.1 Global Market Conditions and Regulations 1

1.1.2 Time Series Model, Risk Measure, and Risk Management 4

1.2 Overview of Dissertation 10

Chapter 2 Systematic Risk Measurement in the Global Banking Stock Market with Time Series Analysis and CoVaR 13

2.1 Introduction 13

2.2 Methodology 16

2.2.1 ARMA–GARCH–MNTS model 16

2.2.2 Systematic Risk Measures 18

2.2.3 Scenario Simulation 23

2.3 Data 24

2.4 Estimation Results 25

2.5 Concluding Remarks 37

Chapter 3 Mean–CoAVaR Optimization for Global Banking Portfolios 39

3.1 Introduction 39

3.2 Mean–CoAVaR Optimization 41

3.3 Empirical Study 43

3.4 Concluding Remarks 46

Chapter 4 Tempered Stable GARCH vs. Markov Switching Approaches for OECD Currency Exchange Rates 50

4.1 Introduction 50

4.2 Methodology 54

4.2.1. AR(1)–GARCH(1,1) Model 54

4.2.2. MS–AR(1) Model 55

4.2.3. VaR Estimation 57

4.3 Data 59

4.4 Empirical Results of Model Performance 60

4.4.1. In-Sample Test 62

4.4.2. Out-of-Sample Test 65

4.5 Multivariate Extension 68

4.6 GARCH Residuals by Regimes 70

4.7 Concluding Remarks 74

Chapter 5 Conclusion 98

Bibliography 103

Appendix A List of 29 G-SIFIs as of November 2011 108

Appendix B Multivariate Normal Tempered Stable Distribution 109

Appendix C VaR and AVaR in Infinitely Divisible Distribution 111

Appendix D Markov Switching Model 112

Appendix E Results of In-Sample Tests on the Basis of Daily Datasets of 18 OECD Currency Exchange Rate Returns (3129 Observations) 114

List of Figures

Figure 1.1: Conditions of Global Markets 3

Figure 2.1: Time Series of the VaR and AVaR of the Equally Weighted Portfolio 28

Figure 2.2: Time Series of JPNM 31

Figure 2.3: Time Series of AVaR, ΔCoAVaR, and CPNM by Region 35

Figure 2.4: Cross-Sectional Linkage among AVaR, ΔCoAVaR, and CPNM 37

Figure 3.1: Cumulative Return of the Portfolios Optimized by Different Strategies (Small Group) 48

Figure 3.2: Cumulative Return of the Portfolios Optimized by Different Strategies (Europe Group) 48

Figure 3.3: Difference in the Cumulative Return between the Mean–CoAVaR and Mean–AVaR Portfolios (Small Group) 49

Figure 3.4: Difference in the Cumulative Return between the Mean–CoAVaR and Mean–AVaR Portfolios (Europe Group) 49

Figure 4.1: Smoothed Probability of a Turbulent Regime in the MS−AR(1) Model 72

List of Tables

Table 2.1: Number of Rejections of Distributional Assumptions for Each Stock on the Basis of the KS test (Out of 2011 Estimations) ... 28

Table 2.2: Number of VaR Violations (Out of 2011 Estimations) ... 29

Table 2.3: p-values of the Christoffersen Test for VaR ... 30

Table 2.4: Rate of CoVaR to VaR Violations ... 33

Table 2.5: p-values of the Christoffersen Test for CoVaR ... 34

Table 2.6: Iterative Single Regression Analysis for 2011 Cross-Sectional Datasets among AVaR, ΔCoAVaR, and CPNM ... 37

Table 3.1: Portfolio Performance of Three Mean–Risk Optimizations (by Size) ... 47

Table 3.2: Portfolio Performance of Three Mean–Risk Optimizations (by Region) ... 47

Table 4.1: AIC and BIC of AR(1) and ARMA(1,1) Models, and Engle's ARCH Test for AR(1) Residuals (625 Weekly Observations) ... 76

Table 4.2: Estimated Parameters of AGNormal Model (625 Weekly Observations) ... 77

Table 4.3: Estimated Parameters of AGFT Model (625 Weekly Observations) ... 78

Table 4.4: Estimated Fat-tailness Parameters of AGFT Standardized Residuals (625 Weekly Observations) ... 79

Table 4.5: KS, AD, and AD^2 Statistics of AR(1)−GARCH(1,1) Standardized Residuals against the Proposed Distributions ... 80

Table 4.6: MS–AR(1) Estimated Parameters (625 Weekly Observations) ... 81

Table 4.7: Log-likelihood ... 83

Table 4.8: p-values of the LB Test for Standardized Residuals 84

Table 4.9: p-values of the LB Test for Squared Standardized Residuals 85

Table 4.10: Variance Forecast Performance (One-week Forecast) 86

Table 4.11: Variance Forecast Performance (Four-week Forecast) 87

Table 4.12: Variance Forecast Performance (Eight-week Forecast) 88

Table 4.13: Numbers of Violations and p-values of the Christoffersen Test for 99% VaR (One-week Forecast) 89

Table 4.14: Numbers of Violations and p-values of the Christoffersen Test for 99% VaR (Four-week Forecast) 90

Table 4.15: Numbers of Violations and p-values of the Christoffersen Test for 99% VaR (Eight-week Forecast) 91

Table 4.16: p-values of the Berkowitz Test for Independence and Tail (One-week Forecast) 92

Table 4.17: p-values of the Berkowitz Test for Independence and Tail (Four-week Forecast) 93

Table 4.18: p-values of the Berkowitz Test for Independence and Tail (Eight-week Forecast)... 94

Table 4.19: Bivariate Tail Dependencies between Exchange Rate Returns......................... 95

Table 4.20: p-values of the KS Test of AGFT Standardized Residuals against the NTS Distribution by Regimes 97

Table A.1: List of 29 G-SIFIs as of November 2011 108

Table E.1: KS, AD, and AD^2 Statistics of AR(1)−GARCH(1,1) Standardized Residuals against the Proposed Distributions (Daily Data) 114

Table E.2: Log-likelihood (Daily Data) 115

Table E.3: p-values of the LB Test for Standardized Residuals (Daily Data) 116

Table E.4: p-values of the LB Test for Squared Standardized Residuals (Daily Data) 117

Acknowledgments

I would like to express my deepest gratitude to my advisor, Professor Svetlozar T. Rachev, for his continuous encouragement, guidance, and valuable comments on the manuscript. I am also grateful to my dissertation committee members, Professor Xiaolin Li, Professor Haipeng Xing, and Professor Noah Smith, for their fruitful discussions and fair evaluation of this dissertation. Without their support, this dissertation could not have been completed.

I would like to acknowledge all faculty members and graduate students at Stony Brook, whose academic expertise and enthusiasm for research have always inspired me. Special thanks must be given to Professor Young Shin Kim for his instructions on tempered stable distributions and his collaborative work on related topics; Professor Jiaoqiao Hu for his comments on the manuscript; Mr. Abhinav Anand and Mr. Tiantian Li for their discussions on related topics; and Japanese peers at the Department of Applied Mathematics and Statistics, Mr. Naoshi Tsuchida, Mr. Shingo Omori, and Mr. Tomoaki Sakamoto for general assistance.

Also, I am grateful to the following persons from outside the campus. Professor Frank J. Fabozzi at EDHEC Business School gave valuable comments on the initial manuscript. Dr. Ivan Mitov and Mr. Daniel Dimitrov at FinAnalytica Inc. instructed me on how to use their company's risk management software, which was critical to my research. Mr. Yuji Sakurai at the University of California, Los Angeles, provided plenty of useful literature; if he had not informed me about the literature on systemic risk, I would not have developed a keen interest in this research field.

In addition, I would like to thank the Bank of Japan for their invaluable sponsorship. Thanks to their financial support, I was able to concentrate on my Ph.D. research at Stony Brook without any financial concerns. Special appreciation has to be expressed to my colleagues in the financial system and bank examination department, who encouraged me to apply for the Bank's scholarship and to go to study abroad.

Finally, I would like to express my gratitude to my family members, especially Kazuo, Kazuko, and Naoko, for their heartfelt support and encouragement.

The text of this dissertation in part is a reprint of the materials as it appears in the journal of Investment Management and Financial Innovations. The co-author listed in the publication directed and supervised the research that forms the basis for this dissertation. The permission to reprint has been granted from the publishing company "Business Perspectives."

Vita

Tetsuo Kurosaki

Fields of Study

Quantitative Finance

Education

- Doctor of Philosophy in Applied Mathematics and Statistics (Quantitative Finance Track), Stony Brook University, New York, U.S.A., August 2011 − December 2013.
- Master of Science in Applied Mathematics and Statistics (Quantitative Finance Track), Stony Brook University, New York, U.S.A., August 2011 − May 2013.
- Master of Science in Physics, University of Tokyo, Tokyo, Japan, April 2005 − March 2007.
- Bachelor of Science in Physics, University of Tokyo, Tokyo, Japan, April 2001 − March 2005.

Working Experience

- Bank of Japan (Full-time, 40 hours per week), Tokyo, Japan, April 2007 − July 2011.

Publications in Refereed Journals

1. Kurosaki, T. and Y. S. Kim, "Mean-CoAVaR optimization for global banking portfolios," *Journal of Investment Management and Financial Innovations*, vol. 10, Issue 2 (2013), 15-20.
2. Kurosaki, T. and Y. S. Kim, "Systematic risk measurement in the global banking stock market with time series analysis and CoVaR," *Journal of Investment Management and Financial Innovations*, vol. 10, Issue 1 (2013), 184-196.
3. Kurosaki, T., "Direct definition of a ternary infinite square-free sequence," *Information Processing Letters*, vol. 106 (2008), 175-179.
4. Kurosaki, T. and M. Wadati, "Matter-Wave Bright Solitons with a Finite Background in Spinor Bose-Einstein Condensates," *Journal of the Physical Society of Japan*, vol. 76 (2007), 084002-1~9.

Chapter 1 Introduction

1.1 Risk Measurement and Management in the Global Markets

Risk measurement and management are critical in modern financial industries from the perspectives of both regulators and investors. In September 2008, the bankruptcy of Lehman Brothers, the fourth largest investment banking firm in the United States (U.S.) at that time, caused a massive chain of defaults and greatly worsened the conditions of all global financial market sectors including stock, bond, currency, credit markets, and so on. The financial crisis triggered by the failure of Lehman Brothers is now referred to as the "Lehman Shock." The recent debt crisis in Greece also had huge and adverse impacts on the global financial markets. In April 2010, because of excessive debt, Standard and Poor's (S&P) downgraded the Greek sovereign credit rating from BBB+ (an investment grade category) to BB+ (a non-investment grade category). A spillover effect into the sovereign debt markets of peripheral countries followed. Because financial institutions typically have large positions in sovereign bonds, which have been regarded as free of credit risk[1], there were great concerns in the market that a systemic downturn would occur following the European sovereign debt crisis. These two financial crises have significantly stimulated the risk measurement and management in the global framework.

1.1.1 Global Market Conditions and Regulations

We show an overview of the recent global market conditions in Figure 1.1. The global stock index dramatically fell after the failure of Lehman Brothers. The yields of government bonds went up right before the failure of Lehman Brothers and rapidly went down after it because of

[1] In Basel II's standard method, home-currency government bonds have zero risk weight regardless of the external credit ratings.

the “zero interest rate” policy of central banks. The yields climbed up again after the downgrading of the Greek sovereign credit rating because concerns were directed toward sovereign creditworthiness. Regarding the foreign currency exchange rates, the Euro (EUR), British Pound Sterling (GBP), and Canadian Dollar (CAD) depreciated against the United States Dollar (USD) during the Lehman shock, while the Japanese Yen (JPY) appreciated because the adverse impact in Japan was relatively moderate compared with other regions such as the Eurozone and the United Kingdom (U.K.). The spread between the London inter-bank offered rate (LIBOR) and the overnight indexed swap (OIS) is regarded as an indicator of liquidity risk in inter-banking trading. The LIBOR-OIS spread was greatly widening during the Lehman shock. The Greek debt crisis also produced an increased risk of liquidity, especially, in the Eurozone. The credit default swap (CDS) spread describes creditworthiness. In the lowest row of Figure 1.1, the CDS spreads of both banking and sovereign sectors are shown. Judging from the movements of CDS spreads, the adverse impacts of the Greek debt crisis are more serious than those of the Lehman shock. In particular, concerns about the sovereign creditworthiness of Southern Europe, including Italy and Spain, have been mounting like never before. As is seen above, we can demonstrate that there was a wide range of adverse impacts of the recent financial turmoil on global markets. In addition, we find strong interconnectedness among market data in different regions and countries.

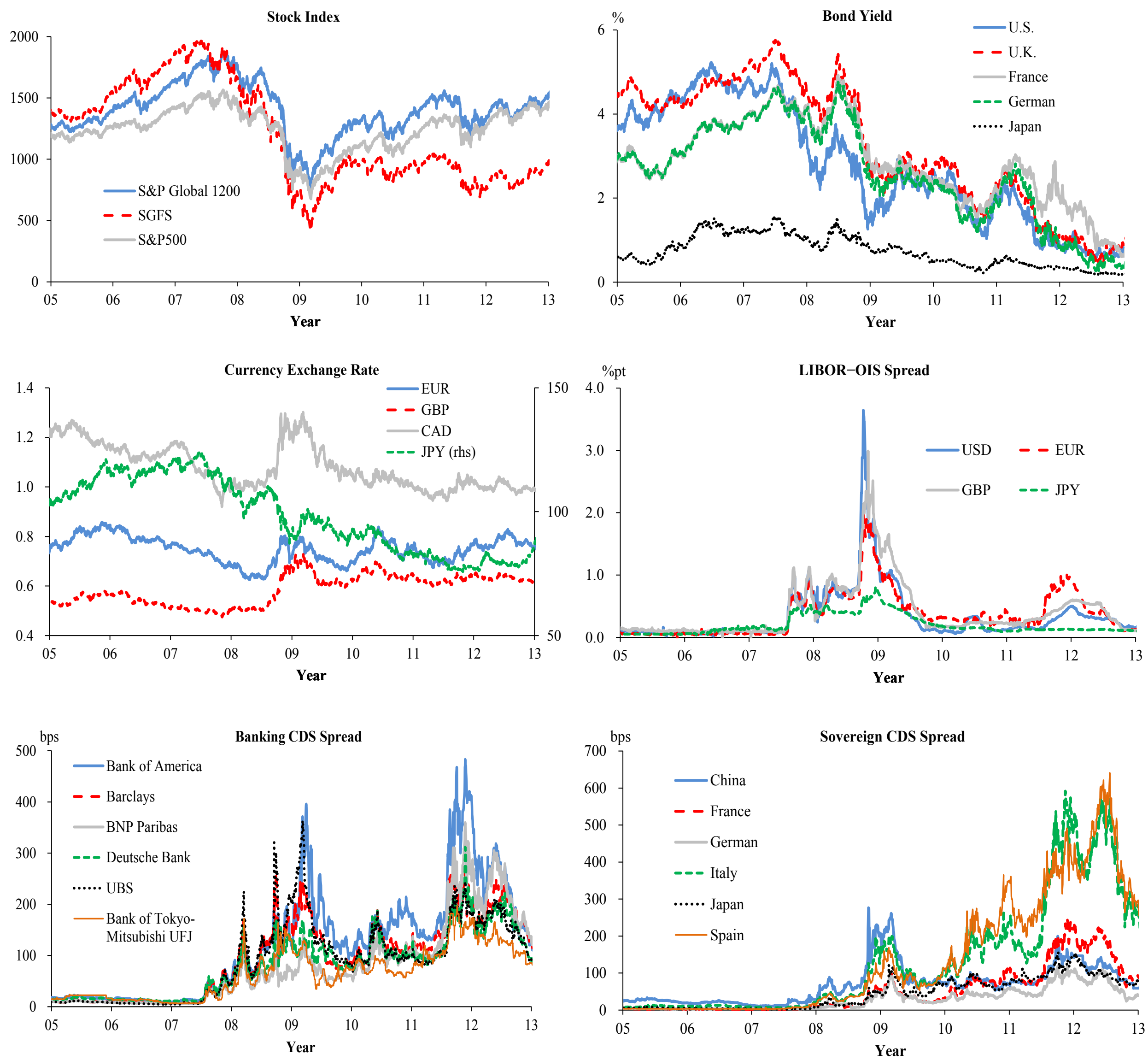

Figure 1.1: Conditions of Global Markets

Note: Regarding stock indexes, SGFS is a financial subsector index of S&P Global 1200. Bond yields are generic indexes of individual government bonds with the five-year maturity. Currency exchange rates are the spot rates for a unit USD. Regarding LIBOR-OIS spreads, the maturity of LIBOR is three months. Regarding banking and sovereign CDS spreads, the maturity of the CDS contract is five years, and it covers senior debts. The U.S. and U.K. sovereign CDS spreads are not shown because of insufficient data availability. All data are downloaded from Bloomberg.

After the Lehman shock, the Basel Committee on Banking Supervision (BCBS) began to formulate a new regulatory framework for international banks known as Basel III in order to prevent such a devastating financial crisis from recurring. The basic framework for Basel III, first published at the end of 2010 and revised in 2011 (Basel Committee on Banking Supervision, 2011a), is still a work in progress at the time of this writing. However, as is clear in the current agreement of Basel III, it is definitely the new mission of regulators to address the so-called "systemic risk," which is the risk of meltdown of an entire financial system. To this end, the BCBS (2011b) has proposed a methodology for measuring how important each financial institution is on the global financial system. More recently, the Financial Stability Board published an initial list of 29 global systemically important financial institutions (G-SIFIs) identified by using the BCBS methodology (Financial Stability Board, 2011). Given the growing importance of systemic risk, when managing their portfolios, investors need to be conscious of the potential huge losses that can be caused by systemic risks or, more specifically, systematic risks[2]. Thus, the risk measurement and management is now a worldwide concept.

1.1.2 Time Series Model, Risk Measure, and Risk Management

Risk measurement and management in the financial industries basically has the following procedures. First, the future scenario that is likely to happen in the market is predicted on the basis of historical data through time series models. Second, by using the predicted future, the risk is measured. Third, referring to the risk measure, the risk is managed. A comprehensive review of traditional risk measurement and management techniques is, for instance, given in the

[2] See the section 2 of Hansen (2013) for a discussion about the distinction between the two terms; *systemic* and *systematic* risk. He explains that *systematic* risk is macroeconomic or aggregate risk that cannot be avoided through diversification, whereas the formal definition of *systemic* risk is much less clear. He also points out three possible notions of systemic risk: "a modern-day counterpart to a bank run triggered by liquidity concerns," "the vulnerability of a financial network in which adverse consequences of internal shocks can spread and even magnify within the network," and "the potential insolvency of a major player in or component of the financial system." We do not dwell further on the issue of this terminological distinction.

textbook of McNeil et al. (2005). With the traditional techniques as the point of departure, we need to develop the time series model, measures to quantify the risk, and management of risk to enhance the risk measurement and management in the globally systemic context. Below, we review the recent developments in the time series model, risk measure, and risk management, and state basic ideas on what we are going to do in this dissertation.

Time Series Model

The generalized autoregressive conditional heteroscedasticity (GARCH) model introduced by Bollerslev (1986) is now standard in the financial industries. A GARCH model is a direct extension of an ARCH model (Engle, 1982). Numerous empirical studies have revealed that a GARCH model well explains volatility clustering phenomena, which asset returns typically show. In some cases, an autoregressive (AR) and/or a moving average (MA) process are embedded in a GARCH model for better model performance.

Since the introduction of the original GARCH model, many multivariate extensions have been proposed[3]. The multivariate extension is essential because the global financial markets are closely interconnected and we need to incorporate such interconnectedness into the time series model. A copula GARCH model is one of the most flexible multivariate extensions[4]. It has two steps in its construction. First, GARCH models are fitted to each marginal time series independently under various distributional assumptions, and subsequently a copula is fitted to all the residuals of univariate GARCH models. The resulting joint distribution is expected to account for the dependent structure of asset returns. Traditionally, the residuals of the GARCH model are assumed to obey the normal distribution for simplicity. However, it has been revealed that the distribution of asset returns typically has fat-tailness and skewness, which the normal

[3] See, for instance, a survey by Bauwens et al. (2006).

[4] See Sun et al. (2008) for more information about a copula GARCH model.

distribution fails to describe. Therefore, non-normal distributions are explored to provide a better description of asset returns[5].

Recently, Kim et al. (2011) proposed replacing the normal distribution with the tempered stable distribution in the assumptions for the residuals of the GARCH model, by which they obtained a much more realistic model for S&P 500 compared with the normal and even student t distributions. The stable and tempered stable distributions are known to be capable of describing fat-tailness and skewness. The stable distribution[6] was applied to asset returns for the first time by Mandelbrot (1963a, 1963b), supplemented by Fama (1963). The main difficulty in the application of the stable distribution to finance is that it has an infinite variance. To overcome this drawback, the tempered stable distribution has been developed. It is derived by introducing the so-called tempering function to the stable distribution. According to the tempering function, Rachev et al. (2011) classify the tempered stable distributions into several variants, such as classical tempered stable (CTS)[7], modified tempered stable (MTS), normal tempered stable (NTS) distributions, and so on. See Rachev et al. (2011) for more information on the stable and tempered stable distributions.

More recently, multivariate extension of the NTS distribution (Kim et al., 2012) and NTS copula (Kim and Volkmann, 2013) have been proposed. These new multivariate models have been clearly shown to explain the interdependencies among asset returns better than the multivariate normal model. Given the aforementioned developments, we now can construct the GARCH model with the multivariate NTS (MNTS) distributed residuals, following a copula

---

[5] See Rachev et al. (2005) for a description and history of fat-tailed and skewed distributions for asset returns.

[6] It is also referred to as α-stable distribution, Lévy distribution, and stable Paretian distribution. Paul Pierre Lévy and Vilfredo Pareto are pioneers in the related fields.

[7] The CTS distribution has been referred to using various names in the literature: the truncated Lévy flights by Koponen (1995), the KoBoL distribution by Boyarchenko and Levendorskiĭ (2000), and the CGMY distribution by Carr et al. (2002). The name of the CTS distribution is first given in Kim et al. (2008).

GARCH framework. The GARCH model with the MNTS distributed residuals is expected to make a more realistic forecast of *simultaneous* asset returns in global markets. In this dissertation, we utilize this new model for the risk analysis.

An alternative time series model for the volatility clustering phenomena is the Markov regime−switching model, or simply, the Markov−switching (MS) model, proposed by Hamilton (1989, 1994). The MS model assumes unobservable regimes whose stochastic dynamics are described by the Markov chain and allow the parameters to switch depending on regimes. In principle, the MS structure can be attached into any traditional time series model, including standard regression, AR, and GARCH models. Since the original paper (Hamilton, 1989) successfully applied the two-regime MS−AR(4) model to postwar U.S. real GNP, numerous extensions and empirical studies have been provided, including the MS−GARCH model (Hamilton and Susmel, 1994; Cai, 1994; Gray, 1996), MS−GARCH with the student t distributed residuals (Marcucci, 2005; Henneke et al., 2011), and multivariate extension (Sims et al., 2008). See Kim and Nelson (1999), Hamilton and Raj (2002), and Bhar and Hamori (2009) for more information on the MS model. We also study the topic of making a comparison of performances between GARCH and MS models.

Risk Measure

Value at Risk (VaR) is the most standard risk measure adopted by financial institutions. The reason for the popularity of VaR is that it is endorsed by the regulators to measure market risk. However, several drawbacks of VaR have been pointed out in spite of its popularity. The main criticism of VaR is that it is not informative about the risk above the VaR level, i.e., the tail risk. Additionally, VaR does not satisfy the axioms of coherent risk measures[8]. Recently, average

[8] See McNeil et al. (2005).

VaR (AVaR), also known as expected shortfall, has attracted more interest because it is free of the above drawbacks of VaR. If fact, the BCBS (2012) proposed to use AVaR instead of VaR in the future to measure market risk.

Moreover, researchers have focused on novel types of risk measures for systemic risk, especially since the Lehman shock. Whereas VaR and AVaR are risk measures based on the premise that an institution is isolated, novel risk measures attempt to measure the risk of meltdown of a whole financial system and/or the risk contribution of each institution to an entire system, i.e., the risk spillover effect. In this dissertation, we consider the CoVaR methodology proposed by Adrian and Brunnermeier (2011) to address systemic risk. CoVaR, or more specifically, $\mathrm{CoVaR}^{j|i}$, is a bivariate risk measure between two institutions, i and j. $\mathrm{CoVaR}^{j|i}$ is the VaR of j on a certain condition of i. Putting j into an entire system, Adrian and Brunnermeier (2011) measure the risk contribution of i to the system. The applications of CoVaR have been studied by several researchers. López-Espinosa et al. (2012) analyze the driving factors of systemic risk in the large international banks with CoVaR approach. Wong and Fong apply CoVaR methodology to the sovereign CDS spreads of Asia Pacific economies (2011) and the Eurozone (2012). Girardi and Ergün (2013) estimate the CoVaR of U.S. financial institutions on the basis of stock prices by using the bivariate GARCH model with dynamic conditional correlation specification and Hansen's skewed t distributed residuals.

Alternative approaches for measuring systemic risk other than CoVaR have also been proposed. Segoviano and Goodhart (2009) assess the stability of the financial system by indicators based on the joint and the conditional default probability, where the joint distribution is derived from a cross-entropy-based optimization technique with CDS data. In line with the work of Segoviano and Goodhart, Zhou (2010) introduces systemic importance indicators based on the conditional default probability, which he calls the "systemic impact index (SII)" and the "vulnerability index (VI)." The multivariate extreme value theory is used for estimations. Huang et al. (2009) quantify systemic risk as a theoretical insurance price against a financial crisis, which is inferred from CDS spread and stock return correlations among financial institutions. Acharya et al. (2010) propose the systemic expected shortfall (SES) and the marginal expected shortfall (MES) as a measure for systemic risk. MES is the institution's expected loss on the condition that the loss of the aggregate banking system is beyond the VaR level, and SES is the institution's expected drop in capital below its target level on the condition that the aggregate banking capital goes below its target level. Schwaab et al. (2011) apply a latent dynamic factor model of mixed-measurement data to systemic risk. Giesecke and Kim (2011) define systemic risk as the conditional probability of defaults of a large proportion of total financial institutions and analyze it by a dynamic hazard model.

Risk Management

We focus on portfolio risk management. There are two distinct methods of portfolio optimization; the mean–risk and reward–risk optimizations. The framework of mean–risk optimization is to construct a portfolio *minimizing* the risk measure with a given desired expected return. The mean–risk optimization originates from Markowitz's mean–variance optimization (1952). Because the variance is not always an appropriate risk measure to minimize, it has been proposed to replace the variance in Markowitz's theory with other risk measures such as VaR and AVaR, which are collectively called mean–risk portfolio optimization. On the other hand, the framework of reward–risk optimization is to construct a portfolio *maximizing* the reward to risk ratio, which is regarded as a performance measure. Whereas the expected return to variance, called the Sharpe ratio (1966), is the most prevailing performance measure, AVaR is also adopted in the reward to risk ratio. The expected return to AVaR ratio is called the stable tail-adjusted return ratio (STARR; Rachev et al., 2008a). See Rachev et al. (2008b) for more information about portfolio optimization. It is worth attempting to use the aforementioned novel risk measures as the objective functions in the framework of the portfolio optimization. In this dissertation, we incorporate CoVaR methodology into the mean−risk framework.

1.2 Overview of Dissertation

In this dissertation, we study several issues related to risk measurement and management in the global financial markets. Specifically, we cover the global stock and foreign currency exchange markets. Our methodology is based on the cutting-edge research outcomes stated in Section 1.1. The rest of this dissertation is structured as follows.

Chapter 2 discusses the measurements of systematic risk in the global banking stock market. Motivated by the growing importance of systemic risk in the global banking system, we measure the risk of the system and the marginal contributions of the institutions in several ways in terms of stock markets. The undiversifiable risk appearing in specific market sectors is called systematic risk rather than systemic risk. We focus on global banking stocks comprising G-SIFIs, and discuss the global systematic risk measurement. To forecast the future joint distribution of returns, we utilize the multivariate ARMA–GARCH model with the MNTS distributed and multivariate normal distributed residuals. We statistically demonstrate that the ARMA–GARCH model with the MNTS distributed residuals is a more realistic model for G-SIFI stocks. In line with previous studies, we estimate four systematic risk measures. We investigate the properties of the measures and relationship among the measures in the time series and cross-section directions. Thereafter, we make some remarks on the systematic risk measurement.

Chapter 3 discusses the management of systematic risk for global banking portfolios. We propose a mean–CoAVaR portfolio optimization to mitigate the potential loss caused by systematic risk. CoAVaR is a natural extension of Adrian and Brunnermeier's CoVaR, and is defined as the AVaR on the condition that the market index is in distress. Similar to CoVaR, CoAVaR also accounts for the extent to which an institution is affected by systematic distress. We expect that the potential loss of the portfolio arising from systematic risk is mitigated by minimizing the CoAVaR of the portfolio against the market index. We investigate the effectiveness of the mean–CoAVaR optimization by using the stocks of G-SIFIs. The reason for choosing G-SIFI stocks as trial samples is that they are both highly interconnected to each other and potentially affected by systematic risk. The joint stock return distribution is predicted by the ARMA–GARCH model with the MNTS distributed residuals, which will be shown to be a better model for G-SIFI stocks in Chapter 2.

Chapter 4 discusses risk management in foreign currency exchange markets. We explore the best models for describing the dynamics of time series of currency exchange rates. Sample currencies are those circulating in the member countries of the Organization for Economic Co-operation and Development. The GARCH and Markov−switching models are compared in terms of both in-sample and out-of-sample tests. In order to consider the fat-tailness and skewness of GARCH residuals, we adopt the student t and NTS distributional assumptions as well as the normal one. Extending on the previous study, we renew sample periods, expand samples, and investigate higher frequency data. For the out-of-sample test, we focus on the accuracy of forecasting of VaR and tail behavior, because they are crucial for risk management, especially during financial turmoil. We also discuss multivariate modeling by using the GARCH model with the MNTS distributed residuals to further examine the effectiveness of the GARCH model when used with exchange rates.

Chapter 5 concludes the dissertation. We summarize the results in each chapter and make some remarks on future works, extending and sophisticating the analysis in this dissertation.

The contents of Chapters 2 and 3 are on the basis of the papers published in refereed journals; Kurosaki and Kim (2013a), and Kurosaki and Kim (2013b), respectively. The content of Chapter 4 is under submission at the time of writing.

Chapter 2 Systematic Risk Measurement in the Global Banking Stock Market with Time Series Analysis and CoVaR

2.1 Introduction

In the modern financial system, global financial institutions become strongly interconnected, leading to awareness of the so-called "systemic risk." According to the definition given by Kaufman and Scott (2003), in contrast to the risk that there will be a breakdown in individual parts or components of the financial system, systemic risk refers to the probability that there will be a breakdown of the entire financial system. Moreover, this risk is evidenced by the co-movements of the different parts of the financial system.

We can observe the applicability of this definition of systemic risk in the case of global financial system in 2008, following the bankruptcy of the United States (U.S.) investment banking firm Lehman Brothers. The financial crisis triggered by the failure of Lehman Brothers, referred to as the "Lehman shock," had a spillover effect in every sector of the global financial market (stock, bond, currency, credit markets, and the like).

Following the Lehman shock, the Basel Committee on Banking Supervision (BCBS) began to formulate a new regulatory framework for international banks known as Basel III to mitigate the risk of a reoccurrence of financial crises due to the problem of large financial institutions. One of the most significant enhancements in Basel III relative to Basel I and II is that of protecting the global financial system from systemic risk. More specifically, within the three pillar framework first introduced in Basel I in 1988, Basel III calls for additional capital requirements for global systemically important financial institutions (G-SIFIs), in contrast to the uniform capital requirement imposed on every bank in Basel II. More recently, an initial list of 29 G-SIFIs (8 from the United States, 17 from Europe, and 4 from Asia) was identified and published based on the BCBS methodology (Financial Stability Board, 2011). See Appendix A for the list of financial institutions.

The recent debt crisis in Greece calls for greater attention to systemic risk in another way. Because financial institutions typically have large positions in sovereign bonds, there was great concern in the market that a systemic downturn would occur because of the European sovereign debt crisis. This, in fact, did occur for one G-SIFI, Dexia Group, because of exposures to these countries[9]. There are some market observers with such a pessimistic view that if Greece collapses, the adverse impact on the financial system would be greater than that of the Lehman shock.

