Bringing out the Playful Side of Mathematics: Using Methods From Improvisational Theater in Professional Development for Urban Middle School Math Teachers

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With such a strong emphasis on testing and accountability in this No Child Left Behind era, and in a nation where more than 60% of the citizens are estimated to be affected by math anxiety (Ashcraft, Krause, & Hopko, 2007), mathematics is not something many people associate with play. In fact, in urban middle school math classrooms, mathematics is typically anything but playful. Such schools are likely to have constrained, procedural-skill-focused mathematics programs in urgent attempts to make quick gains on high-stakes tests (Meier & Wood, 2004; Mintrop, 2004). Many urban districts are underfunded, are low performing by current measures, and serve a high number of students who have special challenges of language, poverty, and cultural differences.

Teachers' interactions with students are typically of a traditional pattern called "teacher initiation, student response, and teacher evaluation" (IRE) (Cazden, 2001; Mehan, 1979). In an IRE interaction, the teacher is the sole authority figure, evaluating the correctness of students' answers and giving feedback when students make mistakes. In classrooms dominated by IRE, the stakes can feel high, correctness can take strong precedence over figuring out concepts, and students are rarely provided an opportunity to generate their own ideas creatively. These students tend to develop a view of their role as passive receivers rather than active constructors and show little confidence in discussing mathematical ideas (Boaler & Greeno, 2000; Chazan, 2000; Lubienski, 2000). In essence, the traditional discursive patterns of the mathematical classroom can foster rigidity in thinking, a lack of conceptual understanding, and low confidence in mathematical skills.

This is in stark contrast to mathematics in the world of the professional mathematician, whose job it is to construct mathematical arguments. Mathematical argumentation is a fundamental mathematical practice that entails making conjectures and justifying them, generating and evaluating the validity of new mathematical ideas. By its very nature, argumentation requires an element of play or playfulness. As Holton, Ahmed, Williams, and Hill (2001) described, professional mathematicians must spend considerable time experimenting and conjecturing—in other words, *playing around* with ideas—before they can establish a formal solution or proof. Mathematicians use play at various stages of argumentation, such as trying to understand a problem, making conjectures about possible pathways to solutions, and, once a solution is found, producing generalizations or extensions of the solution to other domains. For productive solution of complex mathematical problems, there must be times when errors are not only acceptable, but also necessary for the flow of problem-solving. Similarly, play is an intrinsic part of the problem-solving work in all the sciences (Jarrett & Burnley, 2007).

The Bridging Project, our program of design and research of teacher professional development (PD), which began in 2004 as part of the National Science Foundation's Teacher Professional Continuum program, seeks to bring the productive play of mathematical argumentation into urban middle school classrooms. In this program, middle school math teachers in challenging urban settings learn to support students in engaging in this fundamental mathematical practice of argumentation in the discussions in their classrooms. In order for teachers to support the cognitively and socially complex practice of mathematical argumentation in their classrooms, Bridging PD helps them develop a deeper understanding of mathematical content, as well as an element of playfulness and a teaching approach that entails facilitative pedagogy.

In this paper, we focus on one of the core components of the Bridging PD approach: using methods from improvisational theater to help teachers develop skills for facilitating classroom mathematical argumentation. We chose to use methods from improvisational theater because it is a domain in which participants are highly trained in some of the critical aspects of play and collaborative co-construction of discourse. We describe in greater detail what exactly classroom mathematical argumentation is; the knowledge, mind-set, and practices that teachers need in order to facilitate mathematical argumentation; and how we adapted and used methods from improvisational theater to support teachers' development of the key mind-set and practices. We then discuss our research and evaluation, presenting two illustrative case studies of teachers enacting classroom mathematical argumentation using skills acquired in the Bridging PD improv activities. We conclude with a discussion of future directions.

What Is Classroom Mathematical Argumentation?

Mathematics educators, mathematicians, and philosophers agree that mathematical argumentation is essential for learning mathematics (Kuhn, 2005; Lakatos, 1976; Lampert, 1990; Romberg, Carpenter, & Kwako, 2005; Thurston, 1998; Yackel, 2001). It can provide students with opportunities that enable them to construct conceptually rich understandings and develop a sense of ownership in the construction of knowledge. It can also be empowering to students, enabling them to develop intellectual autonomy and confidence as they become active co-constructors of mathematical arguments, formulating and determining the validity of their own justifications (Yackel & Cobb, 1996). Furthermore, making logical connections between abstract ideas and interacting with others to clarify ideas are both important 21st century skills necessary to an increasing number of STEM (science, technology, engineering, and math) jobs (Partnership for 21st Century Skills, 2008). Indeed, mathematical argumentation is highlighted in national math education policy documents such as the National Council of Teachers of Mathematics' *Principles and Standards* (2000) and the National Research Council's *Adding It Up* (Kilpatrick, Swafford, & Findell, 2001), which indicate that generating mathematical conjectures and justifying their validity are essential to reasoning and communication, which in turn support the development of essential mathematical proficiencies—conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition.

To create a clear structure for argumentation that teachers can use readily with their students, we follow the model of researchers who have simplified argumentation to two core elements: conjecture and justification (e.g., Erduran, Simon, & Osborne, 2004; Sowder & Harel, 1998; Zohar & Nemet; 2002). We define *conjecturing* as a process of conscious guessing (Lakatos, 1976) or pattern-finding to create mathematical statements of as-of-yet undetermined mathematical validity (Harel, in press) and *justifying* as a process of explicating one's reasoning to establish the mathematical validity of a conjecture. Finally, we also emphasize *concluding* as the process of coming to consensus or agreement about the validity of the conjecture and its justification.

Although mathematical argumentation can certainly be a solitary activity, we are concerned with what we call *classroom mathematical argumentation*, a process of argumentation that is collaborative in the whole-class discourse among the teacher and students. Argumentation done together within classroom discussion allows students to engage in a rich set of socio-mathematical activities: examining multiple conjectures together, trying out justifying or finding counterexamples for other students' conjectures, co-constructing conjectures and justifications, comparing and contrasting alternative conjectures and justifications, and working together to come to agreement about whether their mathematical statements are true or false.

In the research literature, classroom mathematical argumentation is conceptualized as a dynamic interaction of social and cognitive processes (Simon & Blume, 1996). Williams and Baxter (1996) characterized this type of teaching as “discourse-oriented teaching” because the construction of mathematical knowledge occurs through discourse among all participants in the classroom. Discourse-oriented teaching can be contrasted with a more traditional approach (e.g., IRE). It requires a shift in that the students take on significant mathematical authority. The students become responsible for generating and evaluating mathematical ideas in the classroom. The teacher becomes a representative of the mathematical community (e.g., Lampert, 1990) and a facilitator to scaffold student participation, and the students become more active co-constructors of mathematical knowledge (Ford & Forman, 2006).