Motivated by the growing importance of systemic risk, the purpose of this chapter is to investigate such risk in the global banking system. This is done by focusing on systemic risk observed in stock markets and investigating stocks that are included in G-SIFIs, as of November 2011. Our methodology involves time series analysis to generate a future joint distribution of stock returns, and accordingly we estimate risk measures.

We emphasize that, strictly speaking, we are not going to quantify systemic risk itself given that we exclusively deal with stock returns. There are *systemic* risk and *systematic* risk. Even though both emerge with a downslide of total market returns, systemic risk is considered as the risk that specifically arises from intense interconnectedness and results in a breakdown of the entire system. Aggregate adverse impact in a specific sector of a market should be classified as

[9] Dexia was bailed out by the Belgium, France, and Luxembourg governments in October 2011 and then again in November 2012.

systematic risk. For this reason, we hereafter refer to the risk that we quantify based on stock returns as systematic risk rather than systemic risk[10].

For time series analysis, we use a multivariate autoregressive moving average generalized autoregressive conditional heteroscedasticity (ARMA–GARCH) model, where the innovation terms are assumed to follow the multivariate normal tempered stable (MNTS) and multivariate normal distributions. The MNTS distribution is a relatively new non-Gaussian stock return model proposed by Kim et al. (2012). Each marginal of the MNTS distribution is referred to as a univariate normal tempered stable (NTS) distribution. For systematic risk measures, we use the CoVaR methodology proposed by Adrian and Brunnermeier (2011). CoVaR, or more specifically, $\mathrm{CoVaR}^{j|i}$, is defined between two institutions *i* and *j*. $\mathrm{CoVaR}^{j|i}$ is the Value at Risk (VaR) of j on a certain condition of *i*. Setting j as the market index, we consider the difference between CoVaR on i's distress and "normal" conditions, denoted by $\Delta\mathrm{CoVaR}^{index|i}$. $\Delta\mathrm{CoVaR}^{index|i}$ can be interpreted as the marginal contribution of *i* to the overall market risk.

There are two problems we address in this issue. The first is how to measure and predict systematic risk. The second is how to determine the influence of a financial institution on the entire financial system, i.e., how to quantify the risk spillover effect. From a regulatory perspective, it is critical to recognize signals of a meltdown of the financial system and specify the financial institutions that potentially have considerable influence on the financial system.

For the first problem, we propose the joint probability of negative stock return movements as a measure of systematic risk. This is necessary because although ΔCoVaR can be a measure of marginal contribution to systematic risk, it is not a measure of systematic risk itself. For the second problem, we employ ΔCoVaR to quantify the risk spillover effect. In addition, we extend

[10] The basic measure of systematic risk is beta. Similar to beta, we focus on the co-movement between the entire system and each institution in the global banking stock market.

ΔCoVaR into the counterpart of average VaR (AVaR), which we refer to as ΔCoAVaR. An alternative approach for the risk spillover effect is to describe an institution's power of influence on the system as the probability of a negative co-movement of the market return on the condition that a return of the institution moves downward. The idea underlining the use of conditional probability is parallel to the idea of addressing the first problem via joint probability. We examine the relationship among AVaR, ΔCoAVaR, and conditional probability using regression analysis.

The rest of this chapter is organized as follows. In Section 2.2, we introduce an ARMA–GARCH–MNTS model for time series analysis. Subsequently, we define the following systematic risk measures: the joint probability and conditional probability of negative movements, ΔCoVaR, and ΔCoAVaR. Section 2.3 describes the data to be used. Section 2.4 presents the results and discussion. After we demonstrate that the ARMA–GARCH–MNTS model is a better model for G-SIFI stocks, we present the estimation results of systematic risk measures. We also discuss the relationship among the different types of measures. Section 2.5 concludes the chapter.

2.2 Methodology

Our methodology for the investigation of systematic risk has the following two steps: (1) generating the future joint distribution of stock returns via the ARMA–GARCH model and (2) deriving systematic risk measures from the predicted joint distribution. We also briefly explain our simulation-based estimation methods.

2.2.1 ARMA–GARCH–MNTS model

Our time series model for stock returns is the ARMA(1,1)–GARCH(1,1) model given by

$$R_{t+1}^j = \mu_{t+1}^j + \sigma_{t+1}^j \eta_{t+1}^j,$$

$$\mu_{t+1}^j = a_j R_t^j + b_j \sigma_t^j \eta_t^j + c_j,$$

$$\left(\sigma_{t+1}^j\right)^2 = \omega_j \left(\sigma_t^j\right)^2 \left(\eta_t^j\right)^2 + \xi_j \left(\sigma_t^j\right)^2 + \Psi_j, \tag{2.1}$$

where the index $j = 1,2,\cdots,J$ corresponds to each institution, t represents a time period, R_t^j is the stock return, μ_t^j is the conditional mean, σ_t^j is the conditional standard deviation, η_t^j is the i.i.d. with zero mean and unit variance, called (standardized) innovation, and the other symbols are model parameters. We describe the multivariate distribution whose every marginal has zero mean and unit variance as standard. Thus, $\boldsymbol{\eta}_t = (\eta_t^1, \eta_t^2, \cdots, \eta_t^J)$ forms a standard multivariate distribution. Note that ARMA(1,1)–GARCH(1,1) is a standard specification for financial data in the GARCH framework.

There are several candidate models for each marginal η_t^j. We choose the NTS distribution because it has the ability to capture stylized properties of stock return distributions such as fat-tailness and skewness, which the normal distribution lacks. In addition, we use the normal distribution for the purpose of comparison. The standard NTS distribution is characterized by three parameters: two fat-tailness parameters (α, θ) and one skewness parameter β. If we assume common (α, θ) among NTS marginals with β as a still free parameter for calibration, we can join marginals into MNTS via the variance-covariance matrix of $\boldsymbol{\eta}_t$ without computational difficulty even in a considerably high-dimensional system. See Kim et al. (2012) and Appendix B for the definition and estimation of the MNTS distribution. In the case of the normal model, we can also join marginals into the multivariate distribution via the variance-covariance matrix, because it is the single parameter of the standard multivariate normal distribution. The multivariate distribution of $\boldsymbol{\eta}_t$ accounts for the dependent structures among stock returns.

Following the same approach as Kim et al. (2012), we first estimate the univariate NTS parameters $(\alpha, \theta, \beta) = (\hat{\alpha}, \hat{\theta}, \hat{\beta})$ for the innovation of the representative stock, i.e., the market index. Then, we use the estimated parameters $(\hat{\alpha}, \hat{\theta})$ as those of MNTS. For the CoVaR estimation, Adrian and Brunnermeier (2011) mainly use quantile regressions supplemented with the GARCH model with the normal distributed innovations as a robustness check. Girardi and Ergün (2013) use the GARCH model with Hansen's skewed t distributed innovations. Our methodology is different from the previous studies because we first apply the multivariate tempered stable distribution to the CoVaR estimation. Another advantage of MNTS is that it has the reproductive property; the linear combination of NTS distributed random variables still follows NTS. This property enables us to easily deal with the portfolio of stocks.

Model (2.1) forecasts the joint distribution of stock returns at $t + 1$ period on the basis of the information up to t. We refer to Model (2.1) with the standard MNTS distributed and standard multivariate normal distributed $\boldsymbol{\eta}_t$ as the ARMA–GARCH–MNTS (AGMNTS) model and ARMA–GARCH–multivariate normal (AGMNormal) model, respectively. We primarily use an AGMNTS forecast, whereas we use an AGMNormal forecast as a reference.

2.2.2 Systematic Risk Measures

Before introducing systematic risk measures, we begin with VaR. VaR is the most standard market risk measure used by financial institutions. Consider the VaR of j's stock return R_t^j at the confidence level $1 - q$ $(0 \leq q \leq 1)$, denoted by $\mathrm{VaR}_{q,t}^j$. The definition of $\mathrm{VaR}_{q,t}^j$ is given by

$$\mathrm{VaR}_{q,t}^j = -\inf\{R | \mathrm{Prob}(R_t^j \leq R) \geq q\}. \tag{2.2}$$

If R_t^j is continuous, $\mathrm{VaR}_{q,t}^j$ is the q-quantile of the distribution of R_t^j, which satisfies

$$\mathrm{Prob}(R_t^j \leq -\mathrm{VaR}_{q,t}^j) = q. \tag{2.3}$$

An alternative risk measure is AVaR. The definition of $\mathrm{AVaR}_{q,t}^{j}$ is given by

$$\mathrm{AVaR}_{q,t}^{j} = \frac{1}{q}\int_{0}^{q} \mathrm{VaR}_{p,t}^{j}\,\mathrm{d}p. \tag{2.4}$$

If R_t^j is continuous, AVaR is equivalent to

$$\mathrm{AVaR}_{q,t}^{j} = -\mathrm{E}\left(R_t^j \middle| R_t^j < -\mathrm{VaR}_{q,t}^{j}\right), \tag{2.5}$$

which is called expected tail loss. Henceforth, for simplicity, every stock return distribution is assumed to be continuous. AVaR has more desirable properties than VaR as a risk measure (e.g., the ability to account for risk above the VaR level, often referred to as "tail risk")[11]. In literature, AVaR is also called conditional VaR (CVaR[12]) or Expected Shortfall (ES).

While VaR and AVaR are *micro*-prudential risk measures on the premise of an institution being isolated, alternative *macro*-prudential risk measures for systemic risk have recently been explored in the context of global financial turmoil. While some consider probability-based approaches (Segoviano and Goodhart, 2009; Zhou, 2010; Giesecke and Kim, 2011), others put weight on quantifying systemic risk such as CoVaR (Adrian and Brunnermeier, 2011), SES, and MES (Acharya et al., 2010). In line with the previous studies of systemic risk, we introduce four systematic risk measures in stock markets on the basis of VaR and AVaR, in which two out of four are probability-based indicators: joint and conditional probabilities of negative movements. The other two are measures to quantify the marginal contribution to systematic risk: ΔCoVaR and ΔCoAVaR.

Joint Probability of Negative Movements (JPNM)

We consider systematic risk as simultaneous negative movements of stock returns, where the negative movement simply means the return being less than the conditional mean. Note that this

[11] For further information, see Rachev et al. (2008b).

[12] Note that CoVaR is a different concept from CVaR, despite the analogous name.

definition is consistent with the definition of systemic risk given by Kaufman and Scott (2003). Accordingly, we introduce the joint probability of negative movements (JPNM),

$$\mathrm{JPNM}_t = \mathrm{Prob}\left(\bigcap_{j=1}^{J} R_t^j < \mu_t^j\right), \tag{2.6}$$

as a measure of systematic risk. Because massive simultaneous negative co-movement is a very rare event, the joint probability is low. However, we expect that such a low probability captures the common distress factor among financial institutions and signals crisis. In a previous study, Segoviano and Goodhart (2009) estimate the joint probability of distress among financial institutions from the credit default swap data.

CoVaR

To investigate and quantify the risk spillover effect, we adopt Adrian and Brunnermeier's CoVaR methodology. CoVaR is a bivariate concept between two institutions *i* and *j*. While $\mathrm{VaR}_{q,t}^j$ is the *q*-quantile of the *unconditional* distribution of R_t^j, $\mathrm{CoVaR}_{q,t}^{j|i}$ is the q-quantile of the *conditional* distribution of R_t^j on a certain condition of *i*, more specifically, R_t^i. When we specify the condition of R_t^i as $\mathbb{C}(R_t^i)$, we denote $\mathrm{CoVaR}_{q,t}^{j|\mathbb{C}(R_t^i)}$ instead of $\mathrm{CoVaR}_{q,t}^{j|i}$. The implicit definition of $\mathrm{CoVaR}_{q,t}^{j|\mathbb{C}(R_t^i)}$ for continuous R_t^j is given by

$$\mathrm{Prob}\left(R_t^j \leq -\mathrm{CoVaR}_{q,t}^{j|\mathbb{C}(R_t^i)} \middle| \mathbb{C}(R_t^i)\right) = q. \tag{2.7}$$

Let $\mathbb{C}^d(R_t^i)$ and $\mathbb{C}^n(R_t^i)$ be the distress and "normal" conditions of R_t^i, respectively. Adrian and Brunnermeier (2011) suggest that the difference of $\mathrm{CoVaR}_{q,t}^{j|i}$ between the two conditions $\mathbb{C}^d(R_t^i)$ and $\mathbb{C}^n(R_t^i)$,

$$\Delta\text{CoVaR}_{q,t}^{j|i} = \text{CoVaR}_{q,t}^{j|\mathbb{C}^d(R_t^i)} - \text{CoVaR}_{q,t}^{j|\mathbb{C}^n(R_t^i)}, \quad (2.8)$$

accounts for the risk contribution of *i* to *j*.

For the application of CoVaR to systematic risk in stock markets, we highlight the case of *j* being a market index. $\Delta\text{CoVaR}_{q,t}^{index|i}$ is regarded as the marginal contribution of *i* to the overall systematic risk.

Regarding the conditions, Adrian and Brunnermeier (2011) define the distress and normal conditions as the institution's loss and return being exactly at its VaR and median, respectively,

$$\mathbb{C}^d(R_t^i) = \{R_t^i = -\text{VaR}_{q,t}^i\},$$
$$\mathbb{C}^n(R_t^i) = \{R_t^i = \text{median}_t^i\}. \quad (2.9)$$

However, we adopt the modified definition by Girardi and Ergün (2013), where the distress and normal conditions denote the institution's loss and return being above its VaR and within the range of one standard deviation from its mean, respectively,

$$\mathbb{C}^d(R_t^i) = \{R_t^i \leq -\text{VaR}_{q,t}^i\},$$
$$\mathbb{C}^n(R_t^i) = \{\mu_t^i - \sigma_t^i \leq R_t^i \leq \mu_t^i + \sigma_t^i\}. \quad (2.10)$$

We make the confidence level $1 - q$ of $\mathbb{C}^d$ coincide with that of CoVaR, which is conditioned by $\mathbb{C}^d$. As Girardi and Ergün point out, the modified definition has several merits[13]. First, it focuses on tail risk, i.e., the loss above the VaR level, and thus, the resulting CoVaR becomes more insightful. Second, it allows backtesting of CoVaR. We can apply the ordinary VaR backtesting methods to $\text{CoVaR}_{q,t}^{index|\mathbb{C}^d(R_t^i)}$ for the days during which VaR violation of *i* occurs. Here, the VaR violation of *i* means the event when the observed loss $-R_t^i$ exceeds $\text{VaR}_{q,t}^i$; i.e.,

[13] The modified definition is also supported by Mainik and Schaanning (2012) in terms of the consistency of the response to the dependent parameters.

the condition $\mathbb{C}^d(R_t^i)$ actually occurs[14]. The simplest way of VaR backtesting is to observe how often VaR violations occur. If one attempts to estimate $100(1-q)$ % VaR, violations should occur at $100q$% of whole observations. Following Girardi and Ergün (2013), we shall use the likelihood ratio tests of the unconditional and conditional coverages by Christoffersen (1998) as a more sophisticated VaR backtesting method. The conditional coverage test is more desirable than the unconditional one because it can consider the tendency for consecutive violations, which is observed for ordinary VaR during financial turmoil. We define the CoVaR violation of *i* as the event when the observed $-R_t^{index}$ exceeds $\text{CoVaR}_{q,t}^{index|\mathbb{C}^d(R_t^i)}$ during the VaR violation days of *i*. Through the Christoffersen tests, it can be tested whether CoVaR violation occurs with a reasonable probability during VaR violation days; that is, $\text{CoVaR}_{q,t}^{index|\mathbb{C}^d(R_t^i)}$ is appropriately estimated at the given confidence level. In the conditional test of $\text{CoVaR}_{q,t}^{index|\mathbb{C}^d(R_t^i)}$, the conditions are considered between two adjacent days of the VaR violations of *i*. The last convenience of the modified definition (2.10) for our study is to make scenario simulation-based estimation of CoVaR feasible (See Section 2.2.3).

Conditional Probability of Negative Movements (CPNM)

We can create an alternative probability-based indicator for the risk spillover effect. Given that systematic risk is the simultaneous negative movement of stock returns, the probability of the market index going down contingent on the institution being distressed is regarded as the indicator for systematic risk originating from that institution. Then, we introduce the conditional probability of negative movements (CPNM),

---

[14] Although we can test $\text{CoVaR}_{q,t}^{index|\mathbb{C}^n(R_t^i)}$ in the same way, we concentrate on the distress condition $\mathbb{C}^d$, which is more associated with systematic risk, as Girardi and Ergün (2013) do.

$$\mathrm{CPNM}_{q,t}^{index|i} = \mathrm{Prob}\left(\mathbb{C}^d\left(R_t^{index}\right)\middle|\mathbb{C}^d\left(R_t^i\right)\right). \tag{2.11}$$

We still follow Eq. (2.10) regarding the definition of $\mathbb{C}^d$. In this case, CPNM is proportional to the joint probability of both a market index and an individual institution incurring the loss beyond their respective VaRs. Note that, in contrast to the case of JPNM, negative movement does not stand for the return being less than the conditional mean but rather the loss exceeding VaR in the case of CPNM. This is because the joint probability of returns less than conditional means appears insufficient to inspect bivariate tail dependency.

CoAVaR

We can consider the *Co*-version of AVaR by considering Eqs. (2.4) and (2.5). $\mathrm{CoAVaR}_{q,t}^{j|\mathbb{C}(R_t^i)}$ is defined by

$$\begin{aligned}\mathrm{CoAVaR}_{q,t}^{j|\mathbb{C}(R_t^i)} &= \frac{1}{q}\int_0^q \mathrm{CoVaR}_{p,t}^{j|\mathbb{C}(R_t^i)}\,\mathrm{d}p \\ &= -\mathrm{E}\left(R_t^j\middle|\left\{R_t^j < -Co\mathrm{VaR}_{q,t}^{j|\mathbb{C}(R_t^i)}\right\}\cap\mathbb{C}\left(R_t^i\right)\right).\end{aligned} \tag{2.12}$$

In an analogous fashion to CoVaR, the risk contribution of *i* to *j* in terms of CoAVaR is expressed by

$$\Delta\mathrm{CoAVaR}_{q,t}^{j|i} = \mathrm{CoAVaR}_{q,t}^{j|\mathbb{C}^d(R_t^i)} - \mathrm{CoAVaR}_{q,t}^{j|\mathbb{C}^n(R_t^i)}. \tag{2.13}$$

In Adrian and Brunnermeier (2011), CoAVaR is mentioned as CoES. Because AVaR has some merits compared with VaR, we primarily use CoAVaR rather than CoVaR for the assessment of systematic risk.

2.2.3 Scenario Simulation

We rely on scenario simulation for estimation of systematic risk measures. It flexibly enables the estimations of various risk measures. On the basis of the AGMNTS (AGMNormal) model, we generate a large number S of scenarios about one-period-ahead multivariate stock returns $\boldsymbol{R}_{t+1}^{s} = \left(R_{t+1}^{1,s}, R_{t+1}^{2,s}, \cdots, R_{t+1}^{J,s}\right), 1 \leq s \leq S$ via a Monte Carlo simulation. For the AGMNTS model, the random variables that follow the MNTS distribution are easily simulated using its subordinated representation[15]. The risk measures can be estimated from the selected scenarios, where a relevant or conditioning event like $\mathbb{C}^{d}\left(R_t^i\right)$ or $\mathbb{C}^{n}\left(R_t^i\right)$ is realized out of the overall scenarios. For the estimations of $\Delta\mathrm{CoVaR}_{q,t}^{index|i}$, $\Delta\mathrm{CoAVaR}_{q,t}^{index|i}$, and $\mathrm{CPNM}_{q,t}^{index|i}$, we specify the bivariate ARMA–GARCH model of the market index and institution i.

2.3 Data

For empirical research, we use daily stock logarithmic return data for 28 out of 29 G-SIFIs, as of November 2011. We refer to each stock by its ticker symbol or abbreviation. The list of G-SIFIs is given in Appendix A. The only exclusion is Banque Populaire CdE because it is unlisted. We use the S&P global 1200 financial sector index to represent the global banking stock market. The sample period is from January 1st, 2000 to June 30th, 2012. We exclude the U.S. non-business days from this period, which leads to 3260 observations for each stock. BOC, ACA, and three Japanese G-SIFIs (MUFG, MHFG, and SMFG) do not have sufficient length of historical data to cover the whole sample period. Regarding BOC and ACA, we backfill historical data using Cognity[16]. Regarding the three Japanese G-SIFIs, we extrapolate historical data using those

[15] It is specifically a mixture of the multivariate normal distribution and classical tempered stable subordinator. See Kim et al. (2012).

[16] Risk management software provided by FinAnalytica, Inc. We refer to Shanghai Stock Exchange Composite (SHCOMP) Index and Morgan Stanley Capital International (MSCI) Index of France for backfilling of BOC and ACA, respectively, because these indexes are found to show relatively strong associations with missing stocks.

of their representative affiliates, which had been listed before the establishments of holding companies[17]. All stock return data are downloaded from Bloomberg.

We set the $1 - q = 0.95$ confidence level for risk measures unless otherwise noted. The number of scenarios in the Monte Carlo simulation is $S = 10^6$. The forecast of stock returns is made on a daily basis. Each business day, the model parameters are updated from a moving window of the most recent 1250 days' sample return data. It means that we have 2011 daily parameter estimates starting from October 15th, 2004. In individual model parameter estimations, the variance-covariance matrix of $\boldsymbol{\eta}_t$ is estimated from the most recent 250 days' sample innovations.

2.4 Estimation Results

We present the estimation results of systematic risk measures. The measures are estimated on the basis of the AGMNTS model unless otherwise noted, whereas they are estimated on the basis of the AGMNormal model, if needed for a reference.

First, we validate the usage of the AGMNTS model with G-SIFI stocks. For this validation, we test the standard NTS and normal distributional assumptions for the innovation of each stock in the ARMA(1,1)–GARCH(1,1) model (2.1) through the Kolmogorov–Smirnov (KS) test. Because we have 2011 daily estimations of the ARMA–GARCH model, the KS test is accordingly applied 2011 times for each stock. Table 2.1 reports the number of days on which the NTS and normal assumptions for each stock are rejected at three different significance levels: 1%, 5%, and 10%. The result is that NTS provides much better fitting for innovations than

[17] Japanese major banks restructured their business form into holding companies all together in the beginning of 2000s under financial reforms called Japanese financial Bing Bang. Therefore, stock prices of major affiliate banks can be substituted for those of holding companies before their establishments. Specifically, we substitute Bank of Tokyo-Mitsubishi UFJ (8315 JP) for MUFG, Dai-Ichi Kangyo Bank (8311 JP, until September 2000) and Mizuho Holdings (8305 JP, from October 2000) for MHFG, and Sumitomo Mitsui Banking Corporation (8318 JP) for SMFG.

normal. The only exception is BOC. Both NTS and normal assumptions are rejected by all 2011 estimations for the innovations of BOC. However, except BOC, the rejections of the NTS assumption are much lower than those of the normal assumption at every significance level. The normal assumption is totally rejected by BOC, BK, MUFG, MHFG, STT, and SMFG even at the 1% significance level. These observations support the usage of AGMNTS model with G-SIFI stocks.

To illustrate the basic risk profiles of G-SIFI stocks, we refer to VaR and AVaR. We adopt an equally weighted portfolio as the most representative portfolio, and consider the VaR and AVaR of the portfolio to be equally weighted by the 28 G-SIFI stocks. Figure 2.1 represents the time series plot of the VaR and AVaR of the equally weighted portfolio estimated by the AGMNTS and AGMNormal models. AVaR estimated from the AGMNTS model tends to be higher than the AGMNormal model, especially during financial crisis, because of its capability of accounting for fat-tailness, whereas both models give similar VaR at the 95% confidence level. Through a simple graphic comparison, we find that the AGMNTS model and AVaR is the best combination for the purpose of warning of distress of individual institutions or their portfolios in terms of *micro*-prudential perspective. Subsequently, we apply the unconditional and conditional Christoffersen's likelihood ratio tests to the estimated daily VaR of each stock to clarify whether the estimations of VaR are reasonable. Tables 2.2 and 2.3 report the number of violation days and p-values of the tests for 90% VaR, 95% VaR, and 99% VaR, respectively. Both AGMNTS and AGMNormal models show similar performance on the 90% VaR and 95% VaR estimations. The AGMNTS model gives fewer VaR violations and higher p-values for some stocks, whereas the AGMNormal model does this for other stocks; a higher p-value means less probability of rejection of the VaR estimation. However, this is not the case for the 99% VaR estimation; the

AGMNTS model clearly gives a better forecast of VaR than the AGMNormal model. The AGMNTS model generally has fewer violation days and higher p-values. The number of 99% VaR violations based on the AGMNTS model is lower than the AGMNormal model, except for BOC and MUFG. In addition, the number of rejections of each stock's 99% VaR estimation under the unconditional and conditional tests are 10 and 17 at the 5% significance level for AGMNTS, whereas 22 and 25 for AGMNormal, respectively. The fact that the 99% VaR estimation of the AGMNTS model is relatively more accurate than the 90% VaR and 95% VaR estimations implies that the deeper tail structure of the distribution is better captured by the AGMNTS model than the AGMNormal model. This property of the AGMNTS model is desirable for our study because our main interest CoVaR casts a spotlight on the deeper tail structure. Therefore, the AGMNTS model is preferable in terms of risk measure estimation as well as fitting performance.

Table 2.1: Number of Rejections of Distributional Assumptions for Each Stock on the Basis of the KS test (Out of 2011 Estimations)

	Significance Level: 10%		Significance Level: 5%		Significance Level: 1%	
	AGMNTS	AGMNormal	AGMNTS	AGMNormal	AGMNTS	AGMNormal
BAC	1463	2007	1219	2007	400	1979
BOC	2011	2011	2011	2011	2011	2011
BK	1890	2011	1617	2011	997	2011
BARC	4	1635	4	1017	2	302
BNP	16	1527	4	1345	3	841
C	1446	1960	1224	1846	523	1460
CBK	694	2002	486	2002	59	1963
CSGN	1388	2011	943	2011	152	1548
DBK	367	1898	72	1553	6	864
DEXB	1076	2009	547	1888	6	1176
GS	893	1883	591	1608	3	855
ACA	138	1744	38	1502	0	918
HSBA	159	2011	13	2008	1	1421
INGA	1183	1804	765	1551	4	571
JPM	1437	2011	891	1984	82	1601
LLOY	342	1945	93	1838	15	1092
MUFG	1775	2011	1436	2011	436	2011
MHFG	1558	2011	1288	2011	385	2011
MS	1718	1807	1087	1476	272	1205
NDA	663	2011	577	2010	317	1528
RBS	628	2011	396	1927	100	1556
SAN	1677	1930	1453	1640	654	888
GLE	665	1988	334	1855	9	913
STT	1311	2011	1198	2011	961	2011
SMFG	1695	2011	1352	2011	770	2011
UBSN	1449	2010	1012	1960	245	1111
UCG	74	1256	4	1027	1	555
WFC	722	1425	597	1272	409	1164

Figure 2.1: Time Series of the VaR and AVaR of the Equally Weighted Portfolio

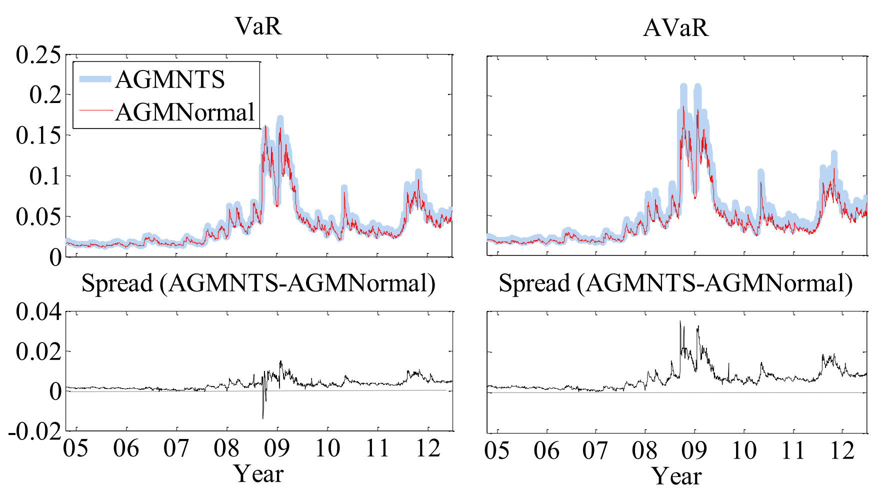

Table 2.2: Number of VaR Violations (Out of 2011 Estimations)

	90% VaR		95% VaR		99% VaR	
	AGMNTS	AGMNormal	AGMNTS	AGMNormal	AGMNTS	AGMNormal
BAC	214	178	111	112	29	46
BOC	155	151	75	79	29	29
BK	188	186	104	104	35	39
BARC	214	196	112	113	31	42
BNP	216	202	118	116	24	35
C	219	203	120	124	34	48
CBK	203	195	102	100	31	34
CSGN	204	189	100	102	21	28
DBK	226	205	126	122	23	36
DEXB	235	216	131	129	33	44
GS	209	187	109	109	25	29
ACA	218	198	109	105	29	35
HSBA	212	188	103	105	31	38
INGA	239	220	126	122	26	35
JPM	211	196	99	94	27	33
LLOY	220	204	107	104	31	39
MUFG	180	167	91	85	26	24
MHFG	186	173	89	80	20	23
MS	217	203	108	111	28	37
NDA	211	180	112	104	27	36
RBS	211	196	104	105	36	45
SAN	239	225	124	130	23	43
GLE	236	206	116	113	32	41
STT	168	141	84	80	22	35
SMFG	179	161	99	87	22	24
UBSN	221	199	118	118	21	34
UCG	247	235	140	139	25	41
WFC	202	195	116	115	33	42

Table 2.3: p-values of the Christoffersen Test for VaR

	90% VaR				95% VaR				99% VaR			
	Unconditional		Conditional		Unconditional		Conditional		Unconditional		Conditional	
	AGMNTS	AGMNormal	AGMNTS	AGMNormal	AGMNTS	AGMNormal	AGMNTS	AGMNormal	AGMNTS	AGMNormal	AGMNTS	AGMNormal
BAC	0.342	0.080	0.175	0.026	0.293	0.250	0.063	0.061	0.062	0.000	0.037	0.000
BOC	0.000	0.000	0.000	0.000	0.006	0.022	0.006	0.012	0.062	0.062	0.043	0.037
BK	0.325	0.256	0.167	0.105	0.726	0.726	0.655	0.457	0.003	0.000	0.001	0.000
BARC	0.342	0.704	0.128	0.052	0.250	0.211	0.061	0.029	0.024	0.000	0.000	0.000
BNP	0.273	0.947	0.040	0.041	0.082	0.122	0.036	0.074	0.398	0.003	0.176	0.001
C	0.189	0.888	0.081	0.394	0.053	0.020	0.050	0.020	0.005	0.000	0.000	0.000
CBK	0.888	0.649	0.015	0.002	0.882	0.955	0.044	0.084	0.024	0.005	0.018	0.004
CSGN	0.830	0.364	0.017	0.008	0.955	0.882	0.364	0.413	0.843	0.095	0.487	0.059
DBK	0.069	0.773	0.068	0.695	0.012	0.033	0.011	0.032	0.527	0.001	0.334	0.001
DEXB	0.014	0.273	0.000	0.001	0.003	0.005	0.000	0.000	0.008	0.000	0.004	0.000
GS	0.559	0.289	0.248	0.048	0.393	0.393	0.393	0.393	0.291	0.062	0.187	0.043
ACA	0.215	0.817	0.003	0.023	0.393	0.651	0.012	0.126	0.062	0.003	0.043	0.002
HSBA	0.421	0.325	0.311	0.294	0.803	0.651	0.019	0.057	0.024	0.000	0.018	0.000
INGA	0.006	0.166	0.001	0.006	0.012	0.033	0.000	0.007	0.207	0.003	0.002	0.000
JPM	0.465	0.704	0.395	0.659	0.874	0.498	0.588	0.307	0.142	0.008	0.087	0.007
LLOY	0.166	0.830	0.000	0.030	0.513	0.726	0.131	0.430	0.024	0.000	0.014	0.000
MUFG	0.111	0.009	0.048	0.003	0.321	0.103	0.320	0.099	0.207	0.398	0.115	0.176
MHFG	0.256	0.033	0.063	0.011	0.228	0.029	0.003	0.009	0.980	0.527	0.015	0.021
MS	0.243	0.888	0.155	0.535	0.451	0.293	0.131	0.193	0.095	0.001	0.062	0.001
NDA	0.465	0.111	0.002	0.027	0.250	0.726	0.232	0.241	0.142	0.001	0.089	0.001
RBS	0.465	0.704	0.019	0.004	0.726	0.651	0.022	0.009	0.001	0.000	0.000	0.000
SAN	0.006	0.081	0.004	0.031	0.020	0.004	0.018	0.004	0.527	0.000	0.334	0.000
GLE	0.011	0.717	0.002	0.062	0.122	0.211	0.025	0.102	0.014	0.000	0.003	0.000
STT	0.012	0.000	0.005	0.000	0.082	0.029	0.078	0.026	0.676	0.003	0.416	0.001
SMFG	0.095	0.002	0.063	0.001	0.874	0.156	0.030	0.120	0.676	0.398	0.212	0.176
UBSN	0.145	0.876	0.006	0.047	0.082	0.082	0.021	0.036	0.843	0.005	0.018	0.000
UCG	0.001	0.014	0.000	0.001	0.000	0.000	0.000	0.000	0.291	0.000	0.020	0.000
WFC	0.947	0.649	0.674	0.608	0.122	0.148	0.052	0.029	0.008	0.000	0.004	0.000
# of p-values less than 5%	7	6	16	19	6	9	16	13	10	22	17	25
# of p-values less than 1%	4	4	11	10	3	3	5	6	5	22	8	22

We now proceed to the estimation results of systematic risk measures. Figure 2.2 illustrates the time series of JPNM[18]. We can see that JPNM has high sensitivity to important financial events. We distinguish three turmoil periods when JPNM rapidly goes up: Period 1 is from July 2007 to September 2008 (subprime loan problem and Lehman's collapse), Period 2 is from April 2010 to March 2011 (dawn of Greek sovereign problem), and Period 3 is from August 2011 to May 2012 (U.S. credit rating downgrading and Greek political turmoil). It is remarkable that JPNM warns the adverse impact of the very recent Greek crisis (Period 3) even more seriously than the Lehman shock (Period 1), whereas VaR or AVaR in Figure 2.1 describes Period 3 relatively moderately. JPNM could be a reference for a forthcoming crisis beyond VaR or AVaR.