As an example of classroom mathematical argumentation, we present an episode drawn from a whole-class discussion about similar rectangles in a middle school mathematics class taught by Ms. Esther, a teacher from the Bridging Project. The discussion illustrates how justifications can be co-constructed between the teacher and many students in the classroom. It was preceded by small group work in which students made sets of five similar rectangles (i.e., rectangles that all have the same ratio of length to width) and were asked to consider the method they used and make a conjecture about why it worked. Here, the teacher works with a group of students to help them justify their conjecture, “All perfect squares are similar.” They have already established the idea that similar rectangles have sides in the same ratio. (The teacher refers to these ratios as fractions.) Note that the teacher asks the students a series of questions that push them to examine, elaborate, and articulate their own thinking. This questioning also affords the participation of multiple students in this process.

SARA All perfect squares are similar.

MS. ESTHER Why?

SARA Because they are all the same. Their lengths and the widths are equal. So if you reduce any square, if you reduce the proportion or ratio, it becomes 1 by 1.

MS. ESTHER I don't know what you mean.

SARA Okay. So let's see. If one of our squares is 3 inches by 3 inches, then the fraction will be 3…uh, okay… 3 is to 3 as 2 is to 2 or whatever. And if you reduce either or both of those, they reduce down to 1.

MS. ESTHER Are you telling me by definition if I reduce or increase the length of one side, the other sides will automatically increase or decrease proportionally? Is that what you're trying to tell me? Because I understand that. Because it's my words. <Teacher laughs.>

SARA It's just like…okay. Let's see.

MS. ESTHER Let me ask. Basically I'm saying you have not convinced me all squares are similar.

EMILY Um, well, it's a perfect square because if you have one of our squares, the smallest one is 2 ½ inches by 2 ½ inches, and it's, like, even on both sides. So we just make all the squares the same, like 18 by 18, 2 ½ by 2 ½. So they are all similar, all the squares.

MS. ESTHER Because?

EMILY Because they are all the same length and …

MS. ESTHER <interrupting> Try using the word "as." Because "as" one side?

EMILY Because …

MS. ESTHER Doesn't help you. Sorry.

GREG	<Interrupts> Okay, like 2 ½ is to 2 ½ as 18 is to 18.
MS. ESTHER	Because?
GREG	Because they are equal. 2 ½ and 2 ½…
MS. ESTHER	<Interrupts> 2 ½ is not equal to 18.
GREG	No, but the proportions are the same. Both the numerator and denominator of the proportions I guess are equal. No matter what, they're going to reduce to the same thing.
MS. ESTHER	Reduce to what thing? In this case?
GREG	By 1.
MS. ESTHER	By 1?
GREG	One is to 1?
MS. ESTHER	Good. How do you get 1 is to 1 from 2 ½ is to 2 ½?
GREG	Divide by 2 ½, divide it by itself.
MS. ESTHER	So in mathematics it's called? Sss <making the "s" sound to give a clue of the first sound of the word> …
GREG	<Whispers to group members>
MS. ESTHER	So are you're telling me if you have 2 ratios, 20 ratios, or there are 100 ratios, and they all simplify to 1, they are equivalent?
GREG	Yes.
MS. ESTHER	Cool. Okay, next group.

Facilitating Classroom Mathematical Argumentation: Knowledge, Mind-set, and Practices

Learning to facilitate classroom mathematical argumentation is challenging for middle school math teachers, especially those accustomed to more a more traditional IRE approach in which the teacher is more of a “sage on the stage” than a “guide on the side.” It requires a constellation of knowledge, mind-set, and practices. First and foremost is mathematical content knowledge. Teachers must have a deep understanding of the mathematics that is the focus of the conversation. They must be able to evaluate student statements for their validity and mathematical precision, interpret unconventional forms or representations that students are likely to make as they construct their understanding, differentiate between colloquial and mathematical uses of language, translate between different representations (e.g., diagrams, algebraic formulas, verbal descriptions), and build connections to mathematics already learned and mathematics to come in the curriculum (Ball, Lubienski, & Mewborn, 2001; Ma, 1999; Shechtman, Roschelle, Knudsen, & Haertel, in press).

However, while improving teachers’ content knowledge is *necessary*, it is not *sufficient* for teachers to enact more effective classroom practices (Empson & Junk, 2004; Sowder, Philipp, Armstrong, & Schappelle, 1998). For example, a professional mathematician may have deep content knowledge but when faced with a classroom of inner-city middle school students may not necessarily fare well.

A second key aspect of facilitating argumentation is being able to create a classroom culture in which students feel comfortable and empowered to make conjectures. This requires, for many teachers, a change in mind-set. In the traditional mathematics classroom, there is a strong emphasis placed on having the “right” answers. However, conjectures by their nature are not necessarily correct, and argumentation is most productive as a process if students feel they have the freedom to make incorrect statements. Therefore, it is important for everyone in the classroom to be able to take risks and make mistakes.

In the Bridging Project, we help teachers develop a new mind-set by approaching classroom mathematical argumentation as a type of *productive mathematical play*. We define mathematical play in terms of the essential behavioral factors and necessary social contexts that play theorists associate with play in general: freedom, spontaneity, an element of uncertainty, generating new possibilities, voluntariness of creativity, and active engagement (e.g., Gordon, 2009; Wood, 2009; Yarnal, Kerstetter, Chick, & Hutchinson, 2009). In play, reality is bracketed within its own context in which there are no real consequences for taking risks or making mistakes (Gordon, 2009). Players know that they are playing and that there is a certain suspension of the constraints of the “real” world in which winning, losing, failing, and succeeding have high stakes and meaning. As Spolin (1963) wrote, “he or she is freed to go out into the environment, to explore, adventure, and face all dangers unafraid” (p. 11). Also in harmonious play, the players agree and cooperate. Players work on teams to accomplish goals together, listening to one another and building off each other’s ideas, and supporting one another in taking risks. Such a social context, in which risk-taking is permitted and all participants are involved and collaborating well, can support the behavioral expressions of play.

Mathematical play is a special case of play in that players move freely about a space that is also necessarily bounded by formal rules. Holton and colleagues (2001) defined mathematical play as “that part of a process used to solve mathematical problems which involves both experimentation and creativity to generate ideas, and using the formal rules of mathematics to follow any ideas to some sort of conclusion” (p. 403). Bringing playfulness to mathematics is by no means intended to undermine its seriousness. In productive mathematical play, students may have fun experimenting with new ideas, but ultimately they are engaged in important processes in support of deep learning. As Holton and colleagues (2001) described, “mathematical play provides a non-threatening environment where incorrect solutions are not read as mistakes and may lead to better understanding of the problem and/or the confrontation of misconceptions” (p. 404).