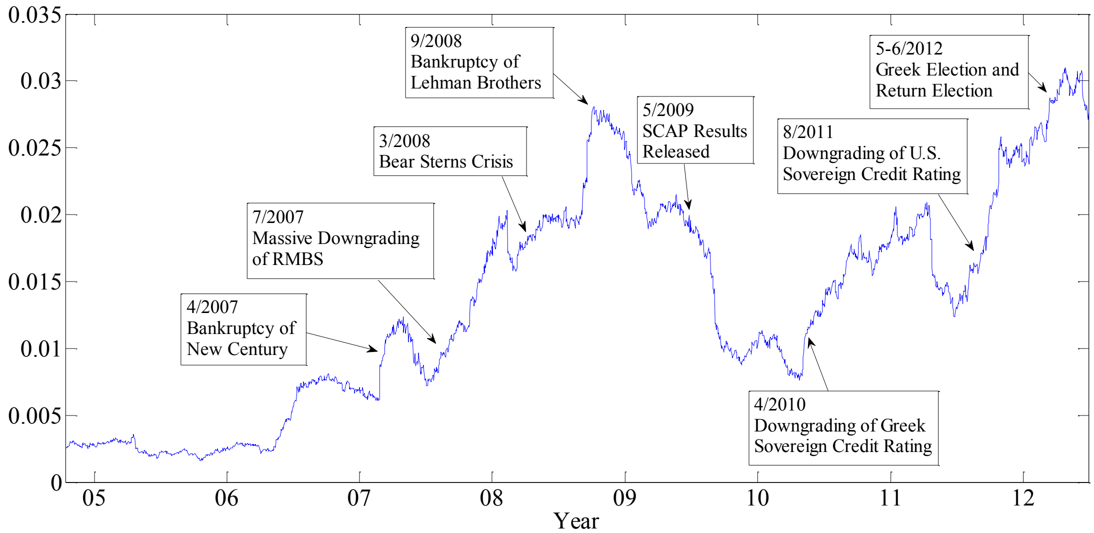

Figure 2.2: Time Series of JPNM

To quantify risk spillover effects, we estimate $\Delta\mathrm{CoVaR}_{q,t}^{index|i}$ and $\Delta\mathrm{CoAVaR}_{q,t}^{index|i}$. We backtest $\mathrm{CoVaR}_{q,t}^{index|\mathbb{C}^d(R_t^i)}$ as well as VaR, on the basis of the Christoffersen tests. Tables 2.4 and 2.5 report the violation rates and p-values of the tests for 90% CoVaR, 95% CoVaR and

[18] The resulting value of JPNM is in the order of 10^{-2}. The number of simulation, $S = 10^6$, is enough for the estimation because the standard deviation of the estimated JPNM is about $\sqrt{\hat{p}(1-\hat{p})/S} \cong 10^{-4}$.

99% CoVaR, respectively[19]. Note that it is not the number of CoVaR violations but the rate of CoVaR violations to VaR violations that is reported in Table 2.4, because the number of VaR violations differs among individual stocks. In general, the rates of CoVaR violations are lower and the p-values of the tests are higher for the AGMNTS model than for the AGMNormal model. The number of rejections of each stock's 95% CoVaR estimation under the unconditional and conditional tests are 3 and 8 at the 5% significance level for AGMNTS, whereas 26 and 27 for AGMNormal, respectively. The AGMNormal estimation of CoVaR is rejected by almost all stocks. These imply that, unlike the case of VaR, the AGMNTS model gives a better forecast of CoVaR than the AGMNormal model regardless of significance levels. As can be observed from the definition, CoVaR addresses tail dependencies among stocks. A better estimation of CoVaR reflects the superior descriptive power for tail dependencies of the MNTS distribution.

[19] We do not deal with the likelihood ratio tests for 99% CoVaR because 99% VaR violations are not frequently observed to test 99% CoVaR.

Table 2.4: Rate of CoVaR to VaR Violations

	90% CoVaR		95% CoVaR		99% CoVaR	
	AGMNTS	AGMNormal	AGMNTS	AGMNormal	AGMNTS	AGMNormal
BAC	0.150	0.213	0.063	0.143	0.034	0.065
BOC	0.142	0.146	0.067	0.139	0.034	0.103
BK	0.186	0.210	0.087	0.135	0.029	0.051
BARC	0.136	0.179	0.071	0.124	0.032	0.048
BNP	0.144	0.163	0.076	0.138	0.000	0.057
C	0.146	0.197	0.067	0.137	0.029	0.021
CBK	0.138	0.190	0.088	0.170	0.032	0.059
CSGN	0.162	0.196	0.090	0.127	0.048	0.071
DBK	0.128	0.156	0.071	0.139	0.043	0.056
DEXB	0.132	0.190	0.084	0.155	0.030	0.091
GS	0.139	0.182	0.083	0.147	0.040	0.069
ACA	0.147	0.182	0.083	0.143	0.034	0.057
HSBA	0.108	0.149	0.087	0.114	0.000	0.053
INGA	0.121	0.164	0.087	0.139	0.000	0.029
JPM	0.142	0.199	0.111	0.191	0.037	0.061
LLOY	0.118	0.157	0.075	0.144	0.000	0.026
MUFG	0.094	0.120	0.055	0.094	0.000	0.042
MHFG	0.086	0.110	0.022	0.088	0.000	0.000
MS	0.147	0.187	0.056	0.135	0.036	0.081
NDA	0.147	0.222	0.089	0.163	0.037	0.083
RBS	0.133	0.173	0.077	0.143	0.000	0.022
SAN	0.121	0.160	0.081	0.131	0.043	0.070
GLE	0.127	0.155	0.095	0.133	0.000	0.024
STT	0.179	0.213	0.095	0.188	0.045	0.086
SMFG	0.101	0.112	0.051	0.103	0.000	0.000
UBSN	0.140	0.181	0.076	0.144	0.048	0.059
UCG	0.134	0.162	0.079	0.122	0.000	0.000
WFC	0.168	0.200	0.095	0.148	0.030	0.071

Table 2.5: p-values of the Christoffersen Test for CoVaR

	90% CoVaR				95% CoVaR			
	Unconditional		Conditional		Unconditional		Conditional	
	AGMNTS	AGMNormal	AGMNTS	AGMNormal	AGMNTS	AGMNormal	AGMNTS	AGMNormal
BAC	0.023	0.000	0.023	0.000	0.543	0.000	0.250	0.000
BOC	0.099	0.078	0.082	0.076	0.528	0.003	0.289	0.002
BK	0.000	0.000	0.000	0.000	0.120	0.001	0.115	0.001
BARC	0.098	0.001	0.079	0.001	0.327	0.002	0.138	0.000
BNP	0.044	0.005	0.012	0.001	0.223	0.000	0.084	0.000
C	0.032	0.000	0.019	0.000	0.424	0.000	0.181	0.000
CBK	0.087	0.000	0.067	0.000	0.108	0.000	0.104	0.000
CSGN	0.006	0.000	0.005	0.000	0.097	0.002	0.033	0.000
DBK	0.172	0.013	0.079	0.013	0.298	0.000	0.115	0.000
DEXB	0.118	0.000	0.042	0.000	0.102	0.000	0.102	0.000
GS	0.076	0.001	0.060	0.001	0.152	0.000	0.055	0.000
ACA	0.030	0.000	0.007	0.000	0.152	0.000	0.055	0.000
HSBA	0.684	0.036	0.026	0.010	0.114	0.009	0.039	0.002
INGA	0.286	0.004	0.172	0.002	0.081	0.000	0.023	0.000
JPM	0.053	0.000	0.038	0.000	0.015	0.000	0.003	0.000
LLOY	0.381	0.012	0.380	0.012	0.272	0.000	0.226	0.000
MUFG	0.802	0.408	0.535	0.353	0.831	0.094	0.426	0.034
MHFG	0.516	0.671	0.454	0.307	0.183	0.162	0.172	0.069
MS	0.028	0.000	0.026	0.000	0.795	0.001	0.377	0.000
NDA	0.032	0.000	0.013	0.000	0.084	0.000	0.084	0.000
RBS	0.130	0.002	0.127	0.002	0.241	0.000	0.205	0.000
SAN	0.286	0.005	0.002	0.002	0.149	0.000	0.144	0.000
GLE	0.181	0.013	0.180	0.013	0.047	0.001	0.012	0.000
STT	0.002	0.000	0.001	0.000	0.089	0.000	0.032	0.000
SMFG	0.980	0.624	0.467	0.313	0.982	0.044	0.463	0.044
UBSN	0.058	0.001	0.037	0.001	0.223	0.000	0.084	0.000
UCG	0.092	0.003	0.006	0.003	0.151	0.001	0.047	0.001
WFC	0.003	0.000	0.002	0.000	0.047	0.000	0.012	0.000
# of p-values less than 5%	10	24	16	24	3	26	8	27
# of p-values less than 1%	4	20	7	21	0	25	1	25

An alternative approach to risk spillover effects is CPNM. We compare ΔCoAVaR and CPNM separately, both in time series and cross-section directions. Recall that CoAVaR is preferable to CoVaR for risk assessment.

To compare time series, we prepare three regional portfolios in the United States, Europe, and Asia. These are equally weighted portfolios comprising G-SIFI stocks belonging to each region, and are intended to represent the time series of stock returns in each region. In Figure 2.3, the AVaR of regional portfolios and ΔCoAVaR and CPNM of each regional portfolio on the market index are plotted in the time series direction. The estimations are made using both AGMNTS and

AGMNormal models. We observe that the AGMNTS model gives more conservative estimations of systematic risk measures than the AGMNormal model because of its superior descriptive power for tail dependencies. From a comparison among risk measures, ΔCoAVaR is found to move significantly parallel to AVaR in the time series direction. It is a natural consequence that higher risk leads to higher risk spillover effects. On the other hand, neither does CPNM show strong linkage with AVaR or ΔCoAVaR, nor it is very sensitive to global adverse impacts. However, ΔCoAVaR and CPNM agree with the magnitude relation; the influence of Asia on the system is relatively lower than that of the United States and Europe. It also follows our assumption regarding the regional power of influence on the global financial system.

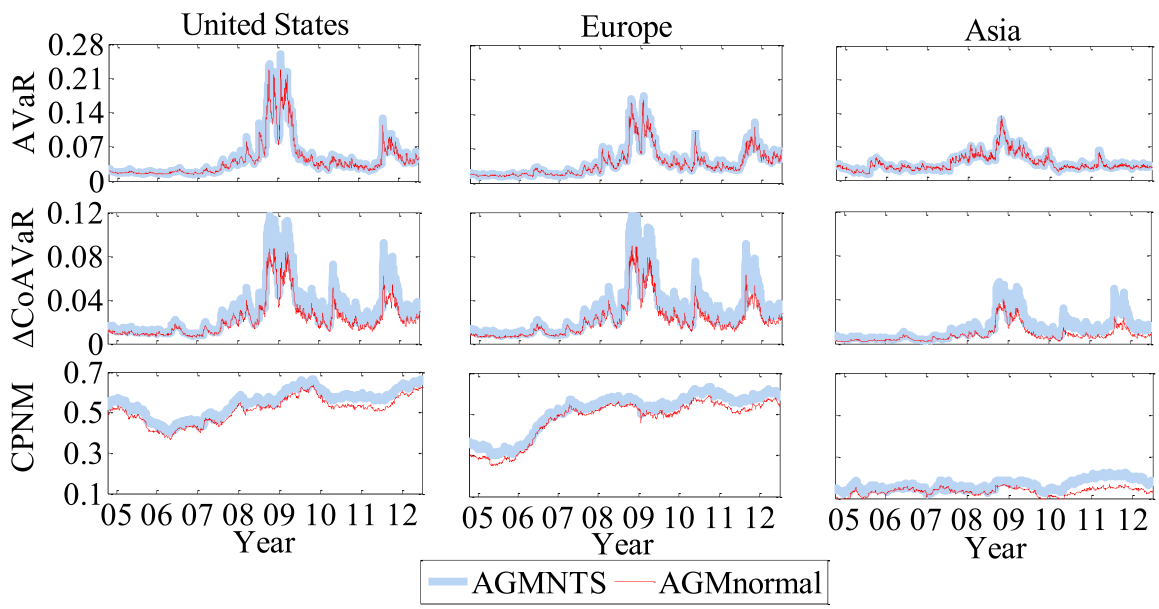

Figure 2.3: Time Series of AVaR, ΔCoAVaR, and CPNM by Region

The situation is different in the cross-section direction. To gain visual understanding, the scatter plots of cross-sectional ΔCoAVaR vs. AVaR and ΔCoAVaR vs. CPNM are depicted in the upper and lower halves of Figure 2.4, respectively, where the average of risk measures is taken over each stock's time series during the three turmoil periods suggested by JPNM in Figure 2.2. It appears that the cross-sectional AVaR has very weak linkage with the cross-sectional ΔCoAVaR. This result supports the idea that the institution that has higher risk is not necessarily the same one as the institution whose risk contribution to the entire system is larger. The contribution to systematic risk should be dependent not only on the institution's stand-alone risk measured by, for example, VaR, but also on other factors such as interconnectedness with other institutions. By contrast, CPNM has strong positive linear linkage with ΔCoAVaR. Though four points corresponding to the Asian G-SIFIs outlie others in each scatter plot, they still appear to be on a line. This suggests that ΔCoAVaR and CPNM are consistent when ranking the power of influence on the entire system among institutions at the same time. This consentience is already observed about the ranking among three regions in Figure 2.3. We further investigate the relationship among AVaR, ΔCoAVaR, and CPNM via the single linear regression, where the explained variable is ΔCoAVaR and the explanatory variables are AVaR and CPNM. Because we have 2011 daily cross-sectional datasets for 28 G-SIFI stocks, we iteratively run the regression 2011 times. Table 2.6 reports the number of significantly non-zero regression coefficients at the 1% level by signs and average R^2 out of 2011 tests by risk measures at three different confidence levels. For AVaR, significantly positive coefficients at the 1% level to ΔCoAVaR are obtained from less than 10% of all trials and R square is, on average, quite low regardless of confidence levels. For CPNM, in contrast, all trials result in a significantly positive coefficient at the 1% level with very high average R^2. Therefore, from statistical evidence, we confirm that AVaR has almost nothing to do with ΔCoAVaR, but that CPNM has very strong positive linkage with ΔCoAVaR in the cross-section direction.

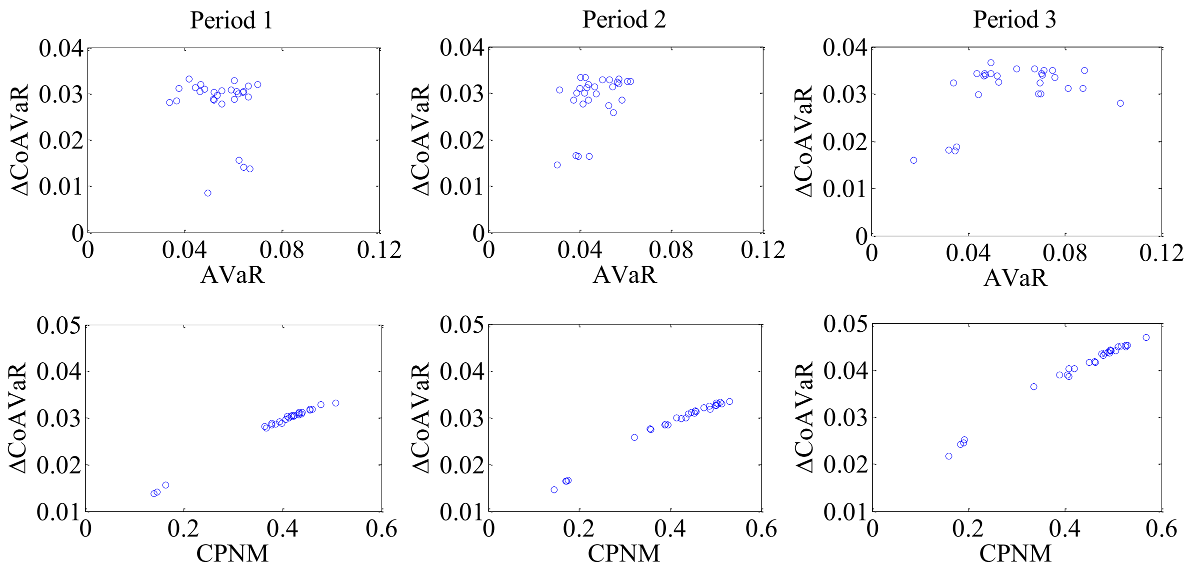

Figure 2.4: Cross-Sectional Linkage among AVaR, ΔCoAVaR, and CPNM

Table 2.6: Iterative Single Regression Analysis for 2011 Cross-Sectional Datasets among AVaR, ΔCoAVaR, and CPNM

Explanatory Variable		Sign of Coefficient	Confidence Level: 10%	Confidence Level: 5%	Confidence Level: 1%
AVaR	# of significant coefficients at the 1% level	positive	155	170	188
		negative	231	218	177
	Average R^2		0.129	0.127	0.124
CPNM	# of significant coefficients at the 1% level	positive	2011	2011	2011
		negative	0	0	0
	Average R^2		0.982	0.973	0.943

2.5 Concluding Remarks

In this chapter, we measure global systematic risk and the marginal contributions to it of the institutions by using stock return data of G-SIFIs, which constitute a large portion of the global banking system. To generate the future joint distribution of stock returns, we utilize the ARMA–GARCH–MNTS and ARMA–GARCH–MNormal models. The statistical tests demonstrate that the ARMA–GARCH–MNTS model is highly preferable to the ARMA–GARCH–MNormal model, mainly because of its capability of describing fat-tailness and skewness of stock return distributions.

We prepare both probability-based indicators and measures to quantify the marginal contribution to systematic risk. To be specific, we estimate the joint probability and conditional probability of negative stock return movements, ΔCoVaR, and ΔCoAVaR against the market index. The joint probability of negative movements turns out to vividly describe a significant increase of systematic risk. It provides information that VaR or AVaR lacks and could be referred to as a signal of financial turmoil. The other measures are for risk spillover effects rather than systematic risk itself. We find that AVaR has very weak linkage with ΔCoAVaR in the cross-section direction, even though both are strongly connected to each other in the time series direction, implying that the institution having higher risk is not necessarily the institution which has a larger power of influence on the entire system. Therefore, exclusively referring to VaR can be misleading for a *macro*-prudential purpose. These results are consistent with those of Adrian and Brunnermeier (2011) for the U.S. financial institutions. On the other hand, the probability of negative movements of the market index on the condition of the instituion's distress tends to provide very similar implications to ΔCoAVaR about the ranking of the institution's power of influence on the entire system. The relative merit of ΔCoAVaR to conditional probability is a stronger sensitivity to adverse impact on the global financial system and the ability to quantify the impact, whereas the relative merit of conditional probability to ΔCoAVaR is the easiness of estimation. From these observations, we conclude that combining AVaR and the conditional probability of negative movements would give a useful reference for ΔCoAVaR-based systematic risk measurement.

Chapter 3 Mean–CoAVaR Optimization for Global Banking Portfolios

3.1 Introduction

The recent financial turmoil has so severely deteriorated the global investment environment that traditional portfolio management theory currently has less effect. The failure of Lehman Brothers in September 2008 and the subsequent financial crisis, referred to as the "Lehman shock," had an adverse impact on global financial markets, causing massive spillover effects and bringing attention to systemic risk. In such a situation, portfolio managers are exposed to and need to address undiversifiable risk; loss is more or less inevitable no matter how they construct a portfolio. Undiversifiable risk is likely to be especially applicable to the banking sector because global banks are now closely interconnected. Undiversifiable risk is also called systematic risk.

The benchmark of modern portfolio theory is Markowitz's mean–variance optimization theory (Markowitz, 1952). The framework of mean–variance optimization is to construct a portfolio minimizing the variance with a given desired expected return. However, it has been revealed that the variance is not always an appropriate risk measure to be minimized. Subsequently, several alternative approaches have been proposed to replace the variance in Markowitz's theory with other risk measures such as Value at Risk (VaR) and average VaR (AVaR), which are called mean–VaR and mean–AVaR optimization, respectively. Such approaches are collectively called mean–risk portfolio optimization. In general, AVaR is preferable to VaR in terms of the optimization problem. While VaR optimization is basically a nonconvex and nonsmooth problem with multiple local minima, AVaR optimization is a convex and smooth problem. See Rachev et al. (2008b) for a general description and history of mean–risk portfolio optimization problems.

Recently, Adrian and Brunnermeier (2011) proposed CoVaR or ΔCoVaR measure for systemic risk. CoVaR, specifically $\text{CoVaR}^{j|i}$, is defined between two institutions i and j. $\text{CoVaR}^{j|i}$ is the VaR of j on a certain condition of i. $\Delta\text{CoVaR}^{j|i}$ is the difference between the VaR of j on the condition of i being distressed and "normal." Note that either i or j can be the entire system. While the case of j being the system usually attracts more attention because $\Delta\text{CoVaR}^{system|i}$ can quantify the marginal risk contribution of i to the overall system, Adrian and Brunnermeier (2011) also mention the case of i being the system. They refer to $\Delta\text{CoVaR}^{j|system}$ as "exposure CoVaR" in the sense that it can be interpreted as j's exposure to systemic risk. CoVaR and ΔCoVaR are directly extended into the counterparts of AVaR, which we call CoAVaR and ΔCoAVaR [20]. We apply the CoVaR methodology to stock markets, where the financial system is approximated by the market index.

In this chapter, we adopt CoAVaR as the objective function and propose mean–CoAVaR portfolio optimization. Even though the loss caused by systematic risk might be inevitable, we attempt to at least mitigate it through CoAVaR optimization. Because $\text{CoAVaR}^{j|index}$ captures j's vulnerability to the overall market risk, we expect to make the portfolio immune to systematic loss by minimizing the CoAVaR of the portfolio against the market index, i.e., $\text{CoAVaR}^{port|index}$. We perform an empirical study by using daily stock return data of 28 listed global systemically important financial institutions (G-SIFIs), as of November 2011. A G-SIFI stock is a good choice for testing the effect of a mean–CoAVaR strategy against systematic risk because G-SIFIs are specified by financial regulators as the institutions with a huge influence on the global financial system, and that potentially experience systemic risk in terms of their size, interconnectedness and so on (Basel Committee on Banking Supervision, 2011b; Financial Stability Board, 2011).

[20] In Adrian and Brunnermeier (2011), CoAVaR is mentioned as CoES, where ES stands for expected shortfall.

By comparing the performance of the portfolio minimizing CoAVaR with that of the portfolio minimizing traditional risk measures such as variance and AVaR, we confirm the effectiveness of mean–CoAVaR optimization. This chapter is a sequel to Chapter 2. We now focus on the management of systematic risk from the perspective of a portfolio manager, whereas we focused on the measurement of systematic risk in Chapter 2. See Chapter 2 for more information on notations, description of datasets, and methodology because some of these are shared with this chapter.

The rest of this chapter is structured as follows. In Section 3.2, we formulate mean–CoAVaR portfolio optimization. Section 3.3 provides an empirical study by using 28 G-SIFI stocks and an ARMA–GARCH[21] forecast. Section 3.4 is devoted to concluding remarks.

3.2 Mean–CoAVaR Optimization

In line with the concept of mean–risk optimization, we propose *mean–CoAVaR* portfolio optimization to minimize a portfolio's potential loss caused by systematic risk. Because exposure CoVaR is a measure of vulnerability to systematic distress, it is quite a natural idea to minimize it for the purpose of a defense against systematic risk. We select CoAVaR as the objective function rather than CoVaR because of the drawbacks of VaR optimization. Note that we also select CoAVaR rather than $\Delta\mathrm{CoAVaR}$ for the following reason. While $\Delta\mathrm{CoAVaR}^{port|index}$ focuses on the increase in the risk of a portfolio in the case of financial crisis, i.e., exposure to systemic distress, $\mathrm{CoAVaR}^{port|index}$ accounts for the portfolio's idiosyncratic risk in addition to exposure. The latter quantity should be minimized in terms of portfolio loss mitigation.

[21] Autoregressive moving average generalized autoregressive conditional heteroscedasticity.

Let R_t^i be the return of stock *i*. The subscript *t* stands for a time period. We assume that any return distribution is continuous. Let $\mathbb{C}(R_t^i)$ be a certain condition of R_t^i. Then, the CoAVaR of stock *j* on the condition $\mathbb{C}(R_t^i)$ at the confidence level $1 - q$ is defined as

$$\begin{aligned} \mathrm{CoAVaR}_{q,t}^{j|\mathbb{C}(R_t^i)} &= \frac{1}{q}\int_0^q \mathrm{CoVaR}_{p,t}^{j|\mathbb{C}(R_t^i)}\,\mathrm{d}p \\ &= -\mathrm{E}\left(R_t^j \middle| \left\{R_t^j < -Co\mathrm{VaR}_{q,t}^{j|\mathbb{C}(R_t^i)}\right\} \cap \mathbb{C}(R_t^i)\right), \end{aligned} \tag{3.1}$$

where $\mathrm{CoVaR}_{q,t}^{j|\mathbb{C}(R_t^i)}$ is the VaR of stock *j* on the condition $\mathbb{C}(R_t^i)$ at the confidence level $1 - q$.

Let $\boldsymbol{w}_t = (w_t^1, w_t^2, \cdots, w_t^J)$ be a set of weights of stock $1,2,\cdots,J$ in the portfolio. We now formulate the *mean–CoAVaR* portfolio optimization as follows:

$$\begin{gathered} \min_{\boldsymbol{w}_t} \ \mathrm{CoAVaR}_{q,t}^{port(\boldsymbol{w}_t)|\mathbb{C}^d(R_t^{index})}, \\ \text{s.t.} \ \sum_{j=1}^J w_t^j \mu_t^j = \bar{\mu}_t, \sum_{j=1}^J w_t^j = 1, w_t^j \geq 0, \forall j, \end{gathered} \tag{3.2}$$

where μ_t^j is a conditional mean of R_t^j on the information up to *t – 1* and $\bar{\mu}_t$ is an expected return of the portfolio. Note that CoAVaR is defined for the portfolio return $R_t^{port(\boldsymbol{w}_t)} = \sum_{j=1}^J w_t^j R_t^j$ against the market index return R_t^{index}. The distress condition $\mathbb{C}^d(R_t^{index})$ is defined as the loss of the index being above its VaR:

$$\mathbb{C}^d(R_t^{index}) = \{R_t^{index} \leq -\mathrm{VaR}_{q,t}^{index}\}. \tag{3.3}$$

Short selling is prohibited in line with common practice. For simplicity, we do not take transaction costs into account.

3.3 Empirical Study

We evaluate a mean–CoAVaR strategy through an empirical study by using daily stock logarithmic return datasets of 28 out of 29 G-SIFIs, as of November 2011, where the only exclusion is Banque Populaire CdE because it is unlisted. The list of G-SIFIs is given in Appendix A. We refer to each stock by its ticker symbol or abbreviation. We use the S&P global 1200 financial sector index (SGFS) to represent the global banking stock market.

The procedure of evaluating a mean–CoAVaR strategy is as follows. First, we generate the one-period-ahead joint stock return distribution using the multivariate ARMA(1,1)–GARCH(1,1) model. We assume that the innovations of the ARMA–GARCH model follow the multivariate normal tempered stable (MNTS) distribution[22] because it is a better model for G-SIFI stocks compared with the Gaussian model in terms of both goodness of fit and accuracy of risk measure estimation, as demonstrated in Chapter 2. Subsequently, under the predicted stock return joint distribution, we find the optimized portfolio $\boldsymbol{w}_t$ through three different strategies: mean–variance, mean–AVaR, and mean–CoAVaR optimization. In other words, we minimize the variance, AVaR, and CoAVaR of the portfolio under the same constraints for the three strategies, respectively, as given in (3.2). We regard an equally weighted portfolio as the benchmark, and thus set the expected return $\bar{\mu}_t$ as the simple average of conditional means μ_t^j. We rebalance the portfolio to the optinum each business day. Finally, we compare the performance among strategies in terms of long-run loss mitigation effects.

The operation period starts at January 1st, 2008 and ends at June 30th, 2012, during which systemic risk is of great concern. The operation days amount to 1174 in line with the United States business days. Each business day, the parameters of the ARMA–GARCH model are

[22] See Kim et al. (2012) and Appendix B for the MNTS distribution.

updated on the basis of the most recent 1250 days' historical stock return data. Historical returns are backfilled where missing in the same manner in Chapter 2. The confidence level of risk measures is set as $1 - q = 0.95$. We use the matlab *fmincon* command for optimization problems.

The portfolio is constructed from G-SIFI stocks. To see whether the effectiveness of strategies depends on portfolio size or regional specificity, we prepare three portfolios constructed from different number of stocks and another three portfolios constructed from different regional stocks. The three different-sized portfolios are referred to as large, middle, and small. The large group includes all 28 G-SIFI stocks; the middle group includes the following 12 stocks: BAC, BARC, BNP, C, CBK, CSGN, DBK, HSBA, MUFG, GLE, SMFG, and UBSN; and the small group includes the following 6 stocks: BAC, BARC, BNP, CBK, MUFG, and UBSN. For the middle and small groups, sample stocks are chosen from six countries, the United States, the United Kingdom, France, Germany, Switzerland, and Japan, which play critical roles in the global banking system in the sense that more than one institutions are selected as G-SIFIs from those countries. The three regional portfolios are constructed from G-SIFI stocks in each region: 8 stocks from the United States, 16 stocks from Europe, and 4 stocks from Asia.

The results are summarized in Tables 3.1 and 3.2 for different-sized portfolios and different regional portfolios, respectively. They report standard deviation, skewness, kurtosis of the realized daily returns of the optimized portfolios, the number of days on which the optimized portfolio outperforms the benchmark regarding the return, and cumulative return in percentage terms. The statistics of the market index and equally weighted portfolio are also presented as a reference. The main remark in Tables 3.1 and 3.2 is that the mean–CoAVaR and mean–AVaR portfolios generally incur smaller cumulative loss than the mean–variance portfolio. In Table 3.1, the mean–variance portfolio incurs an even larger cumulative loss than the simple equally weighted portfolio in the middle and small groups. We frequently observe that the mean–variance portfolio yields at most the same performance as the equally weighted portfolio. This supports the idea that the variance is not necessarily a proper risk measure during financial turmoil. Second, the mean–CoAVaR strategy still has loss mitigation effects compared with the mean–AVaR strategy in most cases in Tables 3.1 and 3.2. In Table 3.1, the loss mitigation effect is the least in the large group and the greatest in the small group. It can be explained by the size of the portfolio. When a portfolio is diversified by incorporating a larger number of stocks, the structure of the portfolio becomes closer to the market index. Therefore, the mean–AVaR optimization for a larger portfolio captures systematic risk well even without explicitly considering the co-movement between the portfolio and entire market as CoAVaR does. In Table 3.2, the Asia group is the only exception out of all six portfolios where the mean–CoAVaR strategy is inferior to the mean–AVaR strategy, and moreover, the mean–AVaR strategy is inferior to the mean–variance strategy in terms of the cumulative loss. However, note that the mean–CoAVaR strategy incurs the smallest cumulative loss among the three strategies in the other five cases.

The time series of the cumulative return of the portfolios optimized by three different mean–risk strategies for the small and Europe groups is plotted in Figures 3.1 and 3.2, respectively. In addition, the difference in the cumulative return between the mean–CoAVaR and mean–AVaR portfolios is also plotted in Figures 3.3 and 3.4. Note that the small and Europe groups constitute the portfolio where the mean–CoAVaR strategy has the most pronounced effect of mitigating the loss among different-sized portfolios and different regional portfolios, respectively. It is observed in the figures that the mean–CoAVaR portfolio has a noticeable difference in the cumulative return from the mean–AVaR portfolio after the collapse of Lehman Brothers, which triggered financial turmoil and concern about systemic risk. From the observations above, we conclude that the mean–CoAVaR optimization is as effective or even better compared with the mean–AVaR optimization, especially when systematic distress is of great concern.

3.4 Concluding Remarks

In this chapter, we propose mean–CoAVaR portfolio optimization to mitigate the potential loss arising from systematic risk. Since the CoAVaR of the portfolio accounts for the intrinsic risk and extent of its vulnerability to systematic downturn, on the condition that the market index is in distress, CoAVaR is expected to be a good candidate for the objective function to be minimized against undiversifiable risk. Note that CoAVaR is more appropriate than CoVaR for the optimization problem because of convexity.

We examine the effectiveness of the proposed mean–CoAVaR optimization by using 28 listed G-SIFI stocks. G-SIFIs are good trial samples to test the mean–CoAVaR strategy because they are both highly interconnected and potentially affected by systematic risk in global financial markets. We utilize the ARMA(1,1)–GARCH(1,1) model with the MNTS distributed innovations to forecast the one-period-ahead joint distribution of stock returns, which is revealed to be a better model for G-SIFI stocks in Chapter 2. Throughout the empirical study, we observe that the mean–CoAVaR portfolio incurs smaller cumulative loss than the mean–AVaR and mean–variance portfolios in most cases. Therefore, we conclude that the mean–CoAVaR optimization is effective during the time of global bear markets. Until now, CoVaR has been considered primarily a macro-prudential tool for measuring the systemic importance of an institution. Our results open its applicability to risk management usage.

Table 3.1: Portfolio Performance of Three Mean–Risk Optimizations (by Size)

Portfolio	Standard Deviation	Skewness	Kurtosis	# of Outperforming Days	Cumulative Return
SGFS	0.022	-0.041	8.035	N.A.	-71.580
Large Group (28 G-SIFIs)					
Benchmark (Equally Weighted)	0.026	0.054	9.186	N.A.	-127.879
Mean−Variance	0.016	-0.060	6.612	359	-124.692
Mean−AVaR	0.014	-0.028	9.123	599	-78.316
Mean−CoAVaR	0.014	0.068	9.384	606	-77.237
Middle Group (12 G-SIFIs)					
Benchmark (Equally Weighted)	0.027	0.206	8.323	N.A.	-126.879
Mean−Variance	0.022	-0.021	6.902	219	-127.338
Mean−AVaR	0.021	0.059	8.373	611	-99.424
Mean−CoAVaR	0.021	0.048	8.997	610	-96.641
Small Group (6 G-SIFIs)					
Benchmark (Equally Weighted)	0.029	0.124	8.312	N.A.	-136.908
Mean−Variance	0.025	-0.052	6.885	179	-167.646
Mean−AVaR	0.023	-0.014	7.390	581	-144.940
Mean−CoAVaR	0.023	0.027	7.241	592	-127.064

Note: The cumulative return is the cumulative amount of the weighted average of logarithmic returns of stocks in the portfolio and is expressed as a percentage. Thus, it can be lower than –100.

Table 3.2: Portfolio Performance of Three Mean–Risk Optimizations (by Region)

Portfolio	Standard Deviation	Skewness	Kurtosis	# of Outperforming Days	Cumulative Return
SGFS	0.022	-0.041	8.035	N.A.	-71.580
United States Group (8 G-SIFIs)					
Benchmark (Equally Weighted)	0.038	-0.041	14.612	N.A.	-87.213
Mean−Variance	0.034	-0.580	18.598	165	-130.506
Mean−AVaR	0.032	0.139	14.051	579	-89.881
Mean−CoAVaR	0.033	0.150	14.232	583	-89.520
Europe Group (16 G-SIFIs)					
Benchmark (Equally Weighted)	0.030	0.162	7.370	N.A.	-155.552
Mean−Variance	0.025	0.099	5.989	313	-104.376
Mean−AVaR	0.024	0.126	6.989	621	-38.265
Mean−CoAVaR	0.024	0.173	6.866	620	-34.009
Asia Group (4 G-SIFIs)					
Benchmark (Equally Weighted)	0.022	0.113	7.674	N.A.	-98.517
Mean−Variance	0.020	0.250	7.027	82	-73.763
Mean−AVaR	0.019	0.013	7.862	571	-75.031
Mean−CoAVaR	0.019	-0.012	7.939	570	-76.736

Note: The cumulative return is the cumulative amount of the weighted average of logarithmic returns of stocks in the portfolio and is expressed as a percentage. Thus, it can be lower than –100.

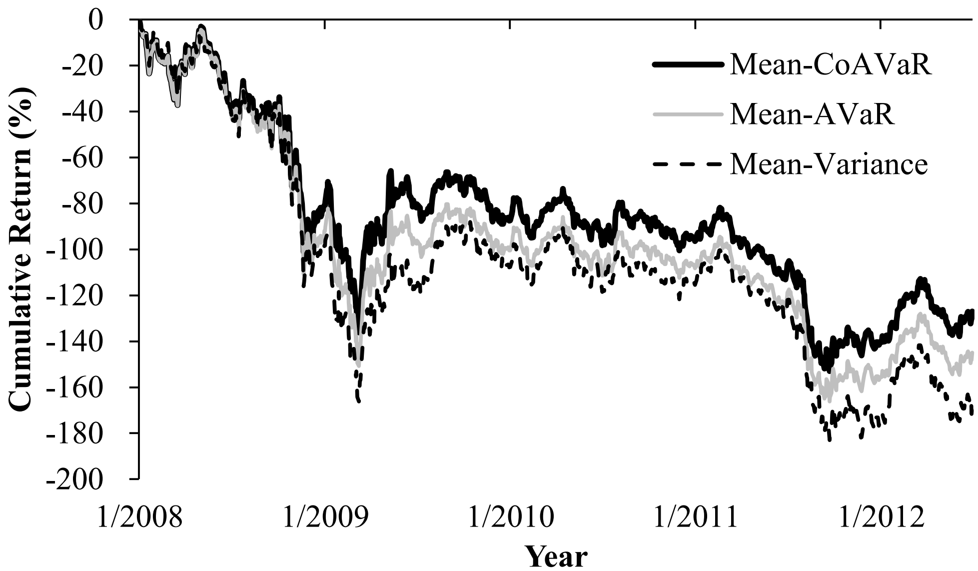

Figure 3.1: Cumulative Return of the Portfolios Optimized by Different Strategies (Small Group)

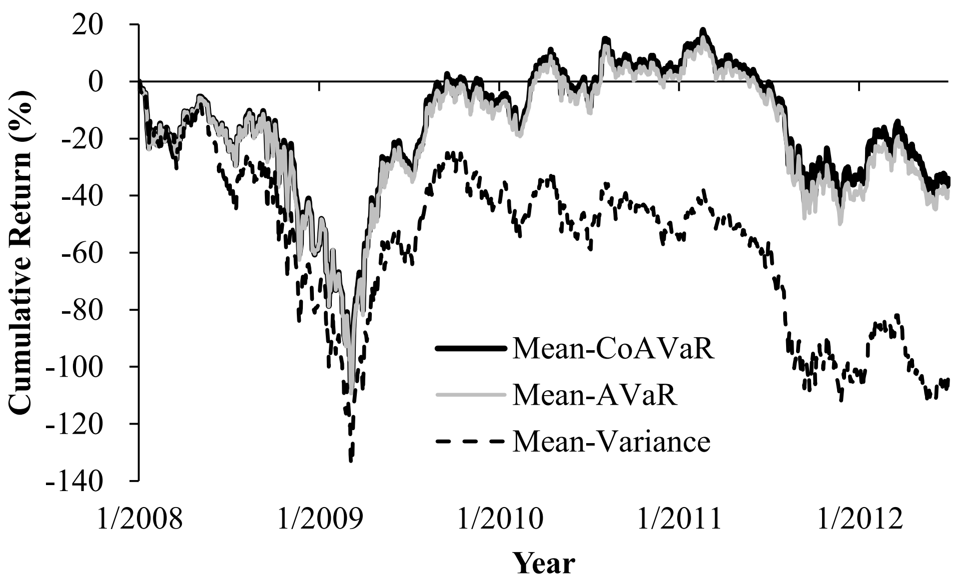

Figure 3.2: Cumulative Return of the Portfolios Optimized by Different Strategies (Europe Group)

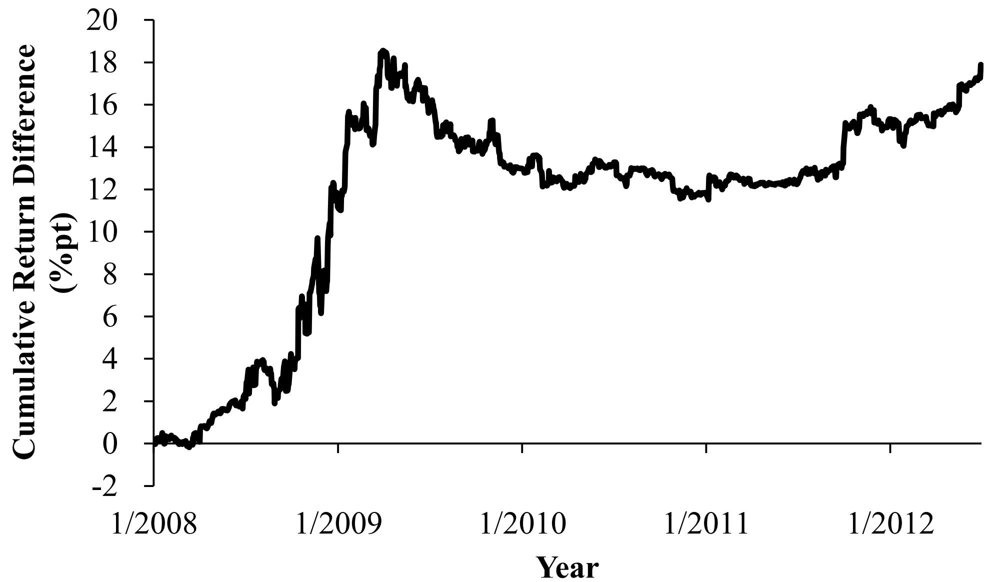

Figure 3.3: Difference in the Cumulative Return between the Mean–CoAVaR and Mean–AVaR Portfolios (Small Group)

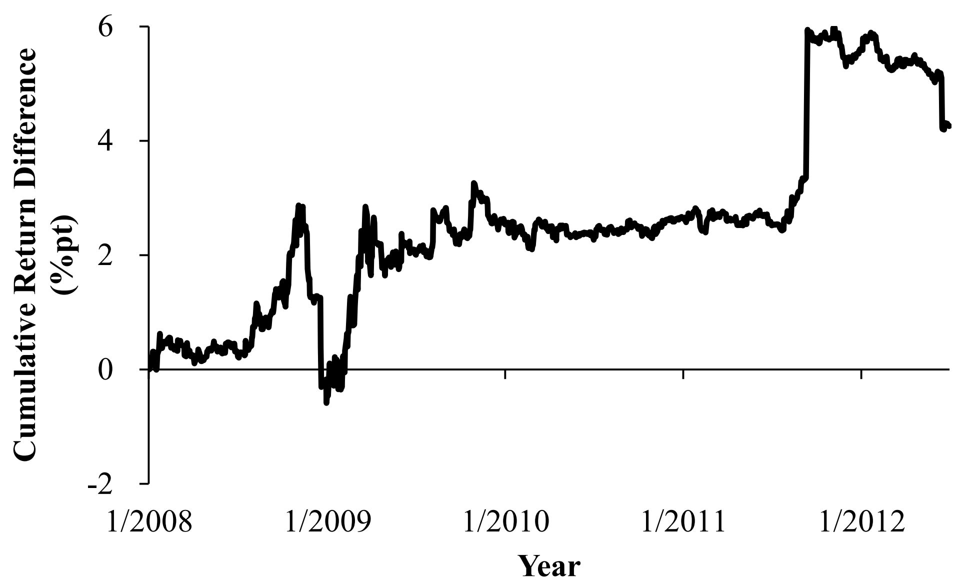

Figure 3.4: Difference in the Cumulative Return between the Mean–CoAVaR and Mean–AVaR Portfolios (Europe Group)

Chapter 4 Tempered Stable GARCH vs. Markov Switching Approaches for OECD Currency Exchange Rates

4.1 Introduction

The foreign currency exchange market is a particular kind of market in the sense that the market has very high liquidity and tradability anytime, and the exchange rate vividly reflects the monetary policy of each country. The exchange rate is not only a speculative object but also something that greatly affects the lives of citizens. Because it is critical for both investors and governments to gain a better understanding of the dynamics of exchange rates after the introduction of the floating rate system, it has a long history of research exploring the realistic time series models for exchange rates.

A generalized autoregressive heteroscedasticity (GARCH) model (Bollerslev, 1986) is currently regarded as the standard time series model for asset returns, because it can well describe the volatility clustering phenomenon which asset returns typically show. However, a GARCH model is not always preferred for exchange rate returns by contrast with stock returns, partly because structures in time series of exchange rate returns show weaker typical patterns. Some studies report negative results for using a GARCH model to describe the dynamics of exchange rates of currencies in some countries on the basis of in-sample tests (Bonilla et al., 2007; Brooks and Hinich, 1998). An alternative description of volatility clustering is provided by a Markov regime–switching model, or simply, the Markov–switching (MS) model (Hamilton, 1989, 1994). An MS model enables us to capture structural breaks in time series, caused by external factors such as policy changes, by introducing unobservable regimes. Engel and

Hamilton (1990) apply an MS model to exchange rate returns, by which they confirm long swings, i.e., exchange rates moving in one direction for long periods of time. Bollen et al. (2000) advocate that an MS model outperforms a GARCH model for describing the dynamics of exchange rates by comparing log-likelihood values, Ljung-Box (LB) statistics and variance error forecasts of both models, in which sample periods are from the beginning of 1973 to the end of 1996, data frequency is weekly, and sample currencies are three of the major ones during the sample periods: the British Pound Sterling, the Deutsche Mark, and the Japanese Yen in terms of United States Dollars.

However, we note that, in most attempts to apply a GARCH model to exchange rate returns, residuals are automatically assumed to follow the normal distribution. Mainly through empirical studies for stock prices, it has been well recognized that asset returns tend to have fat-tailness and skewness which the normal distribution is incapable of capturing. This tendency is especially clear after the recent financial crises with the background of a sudden price crash, which brings awareness of the tail risk. In line with this fact, Kim et al. (2008, 2011) utilize the classical tempered stable (CTS) and other tempered stable distributions for modeling the residuals of a GARCH model. They demonstrate that the GARCH model with the CTS distributed residuals is clearly superior to that with the normal and student t distributed residuals, by using the S&P 500 index. By using the stocks comprising the Dow-Jones index, Kim et al. (2012) also demonstrate that the normal tempered stable (NTS) distribution and its multivariate extension, called the multivariate NTS (MNTS) distribution, are also better models for the market data than the multivariate normal distribution. The effectiveness of using the MNTS structure as a copula is also studied (Kim and Volkmann, 2013). These results prove the superior descriptive power of the tempered stable distributions for fat-tailness and skewness[23]. The success of the CTS or NTS distributions for stock returns motivates us to apply these for exchange rate returns.

The main goal of this chapter is to reevaluate the performance of the GARCH model for exchange rate returns on the basis of the aforementioned developments, comparing the GARCH model against the MS model, which has been often preferred. Noting that the advent of the Euro may have dramatically altered the structure of the foreign currency exchange market, we take sample periods from after it was introduced. The sample periods are also chosen to cover the recent financial crises, because the model performance during the turmoil is relatively more significant than during tranquil time. Also, we expand the samples to all 18 Organization for Economic Co-operation and Development (OECD) currencies from three major currencies in Bollen et al. (2000), which seem far from reliable in a statistical sense. In addition, we investigate not only weekly data but also daily data as a robustness check. More recent sample periods, extended sample currencies, and higher frequency data make the issue worth revisiting.

To be more concrete, the procedure of comparisons of the model performance is as follows. We prepare two popular time series models, i.e., GARCH and MS models. For a GARCH model, we assume three distinct distributions for residuals: normal, student t, and NTS. The reason we prefer NTS to CTS or other tempered stable distributions is the multivariate extendability, which is exploited later for the purpose of multivariate modeling of exchange rates. The performance comparisons are on the basis of both in-sample and out-of-sample tests. Following Bollen et al. (2000), we adopt the log-likelihood values and LB statistics as in-sample tests, and variance forecast errors as an out-of-sample test. More than that, we extend out-of-sample tests by backtesting the Value at Risk (VaR) estimation, and independence and tail behavior of the

[23] For general descriptions of tempered stable distributions and the applications to the time series model, see Rachev et al. (2011).

forecast distribution. The former and the latter can be backtested by the Christoffersen's likelihood ratio (CLR) test (Christoffersen, 1998) and the Berkowitz's likelihood ratio (BLR) test (Berkowitz, 2001), respectively, in the same way as in Kim et al. (2011). These extensions are not only used because the performance of out-of-sample tests usually attracts more attention than that of in-sample tests but also because VaR is a regulatory-endorsed and thus more important risk measure compared with variance, and investigating the tail behavior of asset returns is imperative for risk management during financial crises. We are also conscious of average VaR (AVaR), the risk measure focusing on the tail risk. The BLR tail test leads to backtesting the AVaR estimation.

We further mention two related topics on the applications of GARCH forecasts for exchange rate returns. The first topic is the multivariate extension. We consider the GARCH model with the MNTS distributed residuals for exchange rate returns. Because the MNTS distribution can account for the interdependencies among exchange rate returns, the model is expected to work effectively to describe the co-movements of exchange rates. We also confirm the effectiveness of the GARCH model from this perspective. The second topic is the applicability of embedding an MS process in the GARCH model with the NTS distributed residuals. We consider how well GARCH residuals fit the NTS distribution depending on regimes in an attempt to find evidence of regime-switches of the residuals.

The remainder of the chapter is organized as follows. Section 4.2 introduces our methodology, which mainly consists of using GARCH and MS models. Section 4.3 describes the data to be used. Section 4.4 reports the empirical results of model parameter estimations and performance comparisons. Section 4.5 discusses the multivariate modeling of exchange rates on the basis of the multivariate GARCH model. Section 4.6 argues for the applicability of the MS structure to the GARCH model with the NTS distributed residuals. Section 4.7 is devoted to concluding remarks.

4.2 Methodology

We briefly introduce two popular time series models to be used in this chapter: GARCH and MS models. As a standard model specification for asset returns, we use GARCH(1,1) and two-regime MS models. Moreover, we embed an autoregressive (AR) process in the model, because the conditional mean of asset returns frequently shows it. For fairness of comparison, we assume an AR(1) process for both models, while Bollen et al. (2000) assume an AR(1) process only for a GARCH model. The resulting models are referred to as AR(1)–GARCH(1,1) and MS–AR(1) models, respectively. In addition to both models, we examine the performance of the standard AR(1) model as a benchmark. The parameters of the models are derived using the maximum likelihood estimation (MLE). We also mention how to estimate VaR by using both models.

4.2.1. AR(1)–GARCH(1,1) Model

An AR(1)−GARCH(1,1) model is given by

$$R_t = \mu_t + \sigma_t \eta_t,$$

$$\eta_t \sim \text{ i.i.d. with zero mean and unit variance},$$

$$\mu_t = aR_{t-1} + b,$$

$$\sigma_t^2 = \omega \sigma_{t-1}^2 \eta_{t-1}^2 + \xi \sigma_{t-1}^2 + \Psi, \tag{4.1}$$

where R_t is an asset return, μ_t is a conditional mean, σ_t is a conditional standard deviation, and other symbols are model parameters to be estimated. t $(1 \leq t \leq T)$ stands for a time period. η_t is the i.i.d. with zero mean and unit variance, which we adjectively call *standardized*. Note that we hereafter refer to $\sigma_t \eta_t$ as residuals and η_t as standardized residuals. The key structure of a GARCH model is that the future standard deviation depends on both the current residual (ω) and the current standard deviation itself (ξ). The former and latter are called ARCH and GARCH effects, respectively. While η_t is conventionally assumed to follow the standardized normal i.i.d., we assume the standardized student t and NTS distributions for η_t in addition to the normal one, in order to deal with potential fat-tailness and skewness of asset returns. The standardized NTS distribution is characterized by three parameters (α, θ, β), where α and θ are related to fat-tailness, and β controls the skewness[24]. Smaller α and θ mean a fatter tail. A negative (positive) β signifies the left (right) skewness of the distribution, whereas the distribution is symmetric when $\beta = 0$. The existence of the skewness parameter β makes NTS superior to student t. See Kim et al. (2012) and Appendix B for a definition of the NTS distribution.

We refer to the AR(1)−GARCH(1,1) model with the standardized normal, student t, and NTS distributed residuals as AGNormal, AGT, and AGNTS models, respectively. An AGNTS model is derived by fitting the NTS distribution to the standardized residuals of an AGT model, in the same way as Kim et al. (2008, 2011). We also collectively refer to AGT and AGNTS models as an AGFT model, where FT stands for fat-tail, because both models share the same coefficients and residuals of an AR(1)–GARCH(1,1) model.

4.2.2. MS–AR(1) Model

An MS–AR(1) model with two regimes is given by

$$R_t = a_{s_t} R_{t-1} + b_{s_t} + \sigma_{s_t} \eta_t,$$

$$\eta_t \sim \text{i.i.d. with zero mean and unit variance,} \qquad (4.2)$$

where $s_t \in \{1,2\}$ stands for the latent regime variable and a_{s_t}, b_{s_t}, and σ_{s_t} are regime-dependent AR(1) coefficient, intercept, and standard deviation, respectively. We assume the standardized normal distribution for η_t in line with common practice, by contrast with the modification to the

[24] The ranges of parameters are $0 < \alpha < 2, 0 < \theta, |\beta| < \sqrt{\frac{2\theta}{2-\alpha}}$.

GARCH model[25]. We expect that regime 1 corresponds to a high-mean and low-standard deviation regime, i.e., a tranquil regime, and regime 2 corresponds to a low-mean and high-standard deviation regime, i.e., a turbulent regime. The switching between regimes is described by the following time-homogeneous Markov chain:

$$p_{ij} = \text{Prob}(s_{t+1} = i | s_t = j), i, j \in \{1,2\}. \tag{4.3}$$

The smoothed probability $\text{Prob}(s_t = i | \Omega_T)$, where Ω_T stands for the set of the information available at the end period *T*, gives the information about the probability of the regime $i = 1,2$ in the past and thereby enables us to assess the timing of structural change. This is a conspicuous merit of the MS model which the GARCH model lacks. See Appendix D for more information on the MS model.

We should note that Bollen et al. (2000) adopt not only the standard MS model but also the modified MS model in which mean and standard deviation have independent switching processes, specified by four regimes in total, whereas mean and standard deviation switch simultaneously in the standard two-regime MS model. They show that the modified model even has improvements on the standard model. Moreover, there is also literature which supports the same modification, criticizing the simultaneous switching of mean and standard deviation as too restrictive (Dewachter, 2001; Ichiue and Koyama, 2011). Nonetheless, we still adopt the standard MS−AR(1) model as given in Eq. (4.2) for the following reasons. First, the standard model is literally standard, in the sense that it is the model originally proposed by Hamilton (1989, 1994) and is traditionally employed. The estimation algorithm is well established and even some

[25] Note that, even though the normal distribution is assumed for the residuals in the MS model, the resulting conditional distribution of R_t is given by the normal mixture, which is expected to have better descriptive power at the tail than a single normal distribution.

packages are publicly provided[26]. Second, we focus on the GARCH model, whereas Bollen et al. (2000) focus on the MS model. To assess the superiority of the GARCH model, the standard MS model sufficiently serves as a counterpart model. In fact, the difference in the log-likelihood value of the modified MS model from the standard MS model is much smaller than that of the standard MS model from the GARCH model in Bollen et al. (2000). Third, we embed an AR(1) process into the standard MS model, which might mitigate the restriction of the simultaneous switching to some extent; the mean depends not only on the regime but also on the past mean. Fourth, both the AGNTS and the standard MS–AR(1) models have the same number of parameters (eight). It makes a fair comparison between both models.

4.2.3. VaR Estimation

We estimate VaR on the basis of both AR(1)–GARCH(1,1) and MS–AR(1) models. VaR is the standard risk measure among financial institutions, because it is endorsed by financial regulators. Let $\mathrm{VaR}_\epsilon(R)$ be the VaR at the $1-\epsilon$ confidence level, or $100(1-\epsilon)\%$ VaR, for an asset return *R*. The definition of $\mathrm{VaR}_\epsilon(R)$ is given by

$$\mathrm{VaR}_\epsilon(R) = -\inf\{r | \mathrm{Prob}(R \le r) \ge \epsilon\}. \tag{4.4}$$

If *R* is continuous, $\mathrm{VaR}_\epsilon(R)$ is reduced to the ϵ-quantile of *R*:

$$\mathrm{VaR}_\epsilon(R) = -\hat{F}_R^{-1}(\epsilon), \tag{4.5}$$

where $\hat{F}_R$ is the estimated cumulative distribution function (CDF) of *R*. Because any return distribution is assumed to be continuous in this chapter, it is enough for us to focus on Eq. (4.5).

From the viewpoint of risk management, the accuracy of the VaR forecast is an important criterion of the validity of the time series model. The simplest way to backtest the estimated VaR is to see how often VaR is violated, that is, how often the observed loss falls below the pre-

[26] For instance, the matlab package for the MLE of the standard MS model is provided by Perlin (2012). We exploit it in this chapter.

estimated VaR. If the VaR violation occurs at a frequency of around ϵ in the observations as a whole, the estimation of the $100(1-\epsilon)\%$ VaR is appropriate. In addition to such a simple assessment, we employ the CLR test with unconditional and conditional coverage properties[27] for backtesting of the VaR estimation. The conditional coverage test has the merit of considering the tendency for consecutive VaR violations, which are actually observed during financial turmoil. See Christoffersen (1998) for details.

In spite of its popularity, there are some drawbacks to VaR. The main problem is that it is never informative about the risk above the VaR level, i.e., the tail risk, which attracted a fair amount of attention during the recent financial crises. AVaR is an alternative risk measure to overcome this drawback and thus now gaining in popularity. For the continuous return distribution, AVaR at the $1-\epsilon$ confidence level is simply the expected loss on the condition that the loss is at levels in excess of $100(1-\epsilon)\%$ VaR, which is called the expected tail loss. See Chapter 2 for more information on AVaR.

The accuracy of the AVaR forecast is indirectly backtested through the BLR test. The BLR test uses the transformed observations: $Q_{t+j} = \Phi^{-1}(\hat{F}_{R,t}(R_{t+j}))$, where Φ is the standardized normal CDF, $\hat{F}_{R,t}$ is the CDF of R conditional on the information known up to a period t, and j is the forecast length. By using the BLR test, both the independence of the forecast distribution and the accuracy of the tail behavior forecast can be tested. Then, the accurate tail behavior forecast leads to the accurate AVaR forecast. See Berkowitz (2001) for details.

We derive a straightforward, one-period-ahead forecast of the return distribution by applying both time series models, i.e., Eqs. (4.1) and (4.2). By repeating the one-period-ahead forecast of conditional mean and standard deviation, it is also possible to derive a multi-period-ahead

[27] The conditional test is the joint test of unconditional coverage and independence.

forecast. On the basis of the forecast distributions, we estimate VaR as the forward-looking risk measures. The VaR estimations by using the AGNormal and AGT models are classical. For the AGNTS model, we employ a closed-form formula of VaR for infinitely divisible distributions provided by Kim et al. (2010). See also Appendix C. To implement the BLR test for the AGNTS model and compute the log-likelihood values, we use the formula of the CDF and probability density function (PDF) of the NTS distribution given in Kim and Volkmann (2013). See also Appendix B. In the framework of the standard MS–AR(1) model, VaR estimations are straightforward because the CDF and PDF are explicitly obtained as the normal mixture distribution. See Eqs. (D.3) and (D.4) in Appendix D.

4.3 Data

We use the dataset of foreign exchange spot rates in United States Dollars (USD) per unit of 18 currencies circulating in the OECD member countries as samples. Concretely, our 18 samples are as follows: Australian Dollar (AUD), Canadian Dollar (CAD), Swiss Franc (CHF), Chilean Peso (CLP), Czech Koruna (CZK), Danish Krone (DKK), Euro (EUR), British Pound Sterling (GBP), Hungarian Forint (HUF), Israeli Shekel (ILS), Icelandic Krona (ISK), Japanese Yen (JPY), Korean Won (KRW), Norwegian Krone (NOK), New Zealand Dollar (NZD), Polish Zloty (PLN), Swedish Krona (SEK), and Turkish Lira (TRY). Because the OECD member countries are often regarded as economically developed, circulating currencies are expected to be so liquid in the market that the time series model is immediately applicable. Data frequency is primarily weekly, in line with Bollen et al. (2000). The advantage of weekly data is that we can exclude the so-called day-of-the-week effect[28] from the time series. On the other hand, higher frequency trading is now a trend and a significant price change is more likely to happen in the

[28] There is a tendency towards negative returns during the period between Friday close and the next Monday close. This is often observed in stock markets. See, for instance, Condoyanni et al. (1987).

global market within a week. To address this issue, we supplement our analysis with daily frequency data. We utilize the log difference between consecutive Fridays (to be more precise, the last business days in the week) as an exchange rate return. For daily datasets, the return is the log difference between consecutive business days. Sample periods start with the beginning of 2000 and terminate with the end of 2011, which results in 625 weekly observations and 3129 daily observations for each exchange rate return. Sample periods are chosen to cover the advent of the Euro and two financial crises: the Lehman shock and the Greek sovereign debt crisis. All data are obtained from Bloomberg.

4.4 Empirical Results of Model Performance

We present the empirical results of the performance of AR(1)–GARCH(1,1) and MS–AR(1) models for OECD currency exchange rate returns. Data frequency is weekly, except for the Tables in Appendix E. The estimations are based on the return data within whole sample periods, except for the out-of-sample tests.

Before starting performance comparisons, we address a couple of issues on the model choice. Though we decided to embed an AR(1) process, there is a concern about whether embedding a moving average (MA) process is significant. Table 4.1 reports the Akaike and Baysian information criteria (AIC and BIC, respectively) of AR(1) and ARMA(1,1) models by currencies. All AIC and BIC comparisons are in favor of the AR(1) model, except AUD, CHF, and PLN in AIC. Therefore, we do not adopt an MA process. In the application of a GARCH model, we are concerned about whether the residuals of the AR(1) model have heteroscedasticity. Therefore, Table 4.1 also reports the results of Engle's ARCH test with one lag for AR(1) residuals. We find that the null hypothesis of no heteroscedasticity is rejected by 15 out of 18 currencies at the 10% significance level. This supports the usage of a GARCH model with exchange rate returns.