However, even the most playful of mathematicians still needs a set of practices to use with students in the moment of instruction. A third aspect of facilitating argumentation that we emphasize is the development of a repertoire of “teaching moves.” We use the notion of teaching moves to describe a discrete unit of instructional practice that teachers use to scaffold classroom mathematical argumentation (Jacobs & Ambrose, 2008; Mumme & Carroll, 2007). To support mathematical argumentation, teachers must develop a repertoire of teaching moves that they can use flexibly, responsively, and appropriately given the moment-by-moment needs of the students as they engage in mathematical argumentation.

Adapting Methods From Improvisational Theater to Support the Development of the Mind-set and Practices for Facilitating Argumentation

In the Bridging Project, in training teachers in distressed urban districts to support mathematical argumentation, we weave these key elements together—mathematical content knowledge, argumentation as productive play, and a repertoire of teaching moves. Early in the development of the Bridging Project PD, we drew on traditional PD methods for helping teachers develop requisite content knowledge. Previous studies had shown that intensive work with a handful of teachers, sometimes over multiple years, could support deep change in teaching practices (e.g., Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003; Lampert, 1990). However, little prior work indicated how to frame argumentation as play or support in necessarily short term PD the development of a repertoire of teaching moves for argumentation.

An approach we began to develop was the adaptation of methods from improvisational theater. In improvisational theater, a group of performers works collaboratively to create an unscripted performance that emerges spontaneously on stage in front of an audience. Groups may specialize in doing short-form scenes that take only a few minutes or long-form work in which entire plays are created on the spot. Improvisational actors collaborate skillfully to spontaneously co-construct elaborate characters, scenes, and storylines that unfold in the moment. Through a variety of "improv games," they are trained in the elements of character and narrative, as well as the discipline of spontaneity, active engagement, and productive cooperation. The humor and fun of improvisational theater elicits intense emotional experiences of joy for actors and audience alike. This art form has been popularized in recent years on the television show *Whose Line is It Anyway?* in which improvisers create a series of spontaneous skits in front of a live studio audience. Sawyer (1997), in his book on children's pretend play as improvisational performance, pointed out that the first improvisational theater group in 1955 formed around a series of children's games and later grew into the well-known Chicago-based group *The Second City*, from which eventually grew the television show *Saturday Night Live*.

While the connection to math teaching may not seem obvious at first glance, prior to the Bridging Project a number of researchers had begun using improvisational theater as a metaphor for understanding how discourse and learning unfold in the classroom. For example, Sawyer (2003) framed effective classroom discussion as improvisational because the flow of discourse unfolds in an unpredictable manner and emerges dynamically from the actions of the teacher and students working together. Drawing on observational and interview studies of improvisational actors, he used the term "collaborative emergence" to describe both classroom discussion and improvisational theater because they are both emergent (i.e., the outcomes cannot be predicted in advance) and collaborative (i.e., the outcomes are collectively determined by all participants). Similarly, Borko and Livingston (1989) used the metaphor to capture the fact that teaching requires improvisational responses as lessons unfold in the moment to adapt to what students know and can do. They demonstrated that expert teachers, compared with novice teachers, are better able to both stay on track with a lesson and respond improvisationally to students in a productive manner.

Lobman and Lundquist (2007) discussed and demonstrated how improvisational methods can be used to create productive learning environments. A hallmark of improvisational theater is that it offers a safe environment in which mistakes are acceptable and even expected, and all players must work together through listening to one another and working cooperatively. A core improvisational method is setting up an environment that is collegial, safe for making mistakes, and supportive of collaborative cooperation. Improvisational actors work with a set of guiding principles that underlie all improvisational work, for example: (a) use “Yes, and…” by acknowledging what others have said and then contributing something new; (b) make each other look good; (c) make big mistakes; (d) pay close attention, listen, and remember; (e) participate fully; (f) share responsibility; and (g) take care of each other (Johnstone, 1979; Lobman & Lundquist, 2007; Madson, 2005; Sawyer, 2003). In the classroom, improvisational theater methods could be used to create a learning environment in which students would feel safe to take risks and would have the facility to coconstruct new understandings together.

Likewise, several researchers had already begun to explore how improvisational methods could be adapted for teacher training. For example, Sawyer (2004) and Lobman (2007) used research on improvisational theater to provide practical suggestions and tools for creating social norms in both the classroom and teacher PD supportive of effective collaborative practice, as well as specific activities to establish a classroom environment in which teachers and students learn the skills necessary to interact collaboratively and take on powerful roles for sharing ideas and taking risks. We saw in these associations an opportunity to bring innovative methods into our work to help teachers learn to support classroom mathematical argumentation.

Across the various American improv traditions, there are hundreds of improv games, most of which tend to be intrinsically fun to participate in and watch. Many games are also designed primarily for improvisational training so that players can develop various aspects of their theatrical repertoire. There are games in which improvisers can develop their skills at inventing characters, developing narrative, or creating songs. Many games constrain players to a small set of rules that allow them to experiment with a particular improv skill. For example, in *Word-at-a-Time Story* players stand in a circle and construct a narrative by going around and contributing one word each until a story emerges. This allows actors to practice contributing to the development of a narrative without "stockpiling" (i.e., planning ideas ahead of time).

We saw two uses for improv games in mathematics teachers' professional development: First, together with norms, they can be used to create a nonthreatening environment to support the development of productive mathematical play in professional development and ultimately the classroom. Second, in the same way that improvisational actors develop a toolkit of fundamentals for building scenes together, analogous games could be used to help teachers develop a toolkit of fundamental building blocks to facilitate argumentation—teaching moves.

Parallel to improv games, we created a series of teaching improv games (TIGs) in which teachers could develop flexibility in using the teaching moves required to facilitate classroom mathematical argumentation. We focused on the types of teaching moves that the research literature indicates are important in facilitating argumentation. Such moves include encouraging students to take ownership or responsibility for a mathematical position, helping students to clarify their ideas through questioning, using paraphrasing or re-voicing to help ensure that everyone understands the ideas on the table, eliciting agreement or disagreement, eliciting counterexamples or alternative solutions, providing content, and finally concluding arguments by bringing the correct solution, fully explained and justified, into the discussion (Forman, Larreamendy-Joerns, Stein, & Brown, 1998; Lobato, Clarke, & Ellis, 2005; Silver & Stein, 1996; Stein, 2001; Wood, 1999). The games embedded these moves in the three-part structure of argumentation that we had set up for teachers—conjecturing, justifying, and concluding.