Tables 4.2 and 4.3 give the estimation results of AGNormal and AGFT models. Judging from the p-values, ARCH and GARCH effects are critical whereas AR(1) effects are not necessarily significant. Table 4.4 gives the estimation results of parameters of the student t (ν, degree of freedom) and NTS (α, θ, β) to AGFT standardized residuals. To investigate the goodness of fit of the proposed distributions to standardized residuals, we utilize the Kolmogorov−Smirnov (KS) test, where the null hypothesis is that standardized residuals follow the proposed distribution. To put more emphasis on the fitting at the tail, the Anderson−Darling (AD) and AD square (AD^2) statistics are also referred to. In this chapter, we define both statistics with n samples of η_i as

$$\text{AD}_n = \sup_{1\le i\le n} \frac{\left|F_\eta^{Emp}(\eta_i) - \hat{F}_\eta(\eta_i)\right|}{\sqrt{\hat{F}_\eta(\eta_i)\left(1-\hat{F}_\eta(\eta_i)\right)}},$$

$$\text{AD}_n^2 = -1 - \frac{1}{n^2}\sum_{i=1}^{n}\left[(2i-1)\ln \hat{F}_\eta(\eta_i) + \{2(n-i)+1\}\ln\left\{1-\hat{F}_\eta(\eta_i)\right\}\right], \qquad (4.6)$$

where F_η^{Emp} is the empirical CDF of n samples of η_i. The lower AD and AD^2 statistics mean better fitting at the tail. In Table 4.5, the statistics of KS (with p-values), AD, and AD^2 are reported for the three proposed distributions. The KS test reveals that any distributions are not rejected in most cases. Even the normal distribution is not rejected at the 5% significance level by all 18 currencies except three; CLP, NZD, and TRY[29]. On the other hand, generally speaking, the NTS distribution has the best fitting at the tail whereas the normal distribution has the worst, according to the AD and AD^2 statistics.

Table 4.6 gives the estimation results of an MS–AR(1) model. Regimes 1 and 2 are defined so that $\hat{p}_{11} > \hat{p}_{22}$. Similar to the AR(1)–GARCH(1,1) model, standard deviations are much more

[29] However, this is not the case for daily data; the normal distribution is mostly rejected. See Table E.1 in Appendix E.

significant than AR(1) effects. Because $\hat{b}_1 > 0 > \hat{b}_2$ and $\hat{\sigma}_1 < \hat{\sigma}_2$ in all 18 currencies except JPY, the interpretation is reasonable that regimes 1 and 2 represent a tranquil regime with high-mean and low-standard deviation, and a turbulent regime with low-mean and high-standard deviation, respectively. The transition probabilities $\hat{p}_{11}$ and $\hat{p}_{22}$ are very close to one, implying long swings of exchange rates (Engel and Hamilton, 1990). JPY is exceptional in the sense that the high-mean regime does not coincide with the low-standard deviation regime. See also the footnote 37.

4.4.1. In-Sample Test

We compare both models in terms of the in-sample test, on the basis of log-likelihood values and LB statistics.

The comparisons of log-likelihood values of the models are provided in Table 4.7. The likelihood ratio (LR) statistics on the basis of the benchmark AR(1) model, $-2\ln\Delta$, are also provided. In addition, we calculate the LR statistics on the basis of the AGNormal model for the AGFT model. Compared with the AR(1) model, the LR statistic of the AGNormal model is asymptotically χ^2 distributed with two degrees of freedom. Noting that the 0.99-quantile of the χ^2 distribution with two degrees of freedom is 9.21, all the LR statistics of the AGNormal model versus the AR(1) model are significant at the 1% level. The LR statistic of the AGT model has one degree of freedom versus the AGNormal model. Noting that the 0.95-quantile of the χ^2 distribution with one degree of freedom is 3.84, the LR statistics of the AGT model versus the AGNormal model are significant at the 5% level, except for five currencies; CZK, DKK, EUR, GBP, and SEK. Therefore, it is meaningful to introduce the fat-tailed distribution to the GARCH structure. The AGNTS model improves the AGNormal model in terms of log-likelihood values even more than the AGT model. The MS–AR(1) model also greatly improves the AR(1) model, in general. The simple comparisons of log-likelihood values tell that the MS–AR(1) model outperforms the AGNormal model in 13 out of 18 currencies, which is consistent with Bollen et al. (2000), but is outperformed by the AGNTS model in 16 out of 18 currencies. It is a sign of the superiority of the AGNTS model to the MS–AR(1) model. Statistically speaking, the comparisons of log-likelihood values between the AGNTS and MS–AR(1) models are insignificant because significance levels are unknown[30]. However, noting that both the AGNTS and MS–AR(1) models have the same number of parameters (eight), there is no reason to prefer the model with lower log-likelihood values.

The p-values of LB statistics of the standardized and squared standardized residuals of the models are provided in Tables 4.8 and 4.9, respectively, where we choose three lags: one, five, and twenty. Because the AGT and AGNTS models have the same (standardized) residuals, the LB statistics for both models are presented in a lump as the AGFT model. Through the LB test for standardized and squared standardized residuals, we can examine whether the proposed models eliminate the autocorrelations in the mean and variance processes, respectively. Note that we do not use usual residuals but *standardized* residuals for the LB test, because the GARCH model explicitly forecasts the variance and we are interested in whether the standardized residuals still have the autocorrelations after considering the clustering of the variance (the second moment)[31]. The remarkable result is that the null of no autocorrelation is rejected for the AR(1) model in any lags and most currencies at the 5% significance level, in the case of squared

[30] There are statistical difficulties in the comparisons of log-likelihood values even between the AGNTS and AGNormal models and between the MS-AR(1) and AR(1) models.The asymptotic distribution of the LR statistic of the AGNTS model versus the AGNormal model is no longer χ^2, though the normal distribution is recovered by the NTS distribution with $\alpha = 2$. It is because the case $\alpha = 2$ lies on the boundary of the parameter space, which violates the regularity conditions for the LR statistic to have the asymptotic χ^2 distribution (McCulloch, 1997). The asymptotic distribution of the LR statistic of the MS–AR(1) model versus the AR(1) model also is not χ^2, because the transition probabilities are not identified under the null (Hansen, 1992).

[31] Even in the case of the MS model, using standardized residuals for the LB test is common (see, for instance, the textbook of Bhar and Hamori, 2009). Note that Bollen et al. (2000) appear to use usual residuals for the LB test, though no explicit statement is given. Also note that there is a criticism about using the LB test with the residuals of the MS model, because, strictly speaking, they are not supposed to be i.i.d. (see, for instance, Henneke et al., 2011). However, we dare to use the LB test with the MS model in line with conventional practice.

standardized residuals, whereas it is not rejected for either the AR(1)–GARCH(1,1) or MS–AR(1) models. In other words, both the AR(1)–GARCH(1,1) and MS–AR(1) models describe the variance process much better than the AR(1) constant standard deviation model. Meanwhile, all the models work fairly well for the mean process, though the AR(1) model is slightly negative. Regarding the LB test, no significant differences are found between the AR(1)–GARCH(1,1) and MS–AR(1) models, though the former does a slightly better job in the variance process compared with the latter, in the five and twenty lags.

As a robustness check, we also investigate daily data. The results of the in-sample test for daily data are selectively given in Appendix E. Compared with the case of weekly data, the superiority of the AGNTS to the AGNormal and MS–AR(1) models is clearer. The result of the KS test is that the assumption of the normal distribution to GARCH standardized residuals is rejected in 14 out of 18 currencies at the 5% significance level. The normal distribution also performs much worse than the student t and NTS distributions, regarding AD and AD^2 statistics (Table E.1). The log-likelihood values of the MS–AR(1) model are even smaller than those of the AGNormal model in 14 out of 18 currencies (Table E.2), though the direct comparisons of the log-likelihood values are not statistically valid. The results of the LB test (Tables E.3 and E.4) reveal that the MS–AR(1) model is greatly outperformed by the AR(1)–GARCH(1,1) model in describing the variance dynamics; the null of no autocorrelations with twenty lags of squared standardized residuals of the MS–AR(1) model is rejected in 15 out of 18 currencies at the 5% significance level. From the above observations, we conclude that the MS model is less effective for high frequency data of exchange rate returns.

To summarize the results of the in-sample test, we find that the results tend to be in favor of the AGNTS model against the MS–AR(1) model for weekly data, though the improvements are not so clear. By contrast, the AGNTS model is clearly preferable to the MS–AR(1) model for daily data. Therefore, we further investigate the model performance through the out-of-sample test for weekly data, whereas we omit it for daily data.

4.4.2. Out-of-Sample Test

More important than the in-sample test is the out-of-sample test, which we conduct on the basis of variance and VaR forecasts and independence and tail behavior of the forecast distribution. The basic framework of the out-of-sample test is as follows. First, we prepare a moving window with the length of 260 weeks, i.e., five years. The first window is from the first week of 2000 to the last week of 2004, and proceeds week by week until the last week of 2011, which forms 366 distinct windows. Second, we iteratively estimate the time series models by using the return data within each window, and forecast the future conditional mean and standard deviation, and the future return distribution. We adopt one, four, and eight weeks as forecast lengths, in line with Bollen et al. (2000). Finally, we assess the out-of-sample forecast accuracy of the models.

The accuracy of variance forecasts is measured by the root mean squared forecast errors (RMSE) and mean absolute errors (MAE). The definitions of RMSE and MAE for *j*-week forecasts are given by

$$\mathrm{RMSE}_j = \sqrt{\frac{1}{366}\sum_{t=260}^{625}\left\{\left(R_{t+j}-\mu_{t+j|t}\right)^2-\sigma_{t+j|t}^2\right\}^2},$$

$$\mathrm{MAE}_j = \frac{1}{366}\sum_{t=260}^{625}\left|\left(R_{t+j}-\mu_{t+j|t}\right)^2-\sigma_{t+j|t}^2\right|, \tag{4.7}$$

where $\mu_{t+j|t}$ and $\sigma_{t+j|t}$ are the mean and standard deviation at a period $t + j$ conditional on the information of the window whose end period is t, respectively[32]. Tables 4.10, 4.11, and 4.12 compare the results of the RMSE and MAE of the models for one-week, four-week, and eight-week forecasts, respectively. The AGNormal and AGFT models have a smaller RMSE for one-week forecasts than the AR(1) model in 17 and 16 currencies, respectively. Except for the RMSE for one-week forecasts, we find no comprehensive improvements across the currencies of the AGNormal and even AGFT models from the AR(1) model. The differences in the RMSE and MAE between the AGFT and MS–AR(1) models are also reported in the Tables. The AGFT model has a smaller RMSE (MAE) for one-week, four-week, and eight-week forecasts than the MS-AR(1) model in 16 (12), 15 (9), and 10 (9) currencies, respectively. Thus, a half or more than half of the 18 sample currencies are in favor of the AGFT model rather than the MS–AR(1) model regarding the accuracy of variance forecasts, though the superiority is less noticeable in longer forecast lengths.

The VaR forecasts are assessed by the CLR test with unconditional and conditional coverages. We adopt the 99% confidence level for VaR in line with the Basel accord. Tables 4.13, 4.14, and 4.15 compare the results of the CLR test for the 99% VaR of one-week, four-week, and eight-week forecasts[33], respectively. While both the AGNTS and MS–AR(1) models generally reduce the number of violations compared with the AR(1) model, the excellence of the AGNTS model is much clearer than in the previous tests. The number of currencies on which the 99% VaR of one-week, four-week, and eight-week forecasts is rejected at the 1% (5%) significance level by

[32] Note that the variance of a *j*-week forecast is not defined for the cumulative return for a *j*-week horizon but for a *j*-week-ahead return. This is because it is not straightforward to derive the variance for a cumulative return except using the Gaussian model, where the variance for a cumulative return is simply a summation of the variances at each period. We consider that it is enough to assess the variance forecast at each period, because the accuracy of the variance forecast at each period leads to the accuracy of the variance forecast for a multi-period horizon.

[33] Note that the VaR of a *j*-week forecast is defined for a *j*-week-ahead return. See also the footnote 32.

the conditional test is 2 (7), 4 (12), and 6 (13), respectively, in the AGNTS model, whereas the number is 8 (15), 6 (14), and 7 (15), respectively, in the MS–AR(1) model. Regardless of forecast length, the rejections happen less frequently in the AGNTS model than in the MS–AR(1) model. On the other hand, the corresponding number in the AGNormal model is 11 (15), 11 (15), and 12 (15), respectively. As long as we rely on the normal distribution, the MS model can outperform the GARCH model regarding the VaR forecast. However, if we introduce the tempered stable distribution, the performance of the GARCH model is so enhanced that the MS model is beaten, because of the superior descriptive power for fat-tailness and skewness.

The BLR test is a test for the distribution of $Q_{t+j} = \Phi^{-1}(\hat{F}_{R,t}(R_{t+j}))$, $260 \leq t \leq 625$, in terms of independence and accuracy of tail behavior forecasts, where $\hat{F}_{R,t}$ is estimated by the models fitted to the return data within the window whose end period is *t*. We regard the loss above 99% VaR as the tail for the tail test. Tables 4.16, 4.17, and 4.18 compare the results of the BLR test with one-week, four-week, and eight-week forecast lengths, respectively. While there are no large differences among the models, even including the AR(1) model, in the independence test, the AGNTS model undoubtedly has the best performance in the tail test. The number of currencies on which the tail behavior forecast is rejected at the 1% (5%) significance level is 1 (7), 9 (9), and 11 (13), respectively, in the AGNTS model, whereas the number is 15 (16), 15 (16), and 16 (17), respectively, in the MS–AR(1) model. The corresponding number in the AGNormal model is 17 (17), 17 (17), and 17 (17), respectively. Therefore, similar to the case of the CLR test, the MS-AR(1) model outperforms the AGNormal model, but it is outperformed by the AGNTS model in the BLR tail test. This is also due to the descriptive power of the NTS distribution for the tail behavior.

To summarize the results of the out-of-sample test, we obtain clearer evidence that the AGNTS model is preferable to the MS–AR(1) model for weekly data.

4.5 Multivariate Extension

In this section, we consider the multivariate modeling of OECD currency exchange rates. In Section 4.4, we demonstrated that the AGNTS model is much more effective for risk management, i.e., VaR and tail risk forecasts, compared with the MS–AR(1) model. We note that the significant advantage of the NTS distribution is a multivariate extension, i.e., the MNTS distribution (Kim et al., 2012). The AR(1)–GARCH(1,1) model with the standardized MNTS distributed residuals, denoted by the AGMNTS model, is expected to account for not only the fat-tailness and skewness of each exchange rate return but also interdependencies among exchange rate returns. It is crucial to model the co-movements of asset returns because asset returns are highly correlated with each other in the modern global financial markets and co-movements should be considered for the portfolio management. The multivariate extension of the MS model has a number of computational difficulties for a high-dimensional system, though some studies tackle them (for instance, Sims et al., 2008). By contrast, the MNTS distribution can be used with a considerably high-dimensional system. Therefore, we apply the AGMNTS model to the multivariate dynamics of OECD currency exchange rates to further demonstrate the usefulness of the tempered stable distribution to exchange rate returns. See Kim et al. (2012), Kurosaki and Kim (2013a, 2013b), and Appendix B for the definition and detailed information on the MNTS distribution and AGMNTS model[34].

We briefly explain how to estimate the AGMNTS model for exchange rate returns. After fitting the AGFT structure to each exchange rate return, we describe standardized multivariate

[34] Both Kim et al. (2012) and Kurosaki and Kim (2013a, 2013b) use the MNTS distribution with an around 30 dimensional system.

residuals by using the standardized MNTS distribution. The standardized MNTS distribution has the parameters $(\alpha, \theta, \boldsymbol{\beta}, \boldsymbol{\rho})$. α and θ are common parameters related to fat-tailness. $\boldsymbol{\beta}$ is a vector parameter, each component of which controls the skewness of each marginal. $\boldsymbol{\rho}$ is a matrix parameter to govern correlations among marginals. First, we estimate the univariate NTS parameters $(\alpha, \theta, \beta) = (\hat{\alpha}, \hat{\theta}, \hat{\beta})$ of the standardized residuals of the AGFT model for the time series to represent OECD currency exchange rates. Here, we adopt the equally weighted portfolio of all 18 OECD currencies as the representative. Then, we utilize the estimated parameters $(\hat{\alpha}, \hat{\theta})$ as those of MNTS. Subsequently, we estimate the parameter β of the univariate NTS distribution for the AGFT standardized residuals of each currency exchange rate under the common parameters $(\hat{\alpha}, \hat{\theta})$. Thereby, we obtain the vector parameter $\hat{\boldsymbol{\beta}}$ of MNTS. Finally, we straightforward derive the matrix parameter $\hat{\boldsymbol{\rho}}$ from the variance-covariance matrix of the standardized residuals[35]. The variance-covariance matrix is easily estimated from samples.

We investigate the performance of the AGMNTS model through in-sample tests. To this end, we first estimate the AGMNTS model by using all the weekly return data within whole sample periods. At the same time, we also estimate the AR(1)–GARCH(1,1) model with the standardized multivariate normal distributed residuals, denoted by the AGMNormal model, as the benchmark model. The AGMNormal model is derived through the AGNormal models and the variance-covariance matrix of the standardized residuals. The log-likelihood value of the AGMNTS model is 36689[36], whereas that of the AGMNormal model is 36415. The LR statistic (548) appears large enough, just referring to the 0.99-quantile of the χ^2 distribution with 20 degrees of freedom (37.6). To see the descriptive power of MNTS for interdependencies at the

[35] See the formula (B.5) in Appendix B.

[36] We utilize the formula (B.9) in Appendix B for the calculation of the log-likelihood value of the standardized MNTS distribution.

tail, we adopt the bivariate distress probability, which we define as the joint probability that two exchange rate losses $-R^i$ and $-R^j$, where i and j stand for currencies, will simultaneously exceed their respective 95% VaR levels: $P_{i,j} = \text{Prob}\,(R^i < -\text{VaR}_{0.05}^i \cap R^j < -\text{VaR}_{0.05}^j)$. We compare the values of $P_{i,j}$ estimated by the AGMNTS and AGMNormal models with the empirical values, denoted by $P_{i,j}^{AGMNTS}, P_{i,j}^{AGMNormal}$, and $P_{i,j}^{Emp}$, respectively, where $P_{i,j}^{Emp}$ is estimated from the empirical joint distribution of the AGFT standardized residuals of currencies i and j. Because listing the results for all currency pairs involves excessive work, we confine ourselves to the pairs involving CAD, EUR, GBP, and JPY. These four are selected as major currencies in the sense that the countries where they circulate are members not only of OECD but also of the G7. The results are given in Table 4.19. It is found that the multivariate normal distribution is generally insufficient to describe tail dependencies between exchange rate returns; $P_{i,j}^{AGMNormal}$ is sometimes even less than half of the empirical value. By contrast, the MNTS distribution can capture the tail dependencies much better. $P_{i,j}^{AGMNTS}$ is usually closer to the empirical value, and is always greater than 60 percent of the empirical value, at worst. Now, we observe the better performance of the AGMNTS model.

4.6 GARCH Residuals by Regimes

In this section, we make short remarks on the perspective of associating the AGNTS model with the MS structure.

We have so far shown the greater usefulness of the AGNTS model for the purpose of risk management, in comparison with the MS model. However, we have no intention of suggesting that the MS model is useless. The significant advantage of the MS model is the ability to give insights about the timing of structural changes. Figure 4.1 plots the time series of the smoothed probability of the second regime, considered as the turbulent regime, specified by the MS–AR(1) model, for each exchange rate. The regimes of exchange rates generally switched into the turbulent one around the 2008 financial crisis. In addition, the concern about the recent Greek sovereign debt problem, which was accelerated by the downgrading of Greece in April 2010, created regime turbulent in EUR and some currencies near the Eurozone, whereas CHF and GBP, considered as hard currencies, were not affected by the problem in 2010. Even a simple MS–AR(1) model gives such implications about the structural changes of the market[37].

It is quite a natural idea to combine GARCH and MS models for a better description of time series; the parameters of the GARCH-type model are allowed to switch among regimes. This type of model is referred to as an MS−GARCH model, which has a long history of research (Hamilton and Susmel, 1994; Cai, 1994; Gray, 1996). In addition, Henneke et al. (2011) develop an algorithm for the estimations of a full range of the MS–ARMA–GARCH models with the normal and student t distributed residuals on the basis of Markov chain Monte Carlo methods. Given the results provided in the previous sections, it is also quite a natural expectation that the MS−GARCH model with the NTS distributed residuals, i.e., the MS–GARCH–NTS model, would work very well to describe the dynamics of exchange rates. However, there have so far been no attempts to embed the MS structure in the parameters of the tempered stable distribution, to the best of our knowledge. Though some previous studies have achieved successful results by allowing for the MS structure in the degrees of freedom of the student t distribution which GARCH residuals follow (for example, Marcucci, 2005), even more development seems to be required to overcome the computational difficulty of associating the tempered stable distribution with the MS structure. This problem will be addressed in our future study.

[37] In addition to the strange regime structure mentioned in Section 4.4, JPY shows exceptionally infrequent regime−switching in Figure 4.1, because of a relatively low value of $\hat{p}_{22}$. These suggest that the standard two-regime structure is not necessarily appropriate for JPY at least within our sample periods.

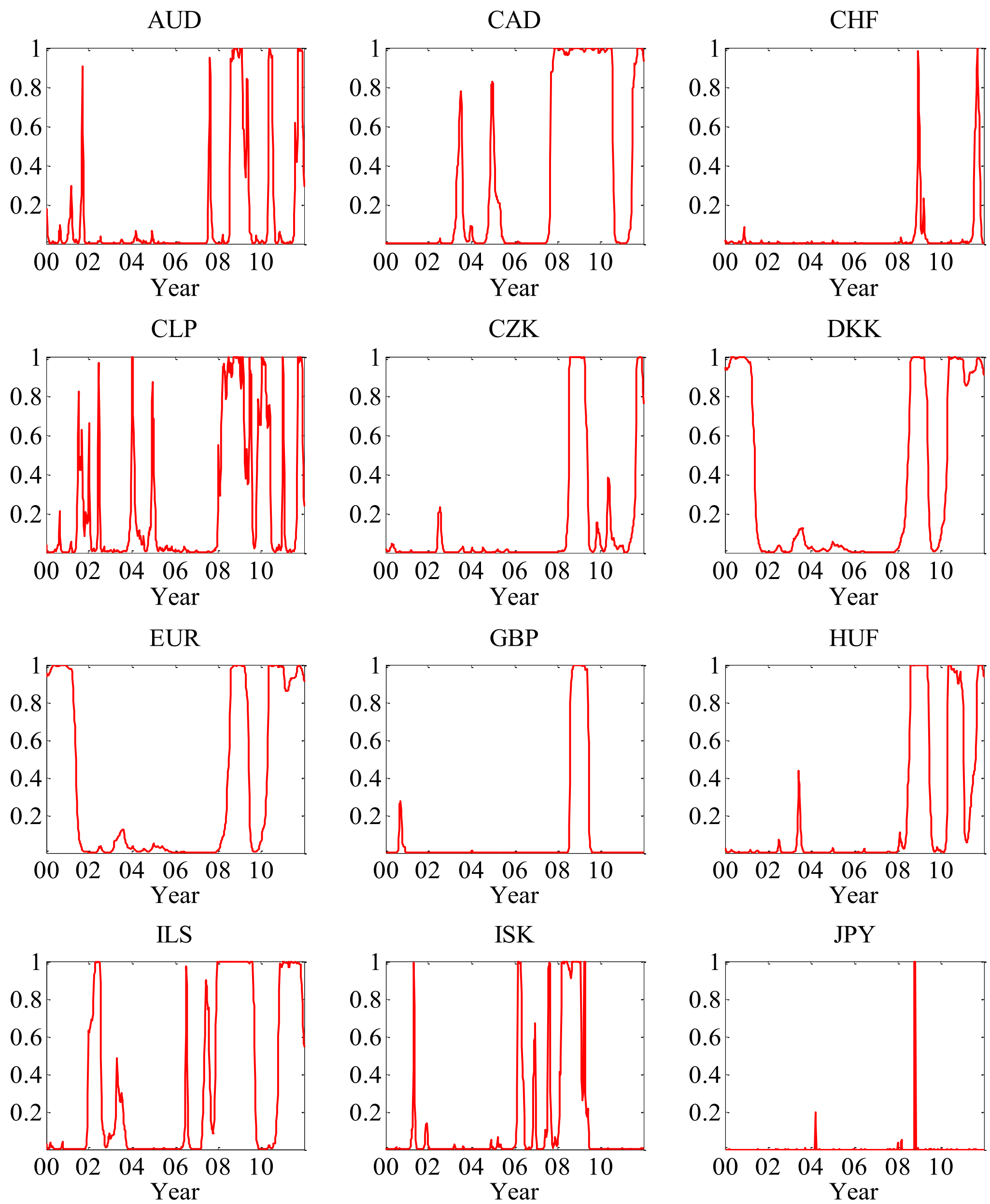

Figure 4.1: Smoothed Probability of a Turbulent Regime in the MS−AR(1) Model

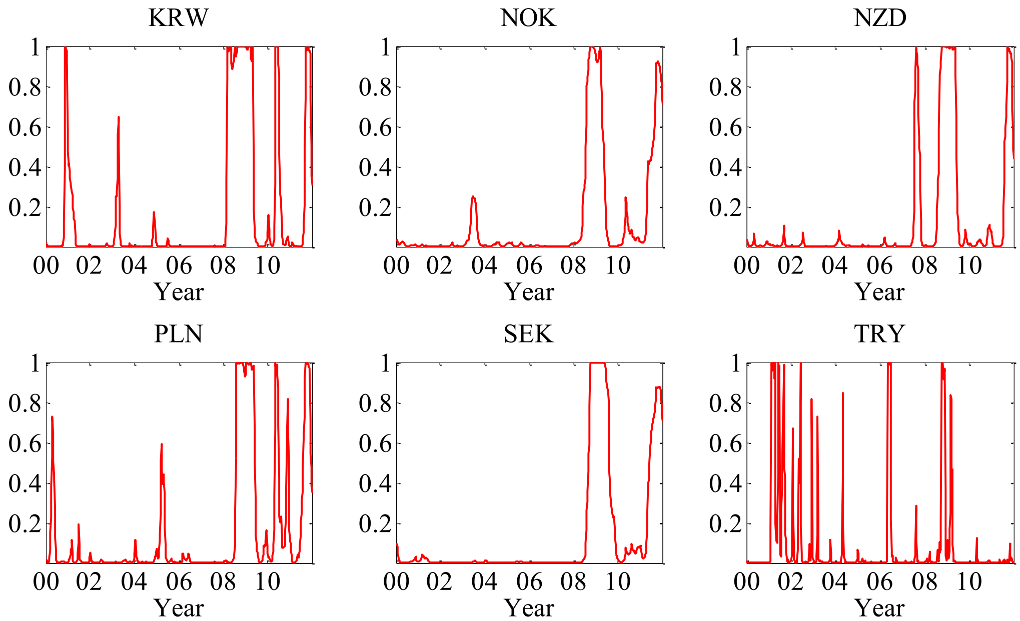

Figure 4.1(Cont.): Smoothed Probability of a Turbulent Regime in the MS−AR(1) Model

We provide simple diagnostic results to motivate the study of the MS–GARCH–NTS model. Let our sample periods including 625 weeks for each exchange rate return be divided into regimes 1 and 2, so that the week on which the smoothed probability of regimes 1 and 2 specified by the MS–AR(1) model is larger than 0.5 belongs to regimes 1 and 2, respectively. Accordingly, let the standardized residuals of the AGFT model for each exchange rate return be separated by the weeks in regimes 1 and 2. Then, we have two different AGFT standardized residuals belonging to regimes 1 and 2 by exchange rates. Table 4.20 reports the results of the KS test of the NTS distribution assumption for the AGFT standardized residuals by regimes, where the assumed NTS distribution is fitted by using the data within whole sample periods[38]. Obviously, the NTS distribution is more frequently rejected in regime 2 compared with regime 1. This implies that the residuals in regime 2 follow a fatter and more skewed NTS distribution and

[38] We omit the results of AD and AD^2 statistics, because comparing those statistics obtained from different numbers of observations is not meaningful.

we may obtain a better description of the dynamics of exchange rates by considering the NTS distributed residuals with the switching parameters in the GARCH framework, i.e., the MS–GARCH–NTS model.

4.7 Concluding Remarks

In this chapter, we investigate the dynamics of time series of OECD foreign currency exchange rates by using both the GARCH and MS models. We employ the NTS distribution as the distribution of standardized residuals in the GARCH model as well as normal and student t distributions, motivated by the recent studies to demonstrate that the modification on the basis of tempered stable distributions greatly improves the performance of the GARCH-type model for stock returns. The success of tempered stable distributions is attributed to the descriptive power of the stylized properties of asset returns: fat-tailness and skewness. Expanding on the previous study, we renew the sample periods to include the recent financial crises, expand the sample currencies, including all 18 OECD currencies, and use not only weekly data but also daily data.

The criteria of model performance are based on both in-sample and out-of-sample tests. The in-sample tests consist of the log-likelihood value and the LB test. The results appear to favor the AGNTS model against the MS–AR(1) model for weekly data. The same conclusion is derived more clearly for daily data; the GARCH model is preferable for high frequency data. More importantly, the out-of-sample tests are based on variance and VaR forecasts, and independence and tail behavior of the forecast distribution. We test the accuracy of forecasts of VaR and tail behavior because they are crucial for the purpose of risk management, especially in our sample periods. The results show that the AGNTS model gives significantly better forecasts of VaR and tail behavior than the MS–AR(1) model, reflecting the descriptive power of the NTS distribution for fat-tailness and skewness.

We also extend the AGNTS model into the multivariate version, i.e., the AGMNTS model, to model the co-movements of OECD currency exchange rates. As an in-sample test, we demonstrate that the AGMNTS model can explain bivariate tail dependencies between exchange rate returns much better than the AGMNormal model by estimating bivariate distress probabilities and comparing them with the empirical probabilities. The better description of the tail dependencies leads to better management of the portfolio tail risk.

To summarize, we show the usefulness of the GARCH model with the NTS distributed residuals in terms of risk management. This fact has been well confirmed for stock returns, and our results reveal that it is also the case for exchange rate returns. Through the empirical results, we find that the GARCH model with the NTS distributed residuals greatly improves the GARCH model with the normally distributed residuals, and is as effective as or even better than the standard MS model. Note that as long as the normal distribution is utilized, the GARCH model has a very high chance of being outperformed by the MS model for weekly data, which agrees with Bollen et al. (2000). The introduction of the NTS distribution reverses the situation, implying the significance of describing tail risk during the financial crises covered by our sample periods. Although the MS model has some descriptive power for the tail behavior of the asset return distribution via the normal mixture distribution, it turns out to be much more insufficient than the descriptive power of the AGNTS model in spite of the same number of parameters. We expect that an even more superior model would be derived by combining the NTS-based GARCH and MS structures. The effectiveness of the MS–GARCH–NTS model is implied by the difference in the fitting performance of the NTS distribution to the GARCH residuals in different regimes, discussed in Section 4.6. This challenging problem will be addressed in our future work.

Table 4.1: AIC and BIC of AR(1) and ARMA(1,1) Models, and Engle's ARCH Test for AR(1) Residuals (625 Weekly Observations)

	AIC		BIC		Engle's ARCH Test for AR(1) Residuals	
	AR(1)	ARMA(1,1)	AR(1)	ARMA(1,1)	Statistic	(p-value)
AUD	-3125.41	-3125.68	-3112.09	-3107.92	33.339	(0.000)
CAD	-3628.28	-3627.33	-3614.97	-3609.58	42.953	(0.000)
CHF	-3400.61	-3401.03	-3387.30	-3383.28	0.516	(0.473)
CLP	-3377.47	-3376.69	-3364.16	-3358.94	27.124	(0.000)
CZK	-3227.45	-3226.26	-3214.14	-3208.51	19.734	(0.000)
DKK	-3490.47	-3489.03	-3477.15	-3471.28	2.731	(0.098)
EUR	-3488.58	-3487.21	-3475.26	-3469.46	3.317	(0.069)
GBP	-3576.66	-3574.97	-3563.34	-3557.21	16.620	(0.000)
HUF	-3007.59	-3005.59	-2994.28	-2987.84	59.147	(0.000)
ILS	-3718.26	-3717.74	-3704.95	-3699.99	10.047	(0.002)
ISK	-2928.04	-2926.05	-2914.72	-2908.30	79.057	(0.000)
JPY	-3529.96	-3528.73	-3516.65	-3510.98	0.186	(0.666)
KRW	-3488.34	-3487.00	-3475.03	-3469.25	78.802	(0.000)
NOK	-3300.32	-3298.34	-3287.01	-3280.59	12.693	(0.000)
NZD	-3119.40	-3117.58	-3106.08	-3099.83	7.136	(0.008)
PLN	-3055.07	-3055.19	-3041.76	-3037.44	31.100	(0.000)
SEK	-3263.01	-3261.94	-3249.70	-3244.19	15.166	(0.000)
TRY	-2632.72	-2630.75	-2619.41	-2612.99	1.236	(0.266)

Note: The lag is one in Engle's ARCH test.