The Bridging PD Workshop

The Bridging PD has been implemented as a 2-week summer workshop designed to foster intensive teacher learning. The training in the first week focuses on deepening mathematical content knowledge, while the training in the second week builds on this knowledge and uses improvisational methods to develop the pedagogical aspects of facilitating mathematical argumentation. We implemented two iterations of this workshop in the summers of 2006 and 2007. Across years, the workshops were designed to be complementary such that teachers could participate in any one summer only or obtain increased learning from participating both summers. Several teachers participated in only one or both.

Week 1: Deepening Mathematical Content Knowledge

The focus of the first week was on deepening mathematical content knowledge. Proportional reasoning and coordinate geometry, central and cross-cutting themes in middle school mathematics (Hiebert & Behr, 1988), were core conceptual themes. Teachers, as adult learners, developed their understanding of mathematical argumentation, proportionality, coordinate geometry, and their roles in the curriculum. As Stein (2001) pointed out, a strong support to classroom argumentation is the selection of activities that prompt students to find different solutions and take different positions. We provided Bridging teachers with curriculum units with a series of activities on proportionality and coordinate geometry with open-ended questions that support conjecturing and justifying. Using adult versions of the classroom units, teachers learned to make mathematical arguments for themselves, while deepening their understanding of the mathematical concepts. They analyzed brief classroom scenarios focusing on students' mathematical thinking. Teachers learned to make their whole-group arguments in terms of conjecturing, justifying, and concluding.

An important principle in this first week was that the workshop facilitators modeled with the teachers how to facilitate argumentation in the classroom. They used the same pedagogical strategies to facilitate the teachers that would be the focus of learning in the second week. This allowed teachers to begin to experience firsthand what it would be like to be a participant in classroom mathematical argumentation. It also provided an important authentic learning opportunity as teachers began to formulate how they might facilitate argumentation.

To begin to structure argumentation as a discussion, the facilitators established a sequence of explicit steps that could be used as guidelines for beginners. We asked teachers to "do" argumentation a step at a time, generating and clarifying conjectures before moving on to justifying them. We thought it important for beginning arguers to get ample opportunity to make conjectures, and this step ensured that several teachers got to make conjectures before we moved into arguing for or against one. Additionally, concluding seemed important as a separate step because arguments can tend to trail off in conversation, leaving conjectures dangling, not yet established as true or false. We concluded arguments by loudly "stamping" each conjecture with a sticker saying “true” or “false.” This seemed to provide some logical, as well as emotional, closure to an argument.

Providing playful opportunities was also an important design consideration in creating the activities in this unit. For example, in *Find Your Similar Rectangle* each teacher is given a paper rectangle and instructed to find the person who had a similar rectangle. When actually doing this activity, teachers had informal definitions of similarity, but they had not been taught a mathematically precise definition. The teachers wandered the room to find the person carrying the rectangle similar to their own. They may have been looking for a match to the skinniness of their own rectangle in another rectangle that was much larger or smaller. There could be quick matches, long searches, and opportunities for humorous mismatches as pairs of teachers held their rectangles up at a distance and then got closer to verify similarity. We watched teachers’ interactions vary from tentative approaches to quick scans—both of which produced spontaneous laughter.

Week 2: Facilitating Classroom Mathematical Argumentation Using Improv Methods

The focus of the second week was on how to use this mathematical knowledge in the actions of teaching to facilitate mathematical argumentation in teachers’ classrooms. We moved between three complementary methods to help teachers observe, experience, analyze, and plan for using teaching moves to facilitate mathematical argumentation in their classroom discussions: (1) traditional improv games and TIGs, in which teachers worked with a more playful mind-set and experimented with new teaching moves; (2) script read-throughs, in which teachers analyzed teaching moves in fictionalized classroom transcripts; and (3) lesson planning, in which teachers composed detailed lesson plans with step-by-step anticipation of classroom action and where they might use their newly learned teaching moves.

We began by first introducing the fundamental principles of improv as ground rules for all the improv activities that teachers would do in the program. As improv was new to all of the teachers and was met with significant trepidation by many, we began with a series of warmup basic improv games that would help teachers gain experience and confidence in their own ability to participate in this new modality. One such game was *Giving a Gift*, which is a traditional improv game in which players present each other with pantomimed gifts and tell stories about them. Another such game was *Freeze*, in which players take turns pantomiming simple two-person scenes. Both of these games help players practice the "Yes, and…" rule, learn to build off of each other's ideas, and begin to gain confidence in their capacity to generate creative ideas spontaneously. Almost all of the teachers participated fully and expressed enjoyment of the activities.

In the workshop, once we had established the supportive, collaborative, and playful environment, teachers engaged in a progression of TIGs that built upon each other. In the first games, teachers further developed their own argumentation skills, emphasizing construction and critiquing of ideas. Next, they tried out constrained sets of classroom facilitation moves, emphasizing questioning and orchestrating participation. In the culminating game of the workshop, teachers integrated these skills to improvise facilitating simulated whole-class mathematical argumentation.

In an early game, *Why, Why, Why?*, triads of teachers experimented with generating explanations and using the one-word inquiry, Why? There were three roles: an explainer, a questioner, and an observer. Each role had associated rules. The explainer had to describe a given mathematical concept. The questioner could ask only why but could ask this question as frequently as desired. The observer was to take notes on what happened, so that the players could stay fully engaged in the game, but all three could participate in a debrief afterwards.

We had teachers play *Why, Why, Why?* with a focus on a concept central to proportionality but sometimes not deeply understood by teachers: a justification for why cross-multiplication works for evaluating the equivalence of fractions. The game often unfolded in an interesting manner. Teachers first had the opportunity to explain their initial conceptions but were then pressed to examine these further. Several rounds of *Why* provided the opportunity to find the gaps in their reasoning, deepen the explanation, and explore different facets of the concept. After a few rounds, the question why encourages mathematical play because it invites a creative explanation for a claim or conjecture. Eventually, many teachers experienced a boundary between “why” as a useful question and “why” as humorous and even absurd. This was often accompanied by playful laughter. Furthermore, the context of the game allowed them to engage in the inquiry without feeling they were exposing a lack of knowledge.

Basic Argumentation was the next TIG. This game allowed teachers to formulate justifications and began to introduce the dynamics of having a “constructor” and a “critiquer” of an argument. As Ford and Forman (2006) discussed in their explication of scientific discourse, in collaborative argumentation participants take on the roles of constructing arguments and critiquing ideas. These roles are dependent upon each other, and participants move fluidly between the roles. In *Basic Argumentation*, teachers worked in groups of three. They were assigned to be constructor, critiquer, or observer. The players chose one untested conjecture that would be the object of their argument. The constructor would begin the game by articulating a justification for the conjecture. The critiquer, whose role was not described in detail, would give some kind of feedback, such as discussing where the reasoning may or may not have made sense, providing a disconfirming counterexample or simply negating the constructor’s statements. Players were to keep their roles until the constructor could no longer argue for the conjecture, and then they would switch roles.

In the workshop, teachers played *Basic Argumentation* with conjectures they had made about proportional relationships among similar geometric shapes. In addition to the initial intentions of the game, an opportunity emerged to explore how the rule “Yes, and…” can be important in collaborative argumentation. In playing the role of critiquer, sometimes teachers presented constructive criticism, but sometimes they simply negated what constructors said. The players could then experience how constructive criticism supported sustained argumentation, even when initial justifications were incorrect, but negation tended to simply close an argument down. This was similar to the improv notion of “blocking,” in which one player fails to take up another player’s bid for the next sequence in a scene. Improvisational scenes cannot flourish when blocking occurs, and argumentation will be curtailed by simple negation.

Progressing further toward classroom practice, proceeding TIGs focused on teaching moves for facilitating *others* to co-construct mathematical arguments. In each game, 4 to 12 teachers would participate. In addition to constructors, critiquers, and an observer, each game now included a facilitator. The role of the facilitator was to draw out reasoning and orchestrate participation for the constructors and critiquers. One or more of the stages of argumentation were included in each game. Constraints on these stages provided different purposes for each game.

In the games *Closed-Ended Only* and *Open-Ended Only*, one teacher facilitated a pair of players who could move between constructing and critiquing roles in developing a justification. A new rule was that the argument had to be mediated by the facilitator. The constructor and critiquer could not respond to each other but only to the facilitator. The games were played successively, each focused on the same mathematical conjecture. The only difference was the rule that in *Closed-Ended Only* the facilitator could only ask closed-ended questions, and in *Open-Ended Only* the facilitator could only ask open-ended questions. Closed-ended questions are those that require simple responses, such as a yes or no, or one number. They are usually designed to elicit a specific piece of information and are often used by teachers to determine whether the student knows the answer. Open-ended questions call for a more elaborate response that the teacher may not be able to fully anticipate in advance. No one would actually teach using only one of those types, but the constraints in the games made it possible for teachers to compare the affordances and drawbacks of each—and have the experience of using them in initiating and sustaining classroom argumentation.

Teachers drew some important conclusions after playing these two games. One teacher noted that it was easy to generate correct statements of content for all to hear by asking closed-ended questions, but she was left unsure about what any individual student actually knew about that information. Another teacher said that using only closed-ended questions left the teacher doing all the work. Teachers had more difficulty with playing and analyzing *Open-Ended Only*. They found themselves asking closed-ended questions despite the rule not to. We wondered why. While they had critiques of using only closed-ended questions, it is possible that this game was closer to their actual practice than the game of asking open-ended questions. Despite the spirit of suspending reality through games, teachers' reality is still an important influence on their participation in games based on their vocation.

In the culminating game, *In the Classroom*, teachers integrated all of the teaching moves in a final improvised simulation of an entire lesson that they would actually teach back in their classrooms. Each teacher was given 30 minutes to practice teaching with the curriculum materials we had used in the first week of the workshop. They chose part of the materials and part of the argumentation process on which to focus. We introduced here another method from improvisational theater, character endowment, which is declaring that someone in the game is a particular person or kind of person. The lead teacher endowed her or his fellow teachers as students with specific qualities that were similar to those of the students in their real classrooms. For example, teachers endowed others as shy, know-it-all, and extremely tired. Endowing teachers with specific student qualities helped avoid a syndrome we had seen in year 1 of our workshops: Our teachers, when asked to play students, automatically played quite ill-behaved students and created a disastrous scene. If working out frustration is one function of play, we can guess at what was going on.

This game served as a transition between games and classroom. Thirty minutes was time enough for teachers to develop one stage of argumentation thoroughly (i.e., conjecturing or justifying) or to practice moving between one stage and another. They could also cover a fair amount of a specific curriculum activity. Most important, this TIG enabled teachers to practice in an unconstrained manner the teaching moves they had analyzed, observed, experienced, and used themselves in previous TIGs.

In each of these games, in combination with the other activities in the workshop—analyzing fictionalized classroom transcripts of idealized episodes of argumentation and making concrete lesson plans for their classrooms—teachers engaged in the playful practice of argumentation and developed teaching moves to facilitate their students' argumentation. We observed that in the process, teachers experienced a wide variety of emotions. The games sometimes evoked strong feelings, ranging from playful joy to sheer frustration. This may be an important part of teaching games: this chance to experience both the excitement and unpleasant feelings that accompany change, within a supportive environment of having adventures in argumentation together.

Research and Evaluation

In this early design phase of the Bridging Project, we implemented the PD within the structure of a small-scale 2-year randomized experiment impact study with complementary in-depth case studies. Data collection focused on teachers' mathematical content knowledge and the mathematical argumentation that occurred in their classrooms when teachers used the Bridging curriculum materials during each school year following summer workshops. We used these data to inform the iterative development of the PD, to investigate broadly the feasibility of the program for impacting teachers' practice, and to understand the different ways that teachers' practices could be affected.

While the more general results are documented elsewhere (e.g., Knudsen, Michalchik, & Kim, 2010; Shechtman, Knudsen, & Stevens, 2010), here we describe the overall research design and present two case studies that trace different ways that teachers enacted in their classrooms some of the playfulness and teaching moves that were the focus of the teaching improv games (TIGs).

Overview of Research Design

Teachers were recruited from high-poverty urban districts in the San Francisco Bay Area and randomly assigned to either a treatment or control group. Each year, teachers in the treatment group received the full 2-week Bridging intervention. Teachers in the control group received only the Bridging week 1 mathematical content training but not the week 2 training using improv; in the second week, these teachers participated in a different workshop of equal duration and professional value, examining how mathematical content is coordinated across grade levels. A total of 35 teachers attended the workshops. During each school year, all teachers in both groups taught the curricular materials used in the week 1 workshop that summer. All teachers were observed and videotaped teaching these units over a 2- to-4-day period. The whole-class discussion portions of these videotapes were transcribed verbatim.