Table 4.2: Estimated Parameters of AGNormal Model (625 Weekly Observations)

	a	(p-value)	b	(p-value)	ω	(p-value)	ξ	(p-value)	ψ	(p-value)
AUD	0.0092	(0.856)	0.0012	(0.073)	0.1513	(0.000)	0.7928	(0.000)	2.2349E-05	(0.009)
CAD	-0.0078	(0.852)	0.0006	(0.174)	0.1134	(0.000)	0.8714	(0.000)	3.4775E-06	(0.040)
CHF	-0.0052	(0.905)	0.0009	(0.130)	0.0668	(0.003)	0.9101	(0.000)	6.5795E-06	(0.236)
CLP	0.0690	(0.142)	0.0000	(0.943)	0.1346	(0.000)	0.8406	(0.000)	8.4936E-06	(0.001)
CZK	0.0255	(0.548)	0.0014	(0.035)	0.0687	(0.000)	0.9117	(0.000)	6.9519E-06	(0.082)
DKK	-0.0074	(0.859)	0.0007	(0.227)	0.0634	(0.001)	0.9159	(0.000)	4.7470E-06	(0.161)
EUR	-0.0059	(0.887)	0.0007	(0.225)	0.0635	(0.001)	0.9157	(0.000)	4.7720E-06	(0.158)
GBP	0.0121	(0.773)	0.0002	(0.743)	0.1047	(0.000)	0.8303	(0.000)	1.1339E-05	(0.011)
HUF	-0.0081	(0.842)	0.0005	(0.528)	0.0826	(0.000)	0.8834	(0.000)	1.5635E-05	(0.035)
ILS	-0.0325	(0.467)	-0.0001	(0.695)	0.1110	(0.000)	0.8804	(0.000)	1.7816E-06	(0.047)
ISK	-0.0461	(0.347)	0.0006	(0.349)	0.1976	(0.000)	0.7749	(0.000)	1.9421E-05	(0.001)
JPY	-0.0729	(0.107)	0.0004	(0.430)	0.0237	(0.054)	0.9544	(0.000)	4.3930E-06	(0.281)
KRW	0.0606	(0.171)	0.0005	(0.154)	0.2173	(0.000)	0.7555	(0.000)	7.6202E-06	(0.000)
NOK	-0.0379	(0.401)	0.0006	(0.364)	0.0462	(0.009)	0.9209	(0.000)	9.8193E-06	(0.126)
NZD	0.0081	(0.866)	0.0009	(0.247)	0.0974	(0.000)	0.8213	(0.000)	3.1255E-05	(0.003)
PLN	-0.0136	(0.754)	0.0011	(0.125)	0.0848	(0.000)	0.8873	(0.000)	1.2364E-05	(0.050)
SEK	0.0092	(0.830)	0.0007	(0.272)	0.0559	(0.000)	0.9256	(0.000)	5.8090E-06	(0.106)
TRY	-0.0198	(0.735)	-0.0014	(0.420)	0.0475	(0.029)	0.6778	(0.000)	2.3298E-04	(0.002)

Table 4.3: Estimated Parameters of AGFT Model (625 Weekly Observations)

	a	(p-value)	b	(p-value)	ω	(p-value)	ξ	(p-value)	ψ	(p-value)
AUD	0.0087	(0.838)	0.0019	(0.003)	0.1021	(0.001)	0.8419	(0.000)	1.9271E-05	(0.039)
CAD	-0.0046	(0.914)	0.0007	(0.112)	0.0911	(0.001)	0.8924	(0.000)	3.2211E-06	(0.089)
CHF	0.0042	(0.920)	0.0009	(0.113)	0.0540	(0.070)	0.9071	(0.000)	9.6661E-06	(0.273)
CLP	0.0617	(0.144)	0.0004	(0.454)	0.1364	(0.000)	0.8392	(0.000)	8.5554E-06	(0.030)
CZK	0.0300	(0.476)	0.0016	(0.016)	0.0730	(0.001)	0.9050	(0.000)	7.7975E-06	(0.118)
DKK	-0.0010	(0.980)	0.0008	(0.144)	0.0624	(0.006)	0.9141	(0.000)	5.2638E-06	(0.184)
EUR	0.0009	(0.982)	0.0008	(0.136)	0.0630	(0.005)	0.9138	(0.000)	5.2373E-06	(0.183)
GBP	0.0111	(0.788)	0.0003	(0.616)	0.0986	(0.001)	0.8340	(0.000)	1.1594E-05	(0.024)
HUF	-0.0077	(0.849)	0.0011	(0.154)	0.0794	(0.002)	0.8927	(0.000)	1.3160E-05	(0.089)
ILS	-0.0429	(0.336)	0.0001	(0.815)	0.1100	(0.001)	0.8828	(0.000)	1.6445E-06	(0.120)
ISK	-0.0417	(0.344)	0.0007	(0.257)	0.1559	(0.000)	0.7726	(0.000)	2.9909E-05	(0.004)
JPY	-0.0620	(0.143)	0.0002	(0.648)	0.0251	(0.099)	0.9567	(0.000)	3.6476E-06	(0.369)
KRW	0.0340	(0.433)	0.0007	(0.060)	0.2566	(0.000)	0.7285	(0.000)	7.5344E-06	(0.006)
NOK	-0.0342	(0.432)	0.0011	(0.095)	0.0499	(0.029)	0.9147	(0.000)	1.0640E-05	(0.200)
NZD	-0.0045	(0.915)	0.0019	(0.010)	0.0747	(0.015)	0.8571	(0.000)	2.6103E-05	(0.066)
PLN	-0.0308	(0.456)	0.0016	(0.018)	0.0879	(0.001)	0.8802	(0.000)	1.4065E-05	(0.091)
SEK	0.0195	(0.644)	0.0009	(0.169)	0.0600	(0.002)	0.9191	(0.000)	6.5651E-06	(0.154)
TRY	-0.0434	(0.282)	0.0005	(0.401)	0.1846	(0.003)	0.7558	(0.000)	4.2025E-05	(0.003)

Table 4.4: Estimated Fat-tailness Parameters of AGFT Standardized Residuals (625 Weekly Observations)

	Student t	NTS		
	v	α	θ	β
AUD	6.5608	1.3798	0.8710	-0.7143
CAD	10.7199	1.8149	0.3492	-0.4619
CHF	14.3737	1.9291	0.1000	0.0087
CLP	5.3602	1.6997	0.1201	-0.1907
CZK	18.8295	0.2500	7.1979	-0.7376
DKK	19.2979	0.2500	7.5198	-1.0000
EUR	18.8617	0.3047	6.9718	-1.0000
GBP	25.2991	0.2500	14.7740	-1.0000
HUF	10.9777	0.3024	4.6867	-1.0000
ILS	11.2103	1.2568	1.5365	-0.4258
ISK	6.9270	1.6836	0.4136	-0.5069
JPY	11.1664	1.7992	0.5669	0.5523
KRW	6.2819	1.6840	0.1831	-0.1604
NOK	12.7760	0.2500	4.4118	-0.9371
NZD	7.0711	0.2500	2.0993	-0.5739
PLN	7.4309	0.2500	2.2330	-0.4152
SEK	18.6338	0.2529	7.0468	-0.9954
TRY	3.7613	0.7195	1.6815	0.0005

Table 4.5: KS, AD, and AD^2 Statistics of AR(1)−GARCH(1,1) Standardized Residuals against the Proposed Distributions

	Normal				Student t				NTS			
	KS		AD	AD^2	KS		AD	AD^2	KS		AD	AD^2
	Statistic	(p-value)	Statistic	Statistic	Statistic	(p-value)	Statistic	Statistic	Statistic	(p-value)	Statistic	Statistic
AUD	0.048	(0.110)	8.288	4.621	0.031	(0.574)	0.146	2.457	0.042	(0.217)	0.091	2.332
CAD	0.028	(0.694)	0.412	1.297	0.028	(0.690)	0.107	0.701	0.020	(0.967)	0.053	0.391
CHF	0.026	(0.780)	31.405	0.905	0.035	(0.424)	0.716	1.197	0.031	(0.570)	0.191	0.869
CLP	0.061	(0.017)	18.840	4.459	0.020	(0.954)	0.104	0.539	0.032	(0.554)	0.063	0.669
CZK	0.024	(0.866)	0.127	1.125	0.021	(0.938)	0.110	0.769	0.026	(0.775)	0.061	0.489
DKK	0.023	(0.880)	0.297	1.109	0.030	(0.607)	0.122	0.989	0.032	(0.543)	0.064	0.600
EUR	0.027	(0.735)	0.333	1.226	0.026	(0.787)	0.132	1.061	0.033	(0.507)	0.066	0.590
GBP	0.024	(0.859)	0.248	1.027	0.022	(0.914)	0.117	0.931	0.020	(0.962)	0.092	0.442
HUF	0.040	(0.273)	0.908	2.565	0.029	(0.665)	0.125	1.765	0.035	(0.422)	0.072	1.336
ILS	0.044	(0.182)	0.239	2.388	0.039	(0.303)	0.104	1.198	0.021	(0.929)	0.090	0.338
ISK	0.042	(0.206)	33.827	3.062	0.024	(0.839)	0.183	1.537	0.037	(0.365)	0.085	2.345
JPY	0.027	(0.726)	2.172	0.900	0.016	(0.996)	0.144	0.338	0.020	(0.956)	0.066	0.335
KRW	0.040	(0.268)	3.706	2.952	0.032	(0.525)	0.126	0.964	0.039	(0.284)	0.081	1.597
NOK	0.040	(0.254)	0.250	3.371	0.034	(0.473)	0.158	2.314	0.028	(0.697)	0.079	0.883
NZD	0.063	(0.013)	1.421	6.036	0.039	(0.296)	0.151	3.377	0.041	(0.242)	0.084	2.227
PLN	0.045	(0.156)	5.674	3.601	0.031	(0.556)	0.126	1.900	0.040	(0.256)	0.104	1.897
SEK	0.029	(0.639)	0.309	1.457	0.032	(0.547)	0.125	1.438	0.033	(0.495)	0.076	0.976
TRY	0.125	(0.000)	4.203E+29	41.758	0.033	(0.499)	1.361	2.616	0.056	(0.036)	0.192	6.268
# of p-values less than 1%		1				0				0		
# of p-values less than 5%		3				0				1		

Table 4.6: MS–AR(1) Estimated Parameters (625 Weekly Observations)

	a_1	(p-value)	a_2	(p-value)	b_1	(p-value)	b_2	(p-value)	σ_1	(p-value)	σ_2	(p-value)
AUD	0.0132	(0.765)	-0.1499	(0.194)	0.0017	(0.015)	-0.0068	(0.153)	0.0147	(0.000)	0.0386	(0.000)
CAD	0.0166	(0.744)	-0.0175	(0.801)	0.0009	(0.035)	-0.0003	(0.833)	0.0088	(0.000)	0.0195	(0.000)
CHF	-0.0015	(0.978)	-0.0212	(0.977)	0.0009	(0.164)	-0.0005	(0.981)	0.0146	(0.000)	0.0366	(0.017)
CLP	0.1145	(0.023)	-0.1366	(0.108)	0.0007	(0.164)	-0.0030	(0.215)	0.0105	(0.000)	0.0269	(0.000)
CZK	0.0377	(0.398)	-0.0370	(0.760)	0.0018	(0.008)	-0.0064	(0.090)	0.0156	(0.000)	0.0302	(0.000)
DKK	-0.0306	(0.553)	0.0500	(0.474)	0.0014	(0.026)	-0.0015	(0.276)	0.0120	(0.000)	0.0188	(0.000)
EUR	-0.0271	(0.603)	0.0482	(0.487)	0.0014	(0.025)	-0.0015	(0.276)	0.0119	(0.000)	0.0189	(0.000)
GBP	0.0140	(0.729)	-0.1376	(0.355)	0.0002	(0.631)	-0.0048	(0.282)	0.0116	(0.000)	0.0289	(0.002)
HUF	0.0027	(0.964)	-0.0619	(0.509)	0.0012	(0.115)	-0.0053	(0.138)	0.0168	(0.000)	0.0353	(0.000)
ILS	0.0448	(0.427)	-0.1678	(0.016)	0.0004	(0.336)	-0.0005	(0.716)	0.0076	(0.000)	0.0182	(0.000)
ISK	-0.0617	(0.097)	-0.1400	(0.200)	0.0009	(0.132)	-0.0126	(0.000)	0.0150	(0.000)	0.0479	(0.089)
JPY	-0.0562	(0.161)	-1.3772	(0.000)	0.0004	(0.479)	0.0568	(0.000)	0.0138	(0.000)	0.0040	(0.391)
KRW	0.0046	(0.922)	0.1033	(0.289)	0.0010	(0.020)	-0.0043	(0.098)	0.0092	(0.000)	0.0289	(0.000)
NOK	-0.0499	(0.264)	-0.0537	(0.647)	0.0012	(0.069)	-0.0049	(0.129)	0.0155	(0.000)	0.0255	(0.000)
NZD	-0.0121	(0.778)	-0.0246	(0.846)	0.0013	(0.044)	-0.0036	(0.386)	0.0165	(0.000)	0.0353	(0.016)
PLN	-0.0438	(0.345)	-0.1739	(0.101)	0.0021	(0.004)	-0.0111	(0.006)	0.0161	(0.000)	0.0358	(0.000)
SEK	0.0222	(0.626)	-0.0398	(0.734)	0.0007	(0.273)	-0.0026	(0.436)	0.0153	(0.000)	0.0288	(0.000)
TRY	-0.0138	(0.741)	-0.3157	(0.021)	0.0004	(0.570)	-0.0350	(0.000)	0.0162	(0.000)	0.0772	(0.000)

Table 4.6(Cont.): MS–AR(1) Estimated Parameters (625 Weekly Observations)

	p_{11}	(p-value)	p_{22}	(p-value)
AUD	0.9859	(0.000)	0.9073	(0.000)
CAD	0.9909	(0.000)	0.9820	(0.000)
CHF	0.9948	(0.000)	0.8472	(0.000)
CLP	0.9720	(0.000)	0.9076	(0.000)
CZK	0.9941	(0.000)	0.9628	(0.000)
DKK	0.9918	(0.000)	0.9888	(0.000)
EUR	0.9919	(0.000)	0.9890	(0.000)
GBP	0.9977	(0.000)	0.9686	(0.000)
HUF	0.9922	(0.000)	0.9714	(0.000)
ILS	0.9858	(0.000)	0.9716	(0.000)
ISK	0.9877	(0.000)	0.9219	(0.006)
JPY	0.9979	(0.000)	0.7061	(0.090)
KRW	0.9887	(0.000)	0.9456	(0.000)
NOK	0.9943	(0.000)	0.9649	(0.000)
NZD	0.9925	(0.000)	0.9496	(0.004)
PLN	0.9868	(0.000)	0.9277	(0.000)
SEK	0.9957	(0.000)	0.9772	(0.000)
TRY	0.9773	(0.000)	0.7620	(0.000)

Table 4.7: Log-likelihood

	AR(1)	AR(1)–GARCH(1,1)								MS–AR(1)	
		AGNormal	LR Statistic vs. AR(1)	AGT	LR Statistic vs. AR(1)	LR Statistic vs. AGNormal	AGNTS	LR Statistic vs. AR(1)	LR Statistic vs. AGNormal		LR Statistic vs. AR(1)
AUD	1565.70	1640.27	149.13	1656.88	182.34	33.22	1666.85	202.29	53.16	1644.56	157.71
CAD	1817.14	1895.21	156.15	1900.11	165.93	9.79	1902.45	170.63	14.48	1891.74	149.20
CHF	1703.31	1719.67	32.72	1725.68	44.75	12.03	1728.65	50.69	17.97	1723.87	41.13
CLP	1691.73	1771.13	158.78	1796.39	209.30	50.52	1798.67	213.87	55.09	1779.70	175.92
CZK	1616.73	1649.75	66.04	1650.83	68.20	2.16	1653.52	73.59	7.54	1648.82	64.19
DKK	1748.23	1765.75	35.03	1767.21	37.95	2.92	1771.78	47.10	12.07	1768.85	41.24
EUR	1747.29	1765.18	35.78	1766.69	38.81	3.03	1771.63	48.69	12.91	1768.68	42.78
GBP	1791.33	1839.85	97.04	1840.66	98.66	1.63	1843.58	104.51	7.47	1845.54	108.42
HUF	1506.80	1555.75	97.91	1560.54	107.49	9.58	1569.10	124.60	26.69	1561.50	109.41
ILS	1862.13	1962.80	201.34	1967.22	210.18	8.84	1969.57	214.87	13.53	1953.46	182.66
ISK	1467.02	1597.33	260.63	1613.06	292.09	31.46	1617.31	300.58	39.96	1603.75	273.46
JPY	1767.98	1774.35	12.74	1779.42	22.88	10.13	1781.29	26.61	13.87	1784.49	33.01
KRW	1747.17	1899.50	304.65	1917.02	339.71	35.05	1918.32	342.30	37.64	1889.86	285.38
NOK	1653.16	1666.26	26.19	1668.91	31.50	5.30	1678.13	49.93	23.74	1668.52	30.72
NZD	1562.70	1594.16	62.92	1603.97	82.54	19.62	1616.44	107.48	44.56	1603.44	81.49
PLN	1530.54	1585.86	110.64	1596.22	131.38	20.74	1601.96	142.85	32.21	1592.69	124.31
SEK	1634.51	1667.11	65.20	1668.72	68.43	3.22	1673.47	77.93	12.73	1665.16	61.30
TRY	1319.36	1339.95	41.17	1584.11	529.50	488.32	1591.66	544.60	503.43	1551.86	464.99

Table 4.8: p-values of the LB Test for Standardized Residuals

	One Lag				Five Lags				Twenty Lags			
	AR(1)	AGNormal	AGFT	MS–AR(1)	AR(1)	AGNormal	AGFT	MS–AR(1)	AR(1)	AGNormal	AGFT	MS–AR(1)
AUD	0.936	0.444	0.483	0.276	0.366	0.966	0.966	0.906	0.291	0.711	0.740	0.660
CAD	0.993	0.514	0.533	0.674	0.439	0.630	0.685	0.769	0.013	0.170	0.179	0.159
CHF	0.995	0.692	0.916	0.886	0.102	0.377	0.361	0.320	0.173	0.296	0.299	0.293
CLP	0.952	0.461	0.374	0.348	0.711	0.499	0.472	0.678	0.163	0.774	0.758	0.678
CZK	0.983	0.544	0.620	0.714	0.774	0.726	0.748	0.886	0.801	0.725	0.735	0.777
DKK	0.996	0.537	0.648	0.802	0.723	0.309	0.338	0.549	0.628	0.571	0.581	0.632
EUR	0.995	0.536	0.656	0.810	0.681	0.259	0.287	0.495	0.603	0.520	0.531	0.596
GBP	0.964	0.464	0.479	0.519	0.017	0.706	0.704	0.770	0.000	0.736	0.727	0.836
HUF	0.999	0.531	0.555	0.327	0.748	0.538	0.549	0.572	0.404	0.699	0.699	0.685
ILS	0.910	0.835	0.634	0.772	0.611	0.150	0.135	0.634	0.017	0.128	0.129	0.485
ISK	0.987	0.446	0.594	0.434	0.307	0.023	0.032	0.172	0.179	0.069	0.080	0.184
JPY	0.957	0.967	0.766	0.904	0.773	0.781	0.766	0.215	0.228	0.390	0.397	0.049
KRW	0.900	0.264	0.096	0.291	0.066	0.438	0.306	0.468	0.000	0.058	0.039	0.014
NOK	0.995	0.715	0.796	0.785	0.905	0.886	0.900	0.942	0.941	0.945	0.948	0.971
NZD	0.996	0.757	0.652	0.549	0.946	0.986	0.983	0.958	0.405	0.747	0.730	0.734
PLN	0.910	0.592	0.363	0.171	0.068	0.504	0.441	0.467	0.110	0.489	0.445	0.169
SEK	0.999	0.594	0.771	0.769	0.990	0.614	0.642	0.558	0.811	0.954	0.957	0.943
TRY	0.974	0.580	0.913	0.970	0.047	0.369	0.897	0.738	0.016	0.157	0.974	0.448
# of p-values less than 5%	0	0	0	0	2	1	1	0	5	0	1	2

Table 4.9: p-values of the LB Test for Squared Standardized Residuals

	One Lag				Five Lags				Twenty Lags			
	AR(1)	AGNormal	AGFT	MS–AR(1)	AR(1)	AGNormal	AGFT	MS–AR(1)	AR(1)	AGNormal	AGFT	MS–AR(1)
AUD	0.000	0.075	0.008	0.063	0.000	0.250	0.055	0.117	0.000	0.638	0.407	0.529
CAD	0.000	0.092	0.035	0.017	0.000	0.385	0.212	0.001	0.000	0.358	0.294	0.016
CHF	0.471	0.719	0.841	0.757	0.028	0.761	0.825	0.789	0.278	0.965	0.977	0.949
CLP	0.000	0.443	0.471	0.700	0.000	0.834	0.842	0.963	0.000	0.993	0.995	0.038
CZK	0.000	0.902	0.857	0.715	0.000	0.972	0.982	0.613	0.000	0.980	0.981	0.898
DKK	0.097	0.993	0.979	0.885	0.002	0.837	0.826	0.905	0.000	0.733	0.728	0.437
EUR	0.068	0.884	0.918	0.819	0.002	0.880	0.873	0.913	0.000	0.801	0.804	0.536
GBP	0.000	0.946	0.877	0.407	0.000	0.407	0.349	0.717	0.000	0.768	0.723	0.854
HUF	0.000	0.364	0.344	0.415	0.000	0.429	0.443	0.681	0.000	0.774	0.779	0.811
ILS	0.001	0.129	0.142	0.288	0.000	0.446	0.458	0.753	0.000	0.907	0.923	0.589
ISK	0.000	0.802	0.427	0.322	0.000	0.885	0.740	0.775	0.000	0.868	0.705	0.000
JPY	0.665	0.263	0.257	0.888	0.012	0.010	0.009	0.002	0.039	0.261	0.246	0.336
KRW	0.000	0.577	0.855	0.141	0.000	0.210	0.400	0.421	0.000	0.407	0.548	0.005
NOK	0.000	0.330	0.253	0.228	0.000	0.724	0.733	0.537	0.000	0.988	0.989	0.918
NZD	0.007	0.485	0.707	0.730	0.000	0.110	0.057	0.021	0.000	0.619	0.584	0.400
PLN	0.000	0.835	0.820	0.804	0.000	0.709	0.711	0.943	0.000	0.988	0.992	0.923
SEK	0.000	0.349	0.303	0.425	0.000	0.721	0.726	0.835	0.000	0.971	0.978	0.906
TRY	0.265	0.932	0.949	0.944	0.755	1.000	1.000	1.000	0.986	1.000	1.000	1.000
# of p-values less than 5%	13	0	2	1	17	1	1	3	16	0	0	4

Table 4.10: Variance Forecast Performance (One-week Forecast)

	RMSE				ΔRMSE	MAE				ΔMAE
	AR(1)	AGNormal	AGFT	MS–AR(1)	MS–AR(1) − AGFT	AR(1)	AGNormal	AGFT	MS–AR(1)	MS–AR(1) − AGFT
AUD	2.023	1.878	1.886	9.299	7.414	0.528	0.530	0.525	1.015	0.490
CAD	0.598	0.567	0.566	1.071	0.505	0.236	0.252	0.251	0.332	0.081
CHF	0.800	0.810	0.806	0.819	0.013	0.258	0.285	0.283	0.275	-0.008
CLP	0.991	0.951	0.950	0.950	0.000	0.364	0.382	0.386	0.379	-0.007
CZK	0.691	0.650	0.654	0.733	0.079	0.375	0.390	0.392	0.423	0.031
DKK	0.430	0.428	0.428	0.443	0.014	0.223	0.243	0.242	0.244	0.002
EUR	0.429	0.428	0.428	0.443	0.015	0.224	0.243	0.244	0.246	0.002
GBP	0.558	0.525	0.524	0.548	0.023	0.231	0.232	0.231	0.241	0.010
HUF	1.173	1.080	1.097	1.308	0.211	0.589	0.588	0.599	0.677	0.078
ILS	0.425	0.397	0.397	0.399	0.002	0.204	0.201	0.200	0.188	-0.012
ISK	2.871	2.596	2.624	2.893	0.270	0.946	0.844	0.873	0.811	-0.062
JPY	0.433	0.430	0.435	0.427	-0.009	0.227	0.233	0.239	0.231	-0.009
KRW	1.010	0.851	0.844	1.071	0.228	0.333	0.323	0.325	0.353	0.027
NOK	0.580	0.576	0.579	0.692	0.113	0.334	0.339	0.341	0.374	0.033
NZD	1.052	1.019	1.039	1.122	0.083	0.482	0.484	0.487	0.517	0.030
PLN	1.418	1.358	1.362	1.380	0.018	0.558	0.607	0.606	0.641	0.035
SEK	0.632	0.591	0.593	0.761	0.168	0.350	0.339	0.343	0.420	0.077
TRY	1.353	1.199	1.289	1.163	-0.126	0.690	0.579	0.638	0.587	-0.051

Note: The figures are scaled by 10^{-3}.

Table 4.11: Variance Forecast Performance (Four-week Forecast)

	RMSE				ΔRMSE	MAE				ΔMAE
	AR(1)	AGNormal	AGFT	MS–AR(1)	MS–AR(1) − AGFT	AR(1)	AGNormal	AGFT	MS–AR(1)	MS–AR(1) − AGFT
AUD	1.997	2.189	2.086	75.851	73.765	0.513	0.628	0.582	4.533	3.951
CAD	0.592	0.613	0.608	0.801	0.193	0.232	0.268	0.265	0.293	0.028
CHF	0.786	0.803	0.797	0.811	0.014	0.253	0.288	0.283	0.265	-0.017
CLP	0.994	1.001	1.000	0.997	-0.003	0.359	0.398	0.403	0.393	-0.010
CZK	0.683	0.664	0.668	0.754	0.086	0.371	0.398	0.399	0.419	0.020
DKK	0.425	0.426	0.425	0.439	0.014	0.219	0.242	0.240	0.240	0.000
EUR	0.425	0.426	0.425	0.438	0.012	0.220	0.243	0.242	0.240	-0.002
GBP	0.555	0.552	0.551	0.562	0.011	0.229	0.236	0.235	0.236	0.002
HUF	1.142	1.101	1.115	1.185	0.069	0.585	0.597	0.612	0.651	0.039
ILS	0.429	0.393	0.393	0.404	0.010	0.205	0.201	0.201	0.190	-0.011
ISK	2.857	2.989	2.859	2.872	0.013	0.935	0.985	0.958	0.841	-0.117
JPY	0.440	0.440	0.444	0.440	-0.003	0.229	0.231	0.237	0.232	-0.005
KRW	0.905	0.897	0.900	0.941	0.041	0.330	0.354	0.356	0.363	0.007
NOK	0.576	0.585	0.585	0.605	0.020	0.332	0.343	0.343	0.351	0.008
NZD	1.054	1.047	1.058	1.058	0.000	0.481	0.499	0.499	0.503	0.004
PLN	1.389	1.417	1.420	1.432	0.012	0.553	0.625	0.623	0.613	-0.010
SEK	0.630	0.592	0.594	0.694	0.100	0.347	0.339	0.342	0.384	0.042
TRY	1.317	1.303	1.478	1.313	-0.165	0.688	0.630	0.770	0.662	-0.108

Note: The figures are scaled by 10^{-3}.

Table 4.12: Variance Forecast Performance (Eight-week Forecast)

	RMSE				ΔRMSE	MAE				ΔMAE
	AR(1)	AGNormal	AGFT	MS–AR(1)	MS–AR(1) − AGFT	AR(1)	AGNormal	AGFT	MS–AR(1)	MS–AR(1) − AGFT
AUD	2.005	2.206	2.090	334.077	331.987	0.519	0.641	0.589	18.016	17.427
CAD	0.594	0.618	0.613	0.610	-0.002	0.235	0.270	0.267	0.265	-0.002
CHF	0.784	0.816	0.807	0.787	-0.020	0.253	0.298	0.294	0.258	-0.035
CLP	1.001	1.037	1.035	1.017	-0.017	0.362	0.427	0.435	0.401	-0.034
CZK	0.684	0.672	0.676	0.737	0.060	0.373	0.409	0.410	0.419	0.009
DKK	0.426	0.426	0.427	0.442	0.015	0.221	0.245	0.242	0.241	-0.001
EUR	0.426	0.426	0.425	0.448	0.023	0.222	0.246	0.243	0.243	0.000
GBP	0.559	0.540	0.539	0.573	0.034	0.231	0.231	0.229	0.243	0.014
HUF	1.147	1.113	1.135	1.202	0.066	0.590	0.598	0.622	0.660	0.038
ILS	0.431	0.389	0.389	0.407	0.017	0.207	0.202	0.202	0.190	-0.011
ISK	2.858	2.907	2.892	2.853	-0.038	0.947	1.005	1.009	0.871	-0.137
JPY	0.440	0.439	0.442	0.439	-0.003	0.230	0.233	0.237	0.232	-0.005
KRW	0.910	0.988	0.989	0.937	-0.053	0.334	0.412	0.416	0.378	-0.038
NOK	0.578	0.587	0.586	0.610	0.024	0.333	0.348	0.347	0.348	0.001
NZD	1.059	1.045	1.060	1.052	-0.008	0.485	0.488	0.495	0.497	0.002
PLN	1.395	1.442	1.439	1.452	0.013	0.559	0.632	0.626	0.633	0.007
SEK	0.634	0.607	0.609	0.712	0.102	0.351	0.355	0.357	0.397	0.040
TRY	1.318	1.335	1.635	1.313	-0.322	0.689	0.666	0.907	0.670	-0.236

Note: The figures are scaled by 10^{-3}.