The main hypothesis of the experiment was: *The classroom discourse of treatment group teachers (compared with that of control group teachers) will have more argumentative talk.* In other words, we hypothesized that additional professional development with a focus on teaching practices using improv techniques would help teachers foster more mathematical argumentation in their classroom than professional development focused mostly on content. To test this hypothesis, we developed a coding protocol to analyze systematically the transcribed classroom discourse, the Mathematical Arguments as Joint Activity in the Classroom (MAJAC) coding protocol (Shechtman, Knudsen, & Kim, 2008). This protocol was used on verbatim transcripts of whole-class discussion to locate and count the number of substantive student statements and teaching moves made in the episodes of mathematical argumentation.

Overall, there was support for the main hypothesis: in both years, we observed more argumentative talk in the treatment classrooms than in the control classrooms, in terms of both student statements and teaching moves. This difference was statistically significant in year 1 and marginally significant in year 2.

Using quantitative methods at such an early stage of a program's design means that our qualitative studies are important for understanding what happened in teachers' classrooms as they returned to them after our workshops and how ideas and techniques from the workshops were taken up in the classroom.

Development of the Case Studies

On the basis of initial impressions from these broad analyses, we selected a subset of teachers whose classroom discourse had important contrasting qualities that would highlight different aspects of the PD and the use of improv methods. Here we present the illustrative cases of Ms. Stephanie and Ms. Peg who exhibited how argumentation can be enacted in ways that were quite different yet both aligned with the intentions of the TIGs. They both taught in the same urban middle school, which had a high level of student poverty and the lowest possible ranking for the state academic performance index. Both told us that argumentation was new to their practice.

In the highlighted excerpts, the teachers' classes were doing the same week 1 Bridging activity from the coordinate geometry unit. Students draw four rectangles (all aligned with the axes) on a coordinate grid and label the coordinates of the vertices (corners). They then look for patterns in the numbers that make up each pair of coordinates. Patterns can then be formulated as conjectures, which serve as the basis for argumentation.

Ms. Stephanie: The Stern Novice Teacher Whose Students Argued Enthusiastically

Ms. Stephanie did not have the odds in her favor for successful enactment of mathematical argumentation. When she attended our workshop, she had been teaching for only one year. As a new teacher in a tough school, she was in the high-risk category for teachers who leave the profession within five years. A typical challenge, and one that we observed for other novice Bridging teachers, is basic classroom management—making sure, for example, that students take turns talking and stay in their seats as necessary. Without this basic classroom order, mathematical argumentation might never be an option. Ms. Stephanie, however, had clearly mastered an effective set of management skills. Her style was quite stern and seemingly unplayful—so much so that one researcher, on seeing a videotape of her establishing order in the beginning of class, predicted that little student argumentation could possibly happen.

Yet a considerable amount of rich argumentation happened in Ms. Stephanie's class—in fact, she told us that she reserved each Friday for argumentation and that her students looked forward to it each week. This was well above and beyond what we had asked the teachers to do. When we observed in her classroom, we saw that the students actually quite enthusiastically participated in elaborate argumentation. She brought many of the workshop moves back to her own classroom, much as the workshop facilitators had modeled and she had rehearsed in the TIGs. She had already established norms that were supportive of argumentation.

It was clear that Ms. Stephanie took the structure of argumentation as we presented it quite seriously and used it with the students as an organizing frame of reference. Prominently displayed in her classroom was her own version of a poster that looked quite similar to the one that we had used in the workshop. As shown in Figure 1, our poster outlined the phases of argumentation according to the project's theoretical framework. Ms. Stephanie used this physical resource in her classroom to guide and structure the discussion as it unfolded.

<INSERT FIGURE 1 NEAR HERE>

Here we discuss an episode that includes all the phases of argumentation and in which Ms. Stephanie used a variety of teaching moves to draw out student reasoning and orchestrate participation. The excerpts also clearly illustrate that Ms. Stephanie had previously created a classroom environment in which the students could be playful in their argumentation and were eager to participate, contribute, and voice opinions. This is supported through a combination of rewards for participating (i.e., points for math argumentation) and teaching moves to orchestrate participation so that students could listen to one another and contribute. Students were playful in coming up with new ideas, and they built off of each other's thinking. They even spontaneously went up and wrote on the board and called on each other to speak. While there was enthusiastic participation, Ms. Stephanie always remained in control of the classroom.

In the previous class period, students had worked in small groups to make observations about patterns they saw in four rectangles they had drawn on a coordinate plane. They then used these observations to make a conjecture about coordinates. The teacher then had each group report out on their conjecture and had the class vote on which conjecture they would like to justify together as a class. The conjecture they chose is illustrated in Figure 2.

<INSERT FIGURE 2 NEAR HERE>

The episode begins with the conjecturing phase of argumentation. Ms. Stephanie has Jason write his group's conjecture on the board. She reminds the class that the purpose of this is for everyone to have a clear understanding of the conjecture before people decide if they agree or disagree.

MS. STEPH Jason, before you write, can you say that out loud again?

JASON If you have a coordinate plane and you put a point to the left and below the origin, then it has to be negative, negative.

MS. STEPH Okay.

<Jason writes this on the board, "If you have a point on a coordinate grid to the left and below, it is a negative, negative.">

DENIS But that's pretty much what the conjecture is.

MS. STEPH Uh, Denis we are clarifying. We are making the sentence clear for everybody. <to the whole class> I want you to think about what Jason is writing on the board and decide if you understand what it says now, so we can move on to justification.

MS. STEPH <after Jason has finished writing> Okay. Read it to us, Jason.

JASON If you have a point on a coordinate grid to the left and below it is a negative, negative.

MS. STEPH Okay. Do we understand what the conjecture is?

STUDENTS Yeah.

MS. STEPH Put your hand up if you don’t understand what the conjecture is. <no hand goes up> Okay let’s move on then to justification. Now you are going to argue that this is true or this is not true.

<Many students raise their hands.>

MS. STEPH You are going to argue this is true or not true. Remember to get your points for math argumentation, you need to be participating.

They then move into the justifying phase. Given the nature of the conjecture, much of the argument is focused around graphing conventions, which are taught almost every school year throughout middle and high school.

VENESSA I think it’s true because they said that this is…I think it’s true because of the positive up there on the right side, the negative number is left side.

MS. STEPH Okay, so you think it’s true? <Many students raise their hands.> Harrison.

HARRISON Um, I think it’s also true because you have your coordinate plane, and this is negative, negative corner right? To make it into a positive, then you’d have to bring it up here. <indicating the upper part of the plane>

MS. STEPH Okay. Ozzy.

OZZY Um, I think it’s true because what Jason said in his statement was like the top top right part is positive, positive and the left lower part is negative, negative.