Table 4.13: Numbers of Violations and p-values of the Christoffersen Test for 99% VaR (One-week Forecast)

	# of Violations (out of 366 Weeks)					p-value (Unconditional Test)					p-value (Conditional Test)				
	AR(1)	AGNormal	AGT	AGNTS	MS−AR(1)	AR(1)	AGNormal	AGT	AGNTS	MS−AR(1)	AR(1)	AGNormal	AGT	AGNTS	MS−AR(1)
AUD	10	12	12	10	10	0.006	0.001	0.001	0.006	0.006	0.003	0.000	0.000	0.003	0.003
CAD	12	9	8	8	9	0.001	0.018	0.049	0.049	0.018	0.000	0.007	0.015	0.015	0.007
CHF	5	5	5	5	4	0.505	0.505	0.505	0.505	0.860	0.456	0.456	0.456	0.456	0.755
CLP	13	5	5	6	7	0.000	0.505	0.505	0.260	0.119	0.000	0.038	0.038	0.036	0.026
CZK	10	10	8	6	8	0.006	0.006	0.049	0.260	0.049	0.005	0.005	0.040	0.226	0.040
DKK	10	8	9	7	7	0.006	0.049	0.018	0.119	0.119	0.005	0.040	0.014	0.103	0.022
EUR	11	8	9	7	8	0.002	0.049	0.018	0.119	0.049	0.001	0.040	0.014	0.103	0.013
GBP	11	7	7	7	6	0.002	0.119	0.119	0.119	0.260	0.000	0.100	0.100	0.100	0.036
HUF	13	12	11	6	9	0.000	0.001	0.002	0.260	0.018	0.000	0.000	0.001	0.231	0.007
ILS	18	8	6	5	7	0.000	0.049	0.260	0.505	0.119	0.000	0.040	0.231	0.456	0.103
ISK	19	13	11	10	10	0.000	0.000	0.002	0.006	0.006	0.000	0.000	0.001	0.004	0.004
JPY	4	4	3	4	5	0.860	0.860	0.720	0.860	0.505	0.729	0.729	0.673	0.729	0.445
KRW	17	11	8	8	11	0.000	0.002	0.049	0.049	0.002	0.000	0.000	0.015	0.015	0.000
NOK	14	13	13	7	10	0.000	0.000	0.000	0.119	0.006	0.000	0.000	0.000	0.026	0.003
NZD	12	13	12	5	10	0.001	0.000	0.001	0.505	0.006	0.000	0.000	0.000	0.445	0.004
PLN	15	11	9	7	11	0.000	0.002	0.018	0.119	0.002	0.000	0.001	0.014	0.100	0.000
SEK	10	11	8	8	8	0.006	0.002	0.049	0.049	0.049	0.005	0.001	0.040	0.040	0.040
TRY	8	13	5	4	5	0.049	0.000	0.505	0.860	0.505	0.015	0.000	0.445	0.729	0.038
# of p-values less than 1%						15	10	5	2	6	15	11	5	2	8
# of p-values less than 5%						16	14	12	5	11	16	15	13	7	15

Table 4.14: Numbers of Violations and p-values of the Christoffersen Test for 99% VaR (Four-week Forecast)

	# of Violations (out of 366 Weeks)					p-value (Unconditional Test)					p-value (Conditional Test)				
	AR(1)	AGNormal	AGT	AGNTS	MS−AR(1)	AR(1)	AGNormal	AGT	AGNTS	MS−AR(1)	AR(1)	AGNormal	AGT	AGNTS	MS−AR(1)
AUD	10	12	12	9	8	0.006	0.001	0.001	0.018	0.049	0.003	0.000	0.000	0.007	0.015
CAD	11	8	8	8	9	0.002	0.049	0.049	0.049	0.018	0.001	0.015	0.015	0.015	0.007
CHF	4	4	5	7	4	0.860	0.860	0.505	0.119	0.860	0.729	0.729	0.445	0.026	0.729
CLP	12	8	7	7	7	0.001	0.049	0.119	0.119	0.119	0.000	0.015	0.026	0.026	0.026
CZK	9	10	8	7	8	0.018	0.006	0.049	0.119	0.049	0.014	0.004	0.039	0.100	0.039
DKK	9	8	7	5	5	0.018	0.049	0.119	0.505	0.505	0.014	0.015	0.026	0.038	0.038
EUR	9	9	6	6	5	0.018	0.018	0.260	0.260	0.505	0.014	0.007	0.036	0.036	0.038
GBP	11	10	10	8	9	0.002	0.006	0.006	0.049	0.018	0.000	0.000	0.000	0.001	0.001
HUF	11	11	8	7	9	0.002	0.002	0.049	0.119	0.018	0.001	0.001	0.015	0.026	0.007
ILS	18	6	5	4	7	0.000	0.260	0.505	0.860	0.119	0.000	0.226	0.445	0.729	0.100
ISK	18	13	11	9	8	0.000	0.000	0.002	0.018	0.049	0.000	0.000	0.001	0.014	0.039
JPY	5	5	2	4	5	0.505	0.505	0.340	0.860	0.505	0.445	0.445	0.334	0.729	0.445
KRW	17	12	12	12	14	0.000	0.001	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000
NOK	12	12	11	8	9	0.001	0.001	0.002	0.049	0.018	0.000	0.000	0.000	0.000	0.000
NZD	12	13	10	4	8	0.001	0.000	0.006	0.860	0.049	0.000	0.000	0.004	0.729	0.039
PLN	14	11	10	6	10	0.000	0.002	0.006	0.260	0.006	0.000	0.000	0.000	0.226	0.000
SEK	9	9	8	7	5	0.018	0.018	0.049	0.119	0.505	0.014	0.014	0.039	0.100	0.445
TRY	9	12	7	5	4	0.018	0.001	0.119	0.505	0.860	0.000	0.000	0.000	0.038	0.027
# of p-values less than 1%						11	10	7	1	2	12	11	8	4	6
# of p-values less than 5%						16	15	11	6	10	16	15	15	12	14

Table 4.15: Numbers of Violations and p-values of the Christoffersen Test for 99% VaR (Eight-week Forecast)

	# of Violations (out of 366 Weeks)					p-value (Unconditional Test)					p-value (Conditional Test)				
	AR(1)	AGNormal	AGT	AGNTS	MS−AR(1)	AR(1)	AGNormal	AGT	AGNTS	MS−AR(1)	AR(1)	AGNormal	AGT	AGNTS	MS−AR(1)
AUD	11	12	12	9	7	0.002	0.001	0.001	0.018	0.119	0.001	0.000	0.000	0.007	0.026
CAD	11	10	9	9	8	0.002	0.006	0.018	0.018	0.049	0.001	0.003	0.007	0.007	0.015
CHF	3	3	1	3	3	0.720	0.720	0.098	0.720	0.720	0.673	0.673	0.097	0.673	0.673
CLP	12	9	9	9	8	0.001	0.018	0.018	0.018	0.049	0.000	0.007	0.007	0.007	0.015
CZK	9	10	9	8	9	0.018	0.006	0.018	0.049	0.018	0.014	0.004	0.014	0.039	0.014
DKK	9	8	7	6	5	0.018	0.049	0.119	0.260	0.505	0.014	0.015	0.026	0.036	0.038
EUR	9	9	7	6	5	0.018	0.018	0.119	0.260	0.505	0.014	0.007	0.026	0.036	0.038
GBP	12	12	12	10	10	0.001	0.001	0.001	0.006	0.006	0.000	0.000	0.000	0.000	0.000
HUF	11	8	8	8	9	0.002	0.049	0.049	0.049	0.018	0.001	0.015	0.015	0.015	0.007
ILS	19	6	5	5	10	0.000	0.260	0.505	0.505	0.006	0.000	0.226	0.445	0.445	0.003
ISK	19	15	11	9	11	0.000	0.000	0.002	0.018	0.002	0.000	0.000	0.001	0.014	0.001
JPY	4	4	1	4	5	0.860	0.860	0.098	0.860	0.505	0.729	0.729	0.097	0.729	0.445
KRW	17	16	13	12	13	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000
NOK	11	12	10	8	9	0.002	0.001	0.006	0.049	0.018	0.000	0.000	0.000	0.000	0.000
NZD	12	12	12	6	8	0.001	0.001	0.001	0.260	0.049	0.000	0.000	0.000	0.226	0.039
PLN	14	12	10	8	10	0.000	0.001	0.006	0.049	0.006	0.000	0.000	0.000	0.015	0.000
SEK	9	9	6	5	4	0.018	0.018	0.260	0.505	0.860	0.014	0.014	0.226	0.445	0.729
TRY	9	17	8	6	4	0.018	0.000	0.049	0.260	0.860	0.000	0.000	0.000	0.036	0.027
# of p-values less than 1%						11	10	7	2	5	12	12	10	6	7
# of p-values less than 5%						16	15	12	10	11	16	15	14	13	15

Table 4.16: p-values of the Berkowitz Test for Independence and Tail (One-week Forecast)

	p-value (Independence Test)					p-value (Tail Test)				
	AR(1)	AGNormal	AGT	AGNTS	MS−AR(1)	AR(1)	AGNormal	AGT	AGNTS	MS−AR(1)
AUD	0.049	0.675	0.997	0.724	0.784	0.000	0.000	0.000	0.013	0.000
CAD	0.475	0.816	0.716	0.698	0.331	0.000	0.000	0.004	0.010	0.000
CHF	0.785	0.672	0.538	0.256	0.932	0.000	0.001	0.033	0.974	0.000
CLP	0.202	0.896	0.931	0.947	0.606	0.000	0.000	0.031	0.031	0.000
CZK	0.734	0.796	0.956	0.996	0.868	0.000	0.010	0.134	0.467	0.003
DKK	0.833	0.506	0.666	0.699	0.514	0.000	0.001	0.010	0.244	0.002
EUR	0.808	0.496	0.654	0.697	0.589	0.000	0.001	0.010	0.242	0.002
GBP	0.651	0.514	0.529	0.552	0.791	0.000	0.006	0.014	0.017	0.000
HUF	0.864	0.410	0.473	0.514	0.176	0.000	0.000	0.003	0.243	0.000
ILS	0.625	0.813	0.933	0.965	0.863	0.000	0.004	0.243	0.523	0.015
ISK	0.387	0.509	0.353	0.275	0.657	0.000	0.000	0.003	0.022	0.000
JPY	0.106	0.153	0.146	0.187	0.566	0.914	0.978	0.024	0.665	0.443
KRW	0.008	0.642	0.404	0.464	0.812	0.000	0.000	0.081	0.054	0.004
NOK	0.820	0.784	0.704	0.664	0.598	0.000	0.000	0.001	0.043	0.003
NZD	0.652	0.953	0.896	0.885	0.776	0.000	0.000	0.002	0.613	0.000
PLN	0.549	0.725	0.721	0.748	0.367	0.000	0.000	0.010	0.137	0.001
SEK	0.768	0.709	0.777	0.754	0.456	0.000	0.000	0.002	0.009	0.003
TRY	0.598	0.779	0.864	0.729	0.686	0.000	0.000	0.612	0.798	0.270
# of p-values less than 1%	1	0	0	0	0	17	17	8	1	15
# of p-values less than 5%	2	0	0	0	0	17	17	14	7	16

Table 4.17: p-values of the Berkowitz Test for Independence and Tail (Four-week Forecast)

	p-value (Independence Test)					p-value (Tail Test)				
	AR(1)	AGNormal	AGT	AGNTS	MS−AR(1)	AR(1)	AGNormal	AGT	AGNTS	MS−AR(1)
AUD	0.153	0.275	0.455	0.000	0.235	0.000	0.000	0.000	0.001	0.000
CAD	0.718	0.569	0.630	0.733	0.736	0.000	0.000	0.000	0.000	0.000
CHF	0.896	0.441	0.385	0.000	0.401	0.000	0.000	0.046	0.406	0.000
CLP	0.331	0.486	0.603	0.645	0.581	0.000	0.000	0.000	0.000	0.000
CZK	0.506	0.352	0.393	0.460	0.167	0.000	0.006	0.120	0.240	0.000
DKK	0.390	0.655	0.734	0.761	0.154	0.000	0.000	0.004	0.327	0.000
EUR	0.360	0.646	0.645	0.708	0.146	0.000	0.000	0.003	0.267	0.000
GBP	0.704	0.543	0.584	0.589	0.636	0.000	0.000	0.000	0.000	0.000
HUF	0.658	0.151	0.228	0.350	0.206	0.000	0.000	0.002	0.055	0.000
ILS	0.130	0.582	0.598	0.620	0.709	0.000	0.004	0.184	0.419	0.075
ISK	0.051	0.008	0.048	0.018	0.021	0.000	0.000	0.000	0.005	0.000
JPY	0.010	0.006	0.019	0.017	0.012	0.585	0.632	0.500	0.632	0.172
KRW	0.000	0.699	0.285	0.436	0.351	0.000	0.000	0.000	0.000	0.000
NOK	0.515	0.955	0.935	0.965	0.978	0.000	0.000	0.000	0.002	0.002
NZD	0.505	0.350	0.490	0.365	0.309	0.000	0.000	0.002	0.064	0.000
PLN	0.378	0.851	0.827	0.971	0.906	0.000	0.000	0.001	0.002	0.000
SEK	0.865	0.357	0.389	0.446	0.301	0.000	0.000	0.000	0.000	0.000
TRY	0.602	0.505	0.493	0.319	0.331	0.000	0.000	0.200	0.348	0.016
# of p-values less than 1%	1	2	0	2	0	17	17	13	9	15
# of p-values less than 5%	2	2	2	4	2	17	17	14	9	16

Table 4.18: p-values of the Berkowitz Test for Independence and Tail (Eight-week Forecast)

	p-value (Independence Test)					p-value (Tail Test)				
	AR(1)	AGNormal	AGT	AGNTS	MS−AR(1)	AR(1)	AGNormal	AGT	AGNTS	MS−AR(1)
AUD	0.149	0.175	0.406	0.074	0.580	0.000	0.000	0.000	0.000	0.000
CAD	0.762	0.786	0.825	0.939	0.548	0.000	0.000	0.000	0.000	0.000
CHF	0.952	0.771	0.609	0.256	0.556	0.000	0.000	0.000	0.557	0.000
CLP	0.334	0.838	0.867	0.893	0.949	0.000	0.000	0.001	0.000	0.000
CZK	0.578	0.597	0.639	0.703	0.372	0.000	0.004	0.055	0.139	0.000
DKK	0.426	0.695	0.794	0.873	0.119	0.000	0.000	0.001	0.002	0.000
EUR	0.395	0.651	0.742	0.774	0.108	0.000	0.000	0.001	0.014	0.000
GBP	0.624	0.709	0.715	0.729	0.518	0.000	0.000	0.000	0.000	0.000
HUF	0.708	0.311	0.398	0.575	0.614	0.000	0.000	0.000	0.001	0.000
ILS	0.140	0.512	0.539	0.597	0.428	0.000	0.003	0.200	0.450	0.007
ISK	0.073	0.010	0.101	0.035	0.037	0.000	0.000	0.000	0.001	0.000
JPY	0.018	0.022	0.058	0.056	0.016	0.489	0.804	0.256	0.887	0.056
KRW	0.000	0.260	0.175	0.327	0.240	0.000	0.000	0.000	0.000	0.000
NOK	0.460	0.735	0.687	0.714	0.880	0.000	0.000	0.001	0.001	0.003
NZD	0.519	0.299	0.443	0.327	0.251	0.000	0.000	0.000	0.013	0.000
PLN	0.279	0.785	0.830	0.768	0.469	0.000	0.000	0.000	0.000	0.000
SEK	0.987	0.606	0.648	0.724	0.478	0.000	0.000	0.000	0.000	0.000
TRY	0.534	0.222	0.474	0.242	0.267	0.000	0.000	0.050	0.159	0.015
# of p-values less than 1%	1	1	0	0	0	17	17	14	11	16
# of p-values less than 5%	2	2	0	1	2	17	17	14	13	17

Table 4.19: Bivariate Tail Dependencies between Exchange Rate Returns

j	$P_{i,j}^{AGMNTS}$	$P_{i,j}^{AGMNormal}$	$P_{i,j}^{Emp}$	$P_{i,j}^{AGMNTS}$	$P_{i,j}^{AGMNormal}$	$P_{i,j}^{Emp}$
		i = CAD			i = EUR	
AUD	0.0207	0.0163	0.0192	0.0194	0.0153	0.0208
CAD	n.a.	n.a.	n.a.	0.0149	0.0111	0.0128
CHF	0.0107	0.0079	0.0128	0.0309	0.0282	0.0352
CLP	0.0101	0.0068	0.0144	0.0099	0.0064	0.0080
CZK	0.0135	0.0100	0.0176	0.0325	0.0300	0.0288
DKK	0.0149	0.0111	0.0128	0.0479	0.0476	0.0480
EUR	0.0149	0.0111	0.0128	n.a.	n.a.	n.a.
GBP	0.0129	0.0092	0.0144	0.0228	0.0193	0.0224
HUF	0.0147	0.0106	0.0176	0.0290	0.0258	0.0224
ILS	0.0113	0.0078	0.0096	0.0124	0.0085	0.0064
ISK	0.0113	0.0077	0.0096	0.0199	0.0164	0.0192
JPY	0.0049	0.0031	0.0064	0.0105	0.0075	0.0096
KRW	0.0110	0.0079	0.0160	0.0112	0.0077	0.0064
NOK	0.0158	0.0116	0.0176	0.0301	0.0270	0.0320
NZD	0.0173	0.0130	0.0224	0.0175	0.0132	0.0160
PLN	0.0149	0.0111	0.0192	0.0243	0.0208	0.0240
SEK	0.0157	0.0118	0.0160	0.0317	0.0290	0.0288
TRY	0.0088	0.0069	0.0144	0.0088	0.0062	0.0096

Table 4.19(Cont.): Bivariate Tail Dependencies between Exchange Rate Returns

j	$P_{i,j}^{AGMNTS}$	$P_{i,j}^{AGMNormal}$	$P_{i,j}^{Emp}$	$P_{i,j}^{AGMNTS}$	$P_{i,j}^{AGMNormal}$	$P_{i,j}^{Emp}$
	i = GBP			i = JPY		
AUD	0.0168	0.0125	0.0160	0.0067	0.0038	0.0080
CAD	0.0129	0.0092	0.0144	0.0049	0.0031	0.0064
CHF	0.0185	0.0155	0.0208	0.0109	0.0095	0.0160
CLP	0.0090	0.0056	0.0096	0.0045	0.0028	0.0032
CZK	0.0205	0.0170	0.0224	0.0088	0.0063	0.0048
DKK	0.0226	0.0191	0.0208	0.0104	0.0075	0.0112
EUR	0.0228	0.0193	0.0224	0.0105	0.0075	0.0096
GBP	n.a.	n.a.	n.a.	0.0075	0.0050	0.0064
HUF	0.0193	0.0153	0.0208	0.0077	0.0047	0.0080
ILS	0.0109	0.0072	0.0112	0.0061	0.0039	0.0064
ISK	0.0158	0.0121	0.0112	0.0070	0.0045	0.0080
JPY	0.0075	0.0050	0.0064	n.a.	n.a.	n.a.
KRW	0.0109	0.0074	0.0112	0.0065	0.0046	0.0032
NOK	0.0194	0.0153	0.0192	0.0092	0.0059	0.0112
NZD	0.0160	0.0117	0.0192	0.0063	0.0035	0.0096
PLN	0.0181	0.0142	0.0208	0.0065	0.0040	0.0048
SEK	0.0203	0.0166	0.0176	0.0087	0.0059	0.0112
TRY	0.0079	0.0057	0.0112	0.0032	0.0020	0.0032

Table 4.20: p-values of the KS Test of AGFT Standardized Residuals against the NTS Distribution by Regimes

	Regime 1	Regime 2
AUD	0.562	0.003
CAD	0.721	0.269
CHF	0.702	0.082
CLP	0.198	0.001
CZK	0.926	0.000
DKK	0.701	0.008
EUR	0.550	0.007
GBP	0.884	0.246
HUF	0.807	0.002
ILS	0.273	0.020
ISK	0.658	0.000
JPY	0.950	n.a.
KRW	0.622	0.002
NOK	0.935	0.027
NZD	0.362	0.002
PLN	0.659	0.000
SEK	0.613	0.001
TRY	0.028	0.000
# of p-values less than 1%	0	12
# of p-values less than 5%	1	14

Note: The result of JPY for regime 2 is not available because the number of observations is insufficient.

Chapter 5 Conclusion

In this dissertation, we discuss risk measurement and management in global markets by using tempered stable distributions. We deal with two sectors of global markets; stocks and currencies. Through empirical studies, it is revealed that tempered stable distributions have wide uses in risk measurement and management. We briefly review the analysis in each chapter as follows.

In Chapter 2, we measure the systematic risk in the global banking sector, focusing on global banking stocks comprising global systemically important financial institutions (G-SIFIs). We estimate four systematic risk measures: joint probability and conditional probability of negative stock return movements, ΔCoVaR, and ΔCoAVaR. We find that the joint probability of negative movements is a good indicator of a significant increase in systematic risk. Subsequently, we investigate the relationship among the other three measures and find the following. Cross-sectional linkages between AVaR and ΔCoAVaR are few, if any, but there is a strong time series linkage. On the other hand, the conditional probability of negative movements and ΔCoAVaR show similar cross-sectional magnitude relations, though their time series linkage is not clear. Thus, we conclude that both AVaR and conditional probability of negative movements reinforce each other and serve as useful references for ΔCoAVaR-based systematic risk measurement. At the same time, we also statistically demonstrate that the multivariate autoregressive moving average generalized autoregressive conditional heteroscedasticity (ARMA–GARCH) model with the multivariate normal tempered stable (MNTS) distributed residuals is a more realistic model for G-SIFI stocks compared with that with the multivariate normal distributed residuals.

In Chapter 3, we propose mean−CoAVaR portfolio optimization to mitigate the potential loss brought by the systematic risk. It is a strategy to minimize the portfolio's CoAVaR with a given desired expected return. We perform empirical studies for six portfolios comprising G-SIFI stocks. The future returns of G-SIFI stocks are forecast by the ARMA–GARCH model with the MNTS distributed residuals because it is a better model to use with G-SIFI stocks, as is demonstrated in Chapter 2. As a result, we observe that the mean–CoAVaR portfolio incurs relatively smaller cumulative loss in most cases compared with the mean–AVaR and mean–variance portfolios. This implies that the mean–CoAVaR strategy is effective during a financial crisis. Our results open the applicability of CoVaR methodology to risk management.

In Chapter 4, we examine the usefulness of the GARCH model for use with 18 Organization for Economic Co-operation and Development (OECD) foreign currency exchange rate returns against United States Dollars, compared with the Markov–switching (MS) model. We introduce the NTS distribution to GARCH residuals of exchange rate returns, in order to capture fat-tailness and skewness. A previous study suggests that the MS model is a better model to use with exchange rate returns than the GARCH model, as long as GARCH residuals are assumed to be the normal i.i.d. in line with convention. We are interested in whether the introduction of the NTS distribution alters the situation. To examine the issue more comprehensively, we renew sample periods and expand samples, using not only weekly data but also daily data. The comparisons of the model performance are based on both in-sample and out-of-sample tests. We find that the results are in favor of the GARCH model with the NTS distributed residuals against the MS model. This is particularly demonstrated by the tests for the accuracy of VaR and tail behavior forecasts, reflecting the superior descriptive power of the NTS distribution for fat-tailness and skewness. Note that the GARCH model with the normally distributed residuals is often outperformed by the MS model, which is consistent with the previous study. However, the introduction of the NTS distribution to the GARCH model inverts the situation. In addition, for higher frequency data, the clear superiority of the MS model disappears even in the in-sample tests. To further demonstrate the usefulness of the GARCH model with exchange rate returns, we apply the GARCH model with the MNTS distributed residuals for 18 OECD currency exchange rate returns. From the in-sample test, we confirm that the MNTS distribution accounts for tail dependencies between exchange rate returns much better than the multivariate normal distribution. Finally, the possibility of combining the GARCH model with the NTS distributed residuals and MS structure is discussed. Given that the residuals in the turbulent regime appear to have a fatter tail than those in the tranquil regime, it is expected that a better model can be derived by introducing the regime-dependent parameters to the NTS distribution.

To summarize, the contribution of this dissertation is threefold. The first is to measure systematic risk through the standpoint of the global markets, the second is to propose portfolio optimization against systematic risk, and the third is to find a better model for global foreign currency exchange rate returns in terms of risk management. Given that risk management and management now need a worldwide perspective, the findings of this dissertation are useful and can serve as a reference when the related topics are studied elsewhere. There are several possible areas for future study that would be in line with this dissertation. We propose these directions in the closing remarks.

One of the most important extensions of our studies would be the improvement of data frequency. Because of the developments of computer technologies, the financial dealings and price changes get more and more frequent. The frequency is now about to reach the nanosecond, i.e., 10^{-9} second level (Barry, 2012). The positive aspect of these advancements is to provide liquidity to the market and thus make trading smooth therein. At the same time, however, it should be noted that it will be critical to measure and manage the intra-day market and liquidity risk in the near future. The significance of the intra-day risk management became apparent in the May 6th, 2010 Flash Crash, when the Dow Jones Industrial Average plunged by about 1000 points within just five minutes. Exchanges, research institutions, and financial regulators are now starting to store and provide intra-day market data to market participants, so that they can analyze the intra-day risk. Through investigating such data, new types of risk will be gradually clarified.

Next, we should pay attention to the cross-cutting linkages among market sectors, whereas our analysis is performed by market sector; stocks and currencies. The influences of devastating systemic events would appear in all sectors of markets simultaneously, and the conditions of individual market sectors would deteriorate even more through interactions among themselves. We could construct a better *early-warning* system for forthcoming financial crises by investigating global financial markets and identifying every symptom of the systemic downturn in a cross-sectional manner. Because not only trading at exchanges but also over-the-counter trading play important roles in the global market, over-the-counter should be considered ultimately in the construction of the early-warning system, overcoming the unavailability of trading data. Moreover, the macroeconomic variables must be significant to the early warning system because they also vividly reflect the market conditions.

Finally, it would be an intriguing and practical problem to perform risk management for more global samples. Given that investment activity now has a global context, the portfolio managers in the largest investment companies can choose from a great number of securities from across the world to construct profitable portfolios. Accordingly, they need to manage the risk of a massive portfolio through analysis of the depending structures among securities. Considering the growing high frequency trading, it is imperative for portfolio managers to analyze the risk profiles of many securities and optimize the portfolio in a very short time. This problem has not only a theoretical aspect to develop advanced techniques to analyze the risk, but also a practical aspect to make such analysis feasible with a given computational resource. We should note that, in this sense, modern quantitative finance is becoming more associated with computer science. It may be easily imagined that researchers in the field of quantitative finance and computer science will collaborate with each other very intensively in the near future.

Bibliography

1. Acharya, V. V., L. H. Pedersen, T. Philippon and M. Richardson (2010), "Measuring Systemic Risk," Technical Report, *NYU Stern School of Business*, New York.

2. Adrian, T. and M. K. Brunnermeier (2011), "CoVaR," *Federal Reserve Bank of New York Staff Reports* 348.

3. Barry, D. J. (2012), "Why Nanosecond Precision is Needed in High Performance Trading Environments," *High Frequency Trading Review*, available at http://www.hftreview.com/pg/blog/napatech/read/64521/why-nanosecond-precision-is-needed-in-high-performance-trading-environments

4. Basel Committee on Banking Supervision (2011a), "Basel III: A global regulatory framework for more resilient banks and banking systems," *Bank for International Settlements*, Basel.

5. Basel Committee on Banking Supervision (2011b), "Global systemically important banks: Assessment methodology and the additional loss absorbency requirement," *Bank for International Settlements*, Basel.

6. Basel Committee on Banking Supervision (2012), "Consultative document: Fundamental review of the trading book," *Bank for International Settlements*, Basel.

7. Bauwens, L., S. Laurent and J. V. K. Rombouts (2006), "Multivariate GARCH Model: A Survey," *Journal of Applied Econometrics* 21 (1), 79-109.

8. Berkowitz, J. (2001), "Testing Density Forecasts, With Applications to Risk Management," *Journal of Business and Economic Statistics* 19 (4), 465-474.

9. Bhar, R. and S. Hamori (2009), *Hidden Markov Models: Applications to Financial Economics*, Kluwer Academic Publishers.

10. Bollen, N. P. B., S. F. Gray and R. E. Whaley (2000), "Regime switching in foreign exchange rates: Evidence from currency option prices," *Journal of Econometrics* 94 (1-2), 239-276.

11. Bollerslev, T. (1986), "Generalized Autoregressive Conditional Heteroskedasticity," *Journal of Econometrics* 31 (3), 307-327.

12. Bonilla, C. A., R. Romero-Meza and M. J. Hinich (2007), "GARCH Inadequacy for Modelling Exchange Rates: Empirical Evidence from Latin America," *Applied Economics* 39 (19), 2529-2533.

13. Boyarchenko, S. I. and S. Z. Levendorskiĭ (2000), ”Option pricing for truncated Lévy processes,” *International Journal of Theoretical and Applied Finance* 3 (3), 549-552.

14. Brooks, C. and M. J. Hinich (1998), ”Episodic nonstationarity in exchange rates,” *Applied Economics* 5 (11), 719-722.

15. Cai, J. (1994), “A Markov Model of Switching-Regime ARCH,” *Journal of Business and Economic Statistics* 12 (3), 309-316.

16. Carr, P., H. Geman, D. B. Madan and M. Yor (2002), ”The Fine Structure of Asset Returns: An Empirical Investigation,” *The Journal of Business* 75 (2), 305-332.

17. Christoffersen, P. F. (1998), “Evaluating interval forecasts,” *International Economic Review* 39 (4), 841-862.

18. Condoyanni, L., J. O'Hanlon and C. W. R. Ward (1987), “Day of the Week Effects On Stock Returns: International Evidence,” *Journal of Business Finance and Accounting* 14 (2), 159-174.

19. Dewachter, H. (2001), “Can Markov switching models replicate chartist profits in the foreign exchange market?,” *Journal of International Money and Finance* 20 (1), 25-41.

20. Engel, C. and J. D. Hamilton (1990), “Long Swings in the Dollar: Are They in the Data and Do Markets Know It?,” *American Economic Review* 80 (4), 689-713.

21. Engle, R. F. (1982), “Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation,” *Econometrica* 50 (4), 987-1007.

22. Fama, E. F. (1963), ”Mandelbrot and the Stable Paretian Hypothesis,” *The Journal of Business* 36 (4), 420-429.

23. Financial Stability Board (2011), “Policy Measures to Address Systemically Important Financial Institutions,” Basel.

24. Financial Stability Board (2012), “Update of group of global systemically important banks (G-SIBs),” Basel.

25. Financial Stability Board (2013), “2013 update of group of global systemically important banks (G-SIBs),” Basel.

26. Giesecke, K. and B. Kim (2011), “Systemic Risk: What Defaults Are Telling Us,” *Management Science* 57 (8), 1387-1405.

27. Girardi, G. and A. T. Ergün (2013), “Systemic Risk Measurement: Multivariate GARCH Estimation of CoVaR,” *Journal of Banking and Finance* 37 (8), 3169-3180.

28. Gray, S. F. (1996), “Modeling the Conditional Distribution of Interest Rates as a Regime-Switching Process,” *Journal of Financial Economics* 42 (1), 27-62.

29. Hamilton, J. D. (1989), ”A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle,” *Econometrica* 57 (2), 357-384.

30. Hamilton, J. D. (1990), ”Analysis of time series subject to changes in regime,” *Journal of Econometrics* 45 (1-2), 39-70.

31. Hamilton, J. D. (1994), *Time Series Analysis*, Princeton University Press.

32. Hamilton, J. D. and B. Raj (2002), *Advances in Markov-Switching Models: Applications in Business Cycle Research and Finance*, Springer-Verlag.

33. Hamilton, J. D. and R. Susmel (1994), "Autoregressive conditional heteroskedasticity and changes in regime," *Journal of Econometrics* 64 (1-2), 307-333.

34. Hansen, B. E. (1992), "The likelihood ratio test under nonstandard conditions: Testing the markov switching model of GNP," *Journal of Applied Econometrics* 7 (S1), 61-82.

35. Hansen, L. P. (2013), "Challenges in Identifying and Measuring Systemic Risk," *Working Paper*, National Bureau of Economic Research.

36. Henneke, J. S., S. T. Rachev, F. J. Fabozzi and M. Nikolov (2011), ”MCMC-based estimation of Markov Switching ARMA-GARCH models,” *Applied Economics* 43 (3), 259-271.

37. Huang, X., H. Zhou and H. Zhu (2009), "A framework for assessing the systemic risk of major financial institutions," *Journal of Banking and Finance* 33 (11), 2036-2049.

38. Ichiue, H. and K. Koyama (2011), "Regime switches in exchange rate volatility and uncovered interest parity," *Journal of International Money and Finance* 30 (7), 1436-1450.

39. Kaufman, G. G. and K. E. Scott (2003), "What is Systemic Risk, and Do Bank Regulators Retard or Contribute to It?," *The Independent Review* 7 (3), Winter, 371-391.

40. Kim, C. J. and C. R. Nelson (1999), *State-Space Models with Regime-Switching: Classical and Gibbs-Sampling Approaches with Applications*, MIT Press.

41. Kim, Y. S. and D. S. Volkmann (2013), ”Normal Tempered Stable Copula,” *Applied Mathematics Letters* 26 (7), 676-680.

42. Kim, Y. S., R. Giacometti, S. T. Rachev, F. J. Fabozzi and D. Mignacca (2012), "Measuring financial risk and portfolio optimization with a non-Gaussian multivariate model," *Annals of Operations Research* 201 (1), 325-343.

43. Kim, Y. S., S. T. Rachev, M. L. Bianchi and F. J. Fabozzi (2008), ”Financial market models with Lévy processes and time-varying volatility," *Journal of Banking and Finance* 32 (7), 1363-1378.

44. Kim, Y. S., S. T. Rachev, M. L. Bianchi and F. J. Fabozzi (2010), ”Computing VaR and AVaR in infinitely divisible distributions,” *Probability and Mathematical Statistics* 30 (2), 223-245.

45. Kim, Y. S., S. T. Rachev, M. L. Bianchi, I. Mitov and F. J. Fabozzi (2011), ”Time series analysis for financial market meltdowns,” *Journal of Banking and Finance* 35 (8), 1879-1891.