And the other two is negative, positive, positive, negative. And it is true. To me.

<Many students raise their hands.>

MS. STEPH Okay. Martin.

MARTIN Yes I agree, too. <A pause. Some students laugh.> Um, all right. That's all I got.

MS. STEPH Irene?

IRENE I think it's true. Can I write up there while someone else talks? Because it's going to take kind a while if I write it.

MS. STEPH Write what?

IRENE Why I think it's true.

Note that in an analysis of argumentation, Sowder and Harel (1998) described appeals to authority (i.e., "this is true because it is the convention") as a particularly weak form of justification. But in many classrooms, the text is the authority—and teachers and students, as well as parents (when helping with homework) accept its authority without question. The majority of what mathematics textbooks demonstrate are the details of mathematical conventions, the way things are commonly done, said, and pictured when doing school mathematics. But in this episode, several students come up with ways or reasons the conventions could be wrong. One student, Kennyn, for example, introduces the idea that there could be a "whacky coordinate plane" on which this conjecture would not be true. Later, when Ms. Stephanie prompts for disagreement with the conjecture, Denis and Andy come up with their own playful ways of falsifying the conjecture.

DENIS If you rotate like this… <Draws arrows on the board showing how if the axes are rotated 90 degrees, the point could still be in the lower left, but the coordinates would be different.>

JASON It’s still the same grid.

<Students discuss this amongst themselves.>

ANDY Denis, call on me.

MS. STEPH Okay. Denis. Thank you. Okay, Andy.

ANDY The thing is that if you rotate it like this, then it won’t be true. And also in other countries, they may have different grids.

SAM What country, Andy?

ANDY [The conjecture] says if it’s left and below the origin, it didn’t say what country. It didn’t say what country we are in.

SAM I don’t think other countries have different grids.

<Students discuss this among themselves.>

ANDY Why not? They have the metric system.

MS. STEPH Thank you. Thank you. Ozzy.

OZZY I agree with Andy, because some other countries do have different ways of doing math or grids, whatever you call it, so yeah.

Eventually, Ms. Stephanie moves to the concluding phase of the argument by pushing the class for a final consensus on the conjecture.

MS. STEPH Okay. We are trying to conclude, and I am asking you basically if you agree with this <points to the conjecture, which has been revised based on the discussion>. Unless you have anything you need to add before we can agree. If you have something you need to add before we can agree, that's what I'm looking for here. Otherwise you can say, do we agree with this. If you have your hand up, I'm going to assume you have a thing to add.

The students discuss the conjecture for another few minutes. Some students take issue with the revision that had been made to the conjecture, saying it was not necessary. Kennyn gets so engrossed in and upset by the discussion that he uses profanity toward Denis. Ms. Stephanie ends the episode with a vote.

MS. STEPH All right, guys. We are going to stop. If you agree with the conclusion, the statement, it is true. This. Our original conjecture was, "If you have a point on a coordinate grid to the left and below, it is a negative comma negative." Raise your hand if you agree with our conclusion that it is true. <Many students raise their hands.> Raise your hand, please, if you agree that it is true. <Counts hands.>

<Many students talk simultaneously and argue whether it is true or not.>

MS. STEPH Okay. Hands down. We have 18 that I counted. We had 18 out of 27 who agreed with it. It is the majority but hardly. It is more than half of you who agreed with it. At this point and time, it's going to be the best we can do. Half of you agreed with it. That's the best thing we can do.

In sum, Ms. Stephanie's case clearly shows how a teacher appropriated both the playfulness and teaching moves that were modeled and developed in the workshop. In this difficult urban setting, she impressively engendered playfulness, eager participation, and critical mathematical thinking. During the workshop, her turn at *In the Classroom* foreshadowed many of the moves we observed in her classroom. Many of her moves for orchestrating participation were among those modeled by the facilitators, but, interestingly, not among those explored in the TIGs. In particular, she herself asked few probing open-ended or closed-ended questions but rather left all mathematical content up to the students. Her class was one of the few we observed in which students were actively arguing with each other. There were also many moves that she invented on her own, such as giving students points. We hypothesize that the moves she learned in the workshop were sufficient for this new teacher to support argumentation because, unlike some other new teachers, she had sufficient classroom management skills to make it possible for argumentation to happen, but her practice was still malleable enough to take on new practices.

Ms. Peg: The Veteran Teacher Who Used Teaching Moves from the TIGs

Ms. Peg was a veteran teacher with over a decade of experience who participated in both years of our program. However, given her fairly traditional background, mathematical argumentation was a new way of doing mathematics for her. She was highly engaged in the workshop, working diligently to learn the basics and asking many questions about the meaning of terms and how to carry out argumentation in her classroom. Her active participation in the workshop was particularly evident during the TIGs and the discussions held just after the games.

When we examined Ms. Peg's classroom discourse, in comparison to Ms. Stephanie she seemed more specifically influenced by her participation in TIGs. In the excerpt from classroom dialogue below, Ms. Peg used a combination of open- and closed-ended questions to help her class make conjectures and justifications. This, of course, echoes our two games *Open-Ended Only* and *Closed-Ended Only*. We had expected the game to enable teachers to contrast the use of these questions in order to find a blend that was suitable for a given situation, and this is in fact what we saw in Ms. Peg's classroom. We know that in our first year observation of her class, her students made only a few arguments and Ms. Peg's support of those arguments seemed tenuous. The excerpt below is from observations made in the second year of the project, after Ms. Peg had participated in new TIGs focused on questioning techniques.

As this episode begins, Milla has examined her four rectangles on the coordinate plane and makes an observation about the vertical lines—specifically about the endpoints of these lines, which are vertices of the rectangles. Her conjecture, stated more precisely than it was in the classroom, was that the x coordinates of the two endpoints of any one vertical line are equal. Ms. Peg's restatement of her conjecture is that the two vertical lines have the same x-coordinate, a different and, as it turns out, false conjecture. Another student makes a move to provide a counterexample.

MS. PEG Okay, go ahead and state your conjecture. What do you think?

MILLA Okay, I say my patterns are -12 and the 10. My other is -12 and the 4. The pattern is -12.

MS. PEG So which … which coordinate is it? The x or y?

MILLA X.

MS. PEG So you're saying that you're noticing that your two x [coordinates] - in the vertical or horizontal line?

MILLA Vertical.

MS. PEG Has to be the vertical, are the same. Can you write that down? <looking at another student> Two vertical lines share the same x. Raise your hand if you made that observation. <Some students raise their hands> Do you agree with what your data is? That your x's are the same. Is that true in every single rectangle?

STACY No.

LUANNE Yeah.

MS. PEG It's not true. Stacy has got a counter. And Stacy we may have you show that on a grid paper, a counter, and I'm going have you do it right here. <hands Stacy a transparency> Somebody else, an observation they made. Mike?

Let us look closely at what questions Ms. Peg uses to elicit and support argumentation. First, she instructs students to state a conjecture and uses the open-ended question, "What do you think?" Milla responds, not with a conjecture (i.e., a statement that could be true for false) but with seemingly disconnected numbers. Ms. Peg asks the student a couple of close-ended questions about which coordinates and which lines on the rectangle she was referring to in her conjecture, thus connecting the numbers to mathematical terms that could form a conjecture. Ms. Peg states her own version of the conjecture: "Two vertical lines share the same x." Then, she asks the entire class to consider the truth of the conjecture, prompting them to find counterexamples with, "Is that true of every single rectangle?" Technically this is a closed-ended question, because the answer is yes or no. In context, however, it occurs as an open-ended question, as if "why" were appended. This suggests that the class had been developing norms of argumentation for some time. Ms. Peg's version of the conjecture is not true, perhaps fortunately, because this provides a student with the opportunity to offer a "counter" (i.e., a counterexample). A counterexample is a particularly efficient form of justification, establishing that a conjecture is false by providing one example where it is not true.

In sum, Ms. Peg combines open- and closed-ended questions to facilitate a classroom argument that started with formulating a conjecture and progressed to a justification for or against the conjecture. Her case indicates promise for the intended purposes of the TIG—in the workshop she had played with the teaching moves introduced in the game, and in her classroom we observed her using these teaching moves flexibly to foster mathematical argumentation.

Discussion

The mission of the Bridging Project is to provide middle school math teachers in challenging urban settings training to support students in engaging in the fundamental mathematical practice of argumentation in the discussions in their classrooms. As classroom discussions in this type of setting are traditionally dominated by IRE types of interactions, the program seeks to leverage scarce professional development resources to help teachers make small shifts in practice that provide significant new opportunities for students to actively participate in constructing their own understandings. The Bridging PD training emphasizes the development of both a deeper understanding of mathematical content and pedagogy for facilitating discussion. Mixed results from our small impact study suggest that the Bridging PD has promise for helping teachers do more argumentation in the classroom.

In this paper, we examined one of the central components of the PD: using methods from improvisational theater to foster mathematical argumentation as productive mathematical play and as an approach to developing teaching moves for the classroom. As part of our 2-week summer program, we developed and implemented a range of games, both drawing from the theater tradition and creating new specialized teaching games. Through close observation of the discourse in teachers' classrooms in the subsequent school year, we found important traces of the approach and tools that teachers learned in the workshops. One case study example showed a teacher who engaged students' enthusiastic participation in argumentation following the structure we had provided, and another case study illustrated at a more fine-grained level how the teaching moves in the teaching games may be enacted in discussion.

This project continues to build the literature illustrating the utility of using improvisational theater methods for teacher professional training. Just as in prior work (e.g., Lobman, 2007; Sawyer, 2004), we found that improvisational games can be used to establish social norms of safety and collaboration in which teachers can be relatively uninhibited in exploring new knowledge and practice. This can be particularly important in developing new practices around facilitating highly dynamic discussions that require teachers to be on their toes, in terms of both mathematical and pedagogical skill. These methods can provide a safe environment in which teachers can experiment and admit where knowledge is lacking.

Another implication is that methods from improvisational theater, particularly the use of strategically designed improvisational games, can be used for the development of specific teaching moves to be used in mathematics classroom discussion. Both teaching and improvisational theater share a collaborative emergence in which there is a complex set of moves and dependencies that can shape the way a discussion or scene unfolds. In the math classroom, such unfolding can have a significant impact on the learning opportunities that students are provided and the ways in which students engage with mathematical ideas. In the Bridging PD, our teaching improv games focused on the moves teachers can make for shaping argumentative discussion in the classroom. There are a multitude of other types of teaching moves that unfold in the mathematics classroom. For example, Hill et al. (2008) discuss the notion of Mathematical Quality of Instruction, which includes, among other aspects of pedagogy, a number of in-the-moment moves that math teachers make that are likely to influence student learning. Such moves include how teachers respond to student errors and misunderstandings, the degree to which teachers acknowledge student ideas, and the use of precise mathematical language in conversation. Furthermore, Pierson (2008) showed that the manner in which teachers responded to student ideas in conversation was directly related to student learning of complex mathematics. Teaching improv games can potentially provide simple and direct ways for teachers to practice applying the mathematical knowledge they have to the process of engaging in important discursive moves that shape student learning.

A next step in the Bridging Project program is to develop improvisational games that teachers can use with their students to enhance mathematical play in the classroom. We will develop a series of student improv games (SIGs) to support students in feeling safe to take risks and collaborating productively with one another. We will focus in particular on developing games that will engage students who are traditionally disenfranchised in mathematics classrooms, such as girls and ethnic minority students. For example, in *Classroom Conject-a-thon* students will brainstorm conjectures about a mathematical object and be encouraged to make some deliberately bad ones so that making a false conjecture loses its stigma. Drawing on our experiences with Ms. Stephanie, we will also create the game *Argument-of-the-Week*. In this game, students will spend an allotted time each week generating a conjecture and justification for some aspect of the content in their current unit. We will also build on the games outlined by Lobman and Lundquist (2007) that support students in taking on new roles in the classroom, such as Compulsive Math Gal/Guy and Captain Precision.

Mathematics is traditionally not associated with play, especially in challenging urban schools in which the focus tends to be on testing and accountability. Yet just as in the work of professional mathematicians, play can have an important place in the math classroom. When teachers are trained to bring an element of serious playfulness to their classrooms, students can be provided the opportunity to work together without fear of failure to experiment with developing their own conceptual understandings.

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Figure Captions

Figure 1. Ms. Stephanie’s Stages of Mathematical Argumentation Poster

Figure 2. The Conjecture Discussed in Ms. Stephanie’s Classroom

Figure 1. Ms. Stephanie's Stages of Mathematical Argumentation Poster

Mathematical argumentation is a *conversation* in 3 parts.

- Conjecturing
    - Boldly stating what MIGHT be true
    - Clarifying (vocabulary and logic)
- Justifying
    - Figuring out together if it is true
    - Logically connecting the dots
- Concluding
    - Deciding if the conjecture is true or false
    - Building new conjectures

Figure 2. The Conjecture Discussed in Ms. Stephanie's Classroom

Conjecture: On a coordinate plane, any point to the left of and below the origin will have two negative coordinates.