46. Koponen, I. (1995), ”Analytic approach to the problem of convergence of truncated Lévy flights towards the Gaussian stochastic process,” *Physical Review E* 52 (1), 1197-1199.

47. Kurosaki, T. and Y. S. Kim (2013a), “Systematic risk measurement in the global banking stock market with time series analysis and CoVaR,” *Journal of Investment Management and Financial Innovations* 10 (1), 184-196.

48. Kurosaki, T. and Y. S. Kim (2013b), “Mean-CoAVaR optimization for global banking portfolios,” *Journal of Investment Management and Financial Innovations* 10 (2), 15-20.

49. López-Espinosa, G., A. Moreno, A. Rubia and L. Valderrama (2012), “Short-term wholesale funding and systemic risk: A global CoVaR approach,” *Journal of Banking and Finance* 36 (12), 3150-3162.

50. Mainik, G. and E. Schaanning (2012), “On dependence consistency of CoVaR and some other systemic risk measures,” *Working paper*, Available at arXiv: http://arxiv.org/abs/1207.3464.

51. Mandelbrot, B. (1963a), ”The Variation of Certain Speculative Prices,” *The Journal of Business* 36 (4), 394-419.

52. Mandelbrot, B. (1963b), ”New methods in statistical economics,” *Journal of Political Economy* 71 (5), 421-440.

53. Marcucci, J. (2005), "Forecasting Stock Market Volatility with Regime-Switching GARCH Models," *Studies in Nonlinear Dynamics and Econometrics* 9 (4), Article 6.

54. Markowitz, H (1952), “Portfolio Selection,” *The Journal of Finance* 7 (1), 77-91.

55. McCulloch, J. H. (1997), “Measuring Tail Thickness to Estimate the Stable Index α: A Critique,” *Journal of Business and Economic Statistics* 15 (1), 74-81.

56. McNeil, A. J., R. Frey and P. Embrechts (2005), *Quantitative Risk Management: Concepts, Techniques and Tools*, Princeton University Press.

57. Perlin, M. (2012), ”MS Regress - The MATLAB Package for Markov Regime Switching Models,” *Working Paper*, Available at SSRN: http://ssrn.com/abstract=1714016.

58. Rachev, S. T., C. Menn and F. J. Fabozzi (2005), *Fat-Tailed and Skewed Asset Return Distributions: Implications for Risk Management, Portfolio Selection, and Option Pricing*, John Wiley and Sons.

59. Rachev, S. T., R. D. Martin, B. Racheva and S. Stoyanov (2008a), “Stable ETL Optimal Portfolios and Extreme Risk Management,” in *Risk Assessment: Decisions in Banking and Finance,* Phisica-Verlag.

60. Rachev, S. T., S. V. Stoyanov and F. J. Fabozzi (2008b), *Advanced Stochastic Models, Risk Assessment, and Portfolio Optimization: The Ideal Risk, Uncertainty, and Performance Measures*, John Wiley and Sons.

61. Rachev, S. T., Y. S. Kim, M. L. Bianchi and F. J. Fabozzi (2011), *Financial Models with Lévy Processes and Volatility Clustering,* John Wiley and Sons.

62. Schwaab, B., S. J. Koopman and A. Lucas (2011), “Systemic Risk Diagnostics; Coincident Indicators and Early Warning Signals,” *European Central Bank Working Paper* 1327.

63. Segoviano, M. A. and C. Goodhart (2009), “Banking Stability Measures,” *IMF Working Paper* WP/09/4.

64. Sharpe, W. F. (1966), “Mutual Fund Performance,” *The Journal of Business* 39 (1), 119-138.

65. Sims, C. A., D. F. Waggoner and T. Zha (2008), “Methods for inference in large multiple-equation Markov-switching models,” *Journal of Econometrics* 146 (2), 255-274.

66. Sun, W., S. Rachev, S. V. Stoyanov and F. J. Fabozzi (2011), “Multivariate Skewed Student’s t Copula in the Analysis of Nonlinear and Asymmetric Dependence in the German Equity Market,” *Studies in Nonlinear Dynamics and Econometrics* 12 (2), Article 3.

67. Wong, A. Y-T. and T. P. W. Fong (2011), “Analysing interconnectivity among economies,” *Emerging Markets Review* 12 (4), 432-442.

68. Wong, A. Y-T. and T. P. W. Fong (2012), “Gauging potential sovereign risk contagion in Europe,” *Economics Letters* 115 (3), 496-499.

69. Zhou, C. (2010), “Are Banks Too Big to Fail? Measuring Systemic Importance of Financial Institutions,” *International Journal of Central Banking* 6 (4), 205-250.

Appendix A List of 29 G-SIFIs as of November 2011

Table A.1: List of 29 G-SIFIs as of November 2011[39]

United States	Europe	Asia
Bank of America (BAC)	Banque Populaire CdE	Bank of China (3988)
Bank of New York Mellon (BK)	Barclays (BARC)	Mitsubishi UFJ FG (8306)
Citigroup (C)	BNP Paribas (BNP)	Mizuho FG (8411)
Goldman Sachs (GS)	Commerzbank (CBK)	Sumitomo Mitsui FG (8316)
JP Morgan Chase (JPM)	Credit Suisse (CSGN)	
Morgan Stanley (MS)	Deutsche Bank (DBK)	
State Street (STT)	Dexia (DEXB)	
Wells Fargo (WFC)	Group Crédit Agricole (ACA)	
	HSBC (HSBA)	
	ING Bank (INGA)	
	Lloyds Banking Group (LLOY)	
	Nordea (NDA)	
	Royal Bank of Scotland (RBS)	
	Santander (SAN)	
	Société Générale (GLE)	
	UBS (UBSN)	
	Unicredit Group (UCG)	

Note: Characters in parentheses stand for the ticker symbols in each domestic market. We refer to G-SIFIs by their ticker symbol except the Asian G-SIFIs. We refer to the Asian G-SIFIs by their abbreviations: BOC (Bank of China), MUFJ (Mitsubishi UFJ FG), MHFG (Mizuho FG), and SMFG (Sumitomo Mitsui FG.)

[39] The most recent list at the time of this writing contains revisions owing to the update on November 2012 and 2013. Compared to the list given here, two institutions were added (BBVA and Standard Chartered) and three institutions were removed (Commerzbank, Dexia, and Lloyds Banking Group) in 2012, and one institution (Industrial and Commercial Bank of China Limited) was added in 2013, which still remains in 29 G-SIFIs in total. The Financial Stability Board announces that the list of G-SIFIs will be updated every November. See Financial Stability Board (2011, 2012, 2013).

Appendix B Multivariate Normal Tempered Stable Distribution

In this appendix, we present the mathematical definition of the multivariate normal tempered stable (MNTS) distribution proposed by Kim et al. (2012). We begin with the univariate NTS distribution. The NTS distributed random variable *X* is defined by

$$X = \mu + \beta(T-1) + \gamma\sqrt{T}\epsilon, \tag{B.1}$$

where $\epsilon \sim N(0,1)$ and *T* is the classical tempered stable (CTS) subordinator whose characteristic function is

$$\phi_T(u) = \exp\left[-\frac{2\theta^{1-\alpha/2}}{\alpha}\left\{(\theta - iu)^{\alpha/2} - \theta^{\alpha/2}\right\}\right]. \tag{B.2}$$

The characteristic function of *X* is given by

$$\phi_X(u) = \exp\left[i(\mu-\beta)u - \frac{2\theta^{1-\alpha/2}}{\alpha}\left\{\left(\theta - i\beta u + \frac{\gamma^2u^2}{2}\right)^{\alpha/2} - \theta^{\alpha/2}\right\}\right]. \tag{B.3}$$

The real parameter set $(\alpha,\theta,\beta,\gamma,\mu)$ characterizes the NTS distribution. The parameters (α,θ) dominate the fat-tailness, β dominates the skewness, γ scales the distribution and μ is the mean $E[X]$, noting that $E[T]=1$. The variance is given by $\mathrm{Var}[X] = \gamma^2 + \beta^2\left(\frac{2-\alpha}{2\theta}\right)$. The constraints of parameters are $0<\alpha<2, 0<\theta, 0<\gamma$. If we set $\mu = 0, \gamma = \sqrt{1-\beta^2\left(\frac{2-\alpha}{2\theta}\right)}$ with $|\beta| < \sqrt{\frac{2\theta}{2-\alpha}}$, the NTS distribution is standardized into zero mean and unit variance, which we call the standard NTS distribution.

Based on Eq. (B.1), the NTS distribution is easily extended into the multivariate version. The *N*-dimensional MNTS distributed random variable $\boldsymbol{X} = (X_1, X_2, \cdots, X_N)$ is defined by

$$\boldsymbol{X} = \boldsymbol{\mu} + \boldsymbol{\beta}(T-1) + \boldsymbol{\gamma}\sqrt{T}\boldsymbol{\epsilon}. \tag{B.4}$$

Here, $\boldsymbol{\mu} = (\mu_1, \mu_2, \cdots, \mu_N)$, $\boldsymbol{\beta} = (\beta_1, \beta_2, \cdots, \beta_N)$, and $\boldsymbol{\gamma} = (\gamma_1, \gamma_2, \cdots, \gamma_N)$ are the *N*-dimensional real vector parameters. The parameter set $(\alpha, \theta, \beta_n, \gamma_n, \mu_n)$ functions as the parameters of the univariate NTS distribution for the *n*-th marginal $(1 \le n \le N)$. Thus, every γ_n is larger than 0. The *N*-dimensional random variable $\boldsymbol{\epsilon} = (\epsilon_1, \epsilon_2, \cdots, \epsilon_N)$ follows the multivariate normal distribution $N(\boldsymbol{0}, \boldsymbol{\rho})$, where $\boldsymbol{\rho} = (\rho_{k,l})$ is the correlation matrix. In summary, the real parameter set $(\alpha, \theta, \boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{\mu}, \boldsymbol{\rho})$ characterizes the MNTS distribution. Note that α and θ are common parameters among every marginal. The variance-covariance matrix of $\boldsymbol{X}$ is given by

$$\mathrm{cov}(X_k, X_l) = \rho_{k,l}\gamma_k\gamma_l + \beta_k\beta_l\left(\frac{2-\alpha}{2\theta}\right), \qquad 1 \le k, l \le N. \tag{B.5}$$

Given the other parameters, we can estimate $\boldsymbol{\rho}$ from the variance and the covariance of $\boldsymbol{X}$ by using Eq. (B.5) in an opposite manner. If we set $\boldsymbol{\mu} = (0,0,\cdots,0)$, $\gamma_n = \sqrt{1-\beta_n^2\left(\frac{2-\alpha}{2\theta}\right)}$ with

$|\beta_n| < \sqrt{\frac{2\theta}{2-\alpha}}, \forall n,$ the MNTS distribution is standardized such that $E[X_n] = 0, \mathrm{Var}[X_n] = 1, \forall n,$ which we call the standard MNTS distribution.

It is critical that the NTS distribution has a reproductive property. Let the joint stock return distribution $\boldsymbol{R} = (R_1, R_2, \cdots, R_N)$ be modeled by the MNTS distribution with the parameters $(\alpha, \theta, \boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{\mu}, \boldsymbol{\rho})$. Consider the portfolio whose weight on stock n is w_n. The portfolio return R is given by

$$\begin{aligned} R = \sum_{n=1}^{N} w_n R_n &= \sum_{n=1}^{N} w_n \mu_n + \sum_{n=1}^{N} w_n \left\{\beta_n (T-1) + \gamma_n \sqrt{T} \epsilon_n\right\} \\ &= \sum_{n=1}^{N} w_n \mu_n + \left(\sum_{n=1}^{N} w_n \beta_n\right)(T-1) + \sqrt{T} \sum_{n=1}^{N} w_n \gamma_n \epsilon_n. \end{aligned} \tag{B.6}$$

Since we have

$$\sum_{n=1}^{N} w_n \gamma_n \epsilon_n = \epsilon_R \sqrt{\sum_{k=1}^{N} \sum_{l=1}^{N} w_k w_l \gamma_k \gamma_l \rho_{k,l}}, \quad \epsilon_R \sim N(0,1), \tag{B.7}$$

by the reproductive property of the normal distribution, R follows the NTS distribution by definition. Thereby, we can estimate the VaR or AVaR of the portfolio under the MNTS model.

It is also worth mentioning the analytic formulae of the cumulative distribution function and probability density function of the MNTS distribution (Kim and Volkmann, 2013). The N-dimensional cumulative distribution function of the standard MNTS distributed random variable $\boldsymbol{X}$ with the parameters $(\alpha, \theta, \boldsymbol{\beta}, \boldsymbol{\rho})$ and $\boldsymbol{\gamma} = \sqrt{1 - \boldsymbol{\beta}^2 \left(\frac{2-\alpha}{2\theta}\right)}$ is given by

$$\begin{aligned} F(a_1, \cdots, a_N) = P(X_1 < a_1, \cdots, X_N < a_N) &= \int_0^\infty G(t; \boldsymbol{a}) f_T(t)\, \mathrm{d}t, \\ f_T(t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-iut} \phi_T(u)\, \mathrm{d}u, \\ G(t; \boldsymbol{a}) &= \prod_{n=1}^{N} \int_{-\infty}^{\frac{a_n - \beta_n (t-1)}{\gamma_n \sqrt{t}}} f_\epsilon(x_1, \cdots, x_N)\, \mathrm{d}x_n, \end{aligned} \tag{B.8}$$

and $f_\epsilon(x_1, \cdots, x_N)$ is the probability density function of the N-dimensional normal distribution $N(\boldsymbol{0}, \boldsymbol{\rho})$.

The N-dimensional probability density function of the standard MNTS distributed random variable $\boldsymbol{X}$ is given by

$$\begin{aligned} f_X(\boldsymbol{x}) &= \prod_{n=1}^{N} \frac{\partial^n}{\partial x_n} F(x_1, \cdots, x_N) \\ &= \int_0^\infty \frac{1}{(2\pi)^{\frac{N}{2}} |\boldsymbol{\Sigma}(t)|^{\frac{1}{2}}} \exp\left[-\frac{1}{2}\big(\boldsymbol{x} - \boldsymbol{m}(t)\big)\big(\boldsymbol{\Sigma}(t)\big)^{-1}\big(\boldsymbol{x} - \boldsymbol{m}(t)\big)\right] f_T(t)\, \mathrm{d}t, \\ \boldsymbol{x} &= (x_1, \cdots, x_N), \qquad \boldsymbol{m}(t) = \boldsymbol{\beta}(t-1) = \big(\beta_1(t-1), \cdots, \beta_N(t-1)\big), \\ &\text{and } \boldsymbol{\Sigma}(t) = \big(\Sigma_{k,l}(t)\big) = \big(\gamma_k \gamma_l \rho_{k,l} t\big),\ 1 \le k, l \le N. \end{aligned} \tag{B.9}$$

Note that the integral kernel of $f_X(\boldsymbol{x})$ is equivalent to the probability density function of the N-dimensional normal distribution $N(\boldsymbol{m}(t), \boldsymbol{\Sigma}(t))$.

Appendix C VaR and AVaR in Infinitely Divisible Distribution

In this appendix, we present the closed-form formulae of Value at Risk (VaR) and average Value at Risk (AVaR) for infinitely divisible distributions[40], including the normal tempered stable (NTS) and classical tempered stable (CTS) distributions. The proof is given in Kim et al. (2010).

Let VaR and AVaR be defined for the infinitely divisible and continuously distributed return R, whose characteristic function is given by $\phi_R(u) = E[e^{iuR}]$. We denote $\mathrm{VaR}_\epsilon(R)$ and $\mathrm{AVaR}_\epsilon(R)$, respectively, where ϵ stands for the significance level ($0 < \epsilon < 1$.) If there is $\delta > 0$ such that $|\phi_R(u + i\delta)| < \infty$ for $\forall u \in \mathbb{R}$, then the cumulative distribution function of R is expressed as

$$F_R(x) = \frac{e^{x\delta}}{\pi} \mathrm{Re} \int_0^\infty e^{-ixu} \frac{\phi_R(u + i\delta)}{\delta - iu} \, \mathrm{d}u. \quad \text{(C.1)}$$

Accordingly, $\mathrm{VaR}_\epsilon(R)$ is derived because, in the continuous case, $\mathrm{VaR}_\epsilon(R)$ is the ϵ-quantile of R.

Under the same condition for δ, $\mathrm{AVaR}_\epsilon(R)$ is derived as

$$\begin{aligned}\mathrm{AVaR}_\epsilon(R) &= -\mathrm{E}(R \mid R < -\mathrm{VaR}_\epsilon(R)) \\ &= \mathrm{VaR}_\epsilon(R) - \frac{e^{-\mathrm{VaR}_\epsilon(R)\delta}}{\pi\epsilon} \mathrm{Re} \int_0^\infty e^{-iu\mathrm{VaR}_\epsilon(R)} \frac{\phi_R(-u + i\delta)}{(-u + i\delta)^2} \, \mathrm{d}u. \quad \text{(C.2)}\end{aligned}$$

We utilize the presented formulae to calculate VaR and AVaR for the NTS distribution in Chapters 2 and 4.

[40] See Rachev et al. (2011) for more information about infinitely divisible distributions.

Appendix D Markov Switching Model

In this appendix, more detail of a two-regime Markov–switching AR(1) model is presented. A two-regime Markov–switching AR(1) model is expressed as

$$R_t = \alpha_{s_t} R_{t-1} + \beta_{s_t} + \sigma_{s_t}\eta_t, \eta_t \sim N(0,1), \tag{D.1}$$

where $s_t \in \{1,2\}$ stands for the latent regime variable and $\alpha_{s_t}, \beta_{s_t}, \sigma_{s_t}$ are regime-dependent parameters. The switching between regimes is described by the following time-homogeneous Markov chain:

$$p_{ij} = \text{Prob}(s_{t+1} = i | s_t = j), i, j \in \{1,2\}. \tag{D.2}$$

In the two-regime Markov–switching model, the conditional probability density distribution $f(R_{t+1}|\Omega_t)$ is given by the mixture distribution of two states:

$$f(R_{t+1}|\Omega_t) = \sum_{i=1}^{2} \text{Prob}(s_{t+1} = i|\Omega_t) f(R_{t+1}|\Omega_t, s_{t+1} = i), \tag{D.3}$$

where Ω_t is the set of information known up to *t*. Under the conditional normal assumption, $f(R_{t+1}|\Omega_t, s_{t+1} = i)$ is in the form of

$$f(R_{t+1}|\Omega_t, s_{t+1} = i) = \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\left[\frac{\{R_{t+1} - (\alpha_i R_t + \beta_i)\}^2}{2\sigma_i^2}\right]. \tag{D.4}$$

At the initial period $t = 1$, we use the ergodic probability for the conditional probability:

$$p_i^* = \text{Prob}(s_1 = i|\Omega_0) = \text{Prob}(s_1 = i) = \frac{1 - p_{ii}}{2 - p_{11} - p_{22}}. \tag{D.5}$$

There are two key probabilities associated with the Markov–switching model: filtered probability $\text{Prob}(s_t = i|\Omega_t)$ and smoothed probability $\text{Prob}(s_t = i|\Omega_T)$, where *T* denotes the end period. Filtered probability is calculated through the forward recurrence formula regarding $\text{Prob}(s_t = i|\Omega_{t-1})$,

$$\begin{aligned}\text{Prob}(s_{t+1} = i|\Omega_t) &= \sum_{j=1}^{2} p_{ij} \text{Prob}(s_t = j|\Omega_t) \\ &= \sum_{j=1}^{2} \frac{p_{ij} \text{Prob}(s_t = j|\Omega_{t-1}) f(R_t|\Omega_{t-1}, s_t = j)}{f(R_t|\Omega_{t-1})}.\end{aligned} \tag{D.6}$$

Given $\text{Prob}(s_{t+1} = i|\Omega_t)$, the filtered probability is updated as follows:

$$\text{Prob}(s_{t+1} = i|\Omega_{t+1}) = \frac{\text{Prob}(s_{t+1} = i|\Omega_t) f(R_{t+1}|\Omega_t, s_{t+1} = i)}{f(R_{t+1}|\Omega_t)}. \tag{D.7}$$

On the other hand, the smoothed probability is calculated through the backward recurrence formula,

$$\begin{aligned}\text{Prob}(s_t = i|\Omega_T) &= \sum_{j=1}^{2} \text{Prob}(s_t = i, s_{t+1} = j|\Omega_T) \\ &= \sum_{j=1}^{2} \frac{p_{ji}\,\text{Prob}(s_{t+1} = j|\Omega_T)\,\text{Prob}(s_t = i|\Omega_t)}{\text{Prob}(s_{t+1} = j|\Omega_t)}. \qquad \text{(D.8)}\end{aligned}$$

Smoothed probability represents the probability of the regime $i = 1,2$ in the past, given all the observations. It gives us insights into the timing of structural changes.

Note that, by combining Eqs. (D.3) - (D.7), the maximum likelihood estimation for the Markov–switching model (D.1) is feasible. The maximum likelihood estimation is easily applicable when the number of regimes is two, as is the case in this dissertation. However, it does not work effectively when the number of regimes increases. In that case, Hamilton (1990) suggests using the EM algorithm instead.

Appendix E Results of In-Sample Tests on the Basis of Daily Datasets of 18 OECD Currency Exchange Rate Returns (3129 Observations)

Table E.1: KS, AD, and AD^2 Statistics of AR(1)–GARCH(1,1) Standardized Residuals against the Proposed Distributions (Daily Data)

	Normal				Student t				NTS			
	KS		AD	AD^2	KS		AD	AD^2	KS		AD	AD^2
	Statistic	(p-value)	Statistic	Statistic	Statistic	(p-value)	Statistic	Statistic	Statistic	(p-value)	Statistic	Statistic
AUD	0.034	(0.002)	0.535	2.042	0.015	(0.502)	0.070	0.708	0.019	(0.186)	0.044	0.717
CAD	0.021	(0.125)	0.056	0.565	0.014	(0.585)	0.044	0.214	0.011	(0.849)	0.041	0.144
CHF	0.033	(0.002)	147.771	1.691	0.018	(0.240)	0.082	0.345	0.012	(0.770)	0.532	0.175
CLP	0.038	(0.000)	6.949E+11	3.512	0.016	(0.363)	0.235	0.247	0.020	(0.155)	0.922	0.304
CZK	0.030	(0.008)	0.301	1.583	0.013	(0.623)	0.031	0.193	0.014	(0.566)	0.030	0.146
DKK	0.025	(0.035)	0.073	0.873	0.018	(0.257)	0.047	0.288	0.016	(0.378)	0.037	0.216
EUR	0.025	(0.038)	0.079	0.889	0.017	(0.312)	0.046	0.280	0.014	(0.590)	0.039	0.193
GBP	0.018	(0.262)	0.078	0.674	0.013	(0.682)	0.058	0.238	0.016	(0.419)	0.045	0.215
HUF	0.031	(0.005)	3.632E+06	1.754	0.013	(0.634)	0.348	0.364	0.022	(0.083)	0.123	0.603
ILS	0.037	(0.000)	201.098	3.686	0.015	(0.474)	0.051	0.182	0.016	(0.412)	0.068	0.161
ISK	0.045	(0.000)	2.217E+04	3.852	0.020	(0.154)	0.053	0.535	0.026	(0.028)	0.053	0.733
JPY	0.033	(0.002)	45.186	2.574	0.011	(0.836)	0.040	0.173	0.017	(0.321)	0.037	0.258
KRW	0.050	(0.000)	2693.934	5.882	0.019	(0.201)	0.082	0.853	0.031	(0.005)	0.068	0.993
NOK	0.019	(0.204)	0.116	0.729	0.012	(0.796)	0.049	0.196	0.016	(0.383)	0.036	0.198
NZD	0.029	(0.010)	0.237	1.439	0.016	(0.380)	0.071	0.614	0.019	(0.230)	0.042	0.532
PLN	0.028	(0.017)	8384.285	1.827	0.017	(0.353)	0.116	0.499	0.030	(0.006)	0.080	0.887
SEK	0.017	(0.296)	0.116	0.594	0.009	(0.966)	0.049	0.138	0.014	(0.524)	0.030	0.157
TRY	0.114	(0.000)	1.755E+129	25.671	0.018	(0.264)	2.338	1.040	0.032	(0.004)	0.147	2.482
# of p-values less than 1%		10				0				3		
# of p-values less than 5%		14				0				4		

Table E.2: Log-likelihood (Daily Data)

	AR(1)	AR(1)−GARCH(1,1)								MS−AR(1)	
		AGNormal	LR Statistic vs. AR(1)	AGT	LR Statistic vs. AR(1)	LR Statistic vs. AGNormal	AGNTS	LR Statistic vs. AR(1)	LR Statistic vs. AGNormal		LR Statistic vs. AR(1)
AUD	10307.49	10794.13	973.28	10835.62	1056.26	82.99	10850.27	1085.57	112.29	10717.77	820.58
CAD	11511.34	11954.32	885.97	11959.62	896.56	10.59	11960.80	898.92	12.96	11857.83	692.99
CHF	10957.80	11136.88	358.16	11178.86	442.11	83.95	11181.14	446.67	88.50	11112.02	308.44
CLP	11350.97	11677.88	653.80	11804.58	907.21	253.40	11801.84	901.74	247.94	11709.48	717.01
CZK	10548.72	10825.33	553.24	10859.95	622.46	69.22	10862.33	627.22	73.98	10779.31	461.19
DKK	11219.78	11393.42	347.26	11404.55	369.53	22.27	11406.86	374.14	26.88	11351.52	263.46
EUR	11225.09	11396.71	343.25	11408.02	365.87	22.62	11410.33	370.49	27.24	11354.95	259.73
GBP	11603.55	11831.96	456.80	11838.97	470.82	14.02	11841.27	475.43	18.62	11791.17	375.22
HUF	10000.60	10319.41	637.61	10384.51	767.81	130.20	10391.39	781.57	143.96	10325.57	649.93
ILS	12105.84	12589.84	968.00	12695.24	1178.80	210.80	12695.83	1179.98	211.99	12569.72	927.75
ISK	9746.02	10781.70	2071.37	10904.05	2316.06	244.70	10904.06	2316.09	244.72	10765.84	2039.65
JPY	11306.38	11454.28	295.81	11547.04	481.32	185.51	11548.37	483.98	188.17	11488.63	364.51
KRW	10926.95	12097.07	2340.24	12241.35	2628.80	288.56	12250.46	2647.01	306.77	11979.68	2105.46
NOK	10637.58	10868.65	462.15	10881.23	487.29	25.15	10885.82	496.48	34.33	10831.55	387.94
NZD	10327.98	10577.27	498.58	10599.80	543.65	45.08	10613.17	570.40	71.82	10558.47	461.00
PLN	10155.24	10644.79	979.10	10698.95	1087.42	108.32	10705.60	1100.73	121.63	10609.45	908.43
SEK	10567.08	10834.74	535.32	10847.58	561.01	25.68	10850.03	565.90	30.58	10791.58	449.01
TRY	9249.32	9755.41	1012.18	10635.82	2773.00	1760.82	10637.63	2776.62	1764.45	10393.57	2288.50

Table E.3: p-values of the LB Test for Standardized Residuals (Daily Data)

	One Lag				Five Lags				Twenty Lags			
	AR(1)	AGNormal	AGFT	MS−AR(1)	AR(1)	AGNormal	AGFT	MS−AR(1)	AR(1)	AGNormal	AGFT	MS−AR(1)
AUD	0.981	0.455	0.370	0.321	0.034	0.126	0.110	0.244	0.002	0.196	0.187	0.179
CAD	0.971	0.904	0.764	0.967	0.030	0.683	0.674	0.592	0.000	0.152	0.153	0.031
CHF	0.998	0.938	0.329	0.947	0.437	0.585	0.403	0.598	0.003	0.202	0.132	0.111
CLP	0.981	0.149	0.177	0.311	0.187	0.032	0.035	0.216	0.001	0.024	0.030	0.038
CZK	0.998	0.883	0.569	0.884	0.111	0.223	0.210	0.597	0.013	0.123	0.126	0.400
DKK	0.993	0.785	0.407	0.477	0.502	0.627	0.536	0.497	0.376	0.581	0.551	0.637
EUR	0.996	0.778	0.407	0.473	0.450	0.540	0.459	0.447	0.388	0.571	0.538	0.638
GBP	0.971	0.590	0.536	0.851	0.756	0.941	0.921	0.944	0.210	0.958	0.954	0.939
HUF	0.990	0.929	0.621	0.772	0.668	0.975	0.972	0.835	0.265	0.473	0.422	0.392
ILS	0.991	0.603	0.538	0.756	0.267	0.697	0.678	0.661	0.031	0.419	0.417	0.411
ISK	0.964	0.353	0.338	0.249	0.000	0.540	0.506	0.424	0.000	0.148	0.145	0.109
JPY	0.950	0.740	0.493	0.519	0.433	0.888	0.856	0.727	0.017	0.636	0.706	0.457
KRW	0.993	0.162	0.003	0.108	0.002	0.015	0.001	0.041	0.000	0.248	0.065	0.090
NOK	0.978	0.728	0.568	0.379	0.425	0.609	0.597	0.439	0.203	0.819	0.806	0.726
NZD	0.997	0.675	0.394	0.447	0.207	0.211	0.181	0.319	0.096	0.678	0.658	0.720
PLN	0.996	0.984	0.318	0.697	0.057	0.577	0.422	0.503	0.006	0.688	0.602	0.772
SEK	0.960	0.826	0.505	0.547	0.053	0.424	0.371	0.395	0.050	0.598	0.575	0.691
TRY	0.929	0.281	0.272	0.197	0.105	0.176	0.205	0.269	0.000	0.572	0.719	0.456
# of p-values less than 5%	0	0	1	0	4	2	2	1	11	1	1	2

Table E.4: p-values of the LB Test for Squared Standardized Residuals (Daily Data)

	One Lag				Five Lags				Twenty Lags			
	AR(1)	AGNormal	AGFT	MS−AR(1)	AR(1)	AGNormal	AGFT	MS−AR(1)	AR(1)	AGNormal	AGFT	MS−AR(1)
AUD	0.000	0.076	0.014	0.004	0.000	0.095	0.009	0.000	0.000	0.267	0.047	0.000
CAD	0.000	0.077	0.082	0.016	0.000	0.461	0.478	0.000	0.000	0.417	0.416	0.000
CHF	0.051	0.324	0.768	0.104	0.000	0.594	0.417	0.439	0.000	0.000	0.000	0.000
CLP	0.000	0.296	0.726	0.537	0.000	0.945	0.973	0.772	0.000	1.000	1.000	0.576
CZK	0.000	0.154	0.200	0.571	0.000	0.749	0.823	0.118	0.000	0.501	0.529	0.004
DKK	0.000	0.062	0.059	0.694	0.000	0.211	0.180	0.564	0.000	0.723	0.723	0.000
EUR	0.000	0.037	0.036	0.606	0.000	0.107	0.092	0.527	0.000	0.581	0.580	0.000
GBP	0.000	0.782	0.895	0.961	0.000	0.582	0.651	0.537	0.000	0.493	0.487	0.000
HUF	0.000	0.818	0.444	0.581	0.000	0.922	0.844	0.566	0.000	0.998	0.999	0.737
ILS	0.000	0.130	0.082	0.065	0.000	0.171	0.107	0.010	0.000	0.479	0.404	0.002
ISK	0.000	0.061	0.025	0.000	0.000	0.015	0.004	0.000	0.000	0.177	0.112	0.000
JPY	0.000	0.986	0.893	0.459	0.000	0.902	0.898	0.502	0.000	0.981	0.987	0.000
KRW	0.000	0.336	0.940	0.245	0.000	0.862	0.750	0.001	0.000	0.002	0.000	0.000
NOK	0.000	0.214	0.109	0.005	0.000	0.600	0.485	0.050	0.000	0.303	0.219	0.001
NZD	0.000	0.331	0.262	0.105	0.000	0.874	0.830	0.090	0.000	0.418	0.459	0.000
PLN	0.000	0.111	0.056	0.045	0.000	0.602	0.454	0.001	0.000	0.903	0.837	0.000
SEK	0.000	0.617	0.597	0.856	0.000	0.281	0.304	0.062	0.000	0.641	0.682	0.005
TRY	0.002	0.935	0.964	0.915	0.004	1.000	1.000	1.000	0.011	1.000	1.000	1.000
# of p-values less than 5%	17	1	3	5	18	1	2	6	18	2	3	15